Putting Theory into Practice Through the Lens of the Saskatchewan Grade 9 Curriculum

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Introduction

This project is a living document. It will be continually changed and amended and may look entirely different in 10 years. My goal with this project was to create something useful and helpful for teachers. Ultimately, I am hoping that it gives teachers the opportunity to apply some of the pedagogical changes the province and school divisions are asking of teachers without them having to give up time from their families. By taking current research in the teaching and learning of mathematics, I have done my best to put theory into practice.

I have only included two of the four possible strands in the grade mathematics curricula. Those being numbers and patterns and relations. Within those strands are seven outcomes: N9.1, N9.2, N9.3, P9.1, P9.2, P9.3, P9.4. The curriculum document was never meant to be followed in a linear fashion and as a result I changed the order of the patterns and relation outcomes in the student workbook to be P9.4, P9.2, P9.3, P9.1.

This document includes the teacher resource, research notes, and student workbook. The format of each is:

1. Teacher Resource:
    a. What are students' previous experience with this topic
    b. A message from me
    c. Outcome of each lesson
    d. Research applied to each lesson
    e. Lesson process
    f. Formative assessment ideas (whiteboard questions/exit questions)
2. Research Notes
    There are eleven research notes in this project. They are listed numerically throughout the document (i.e. RN #3 Learner Generated Examples). In this section I talk about them more in-depth and provide the references for each.
3. Student Workbook:
    The student workbook includes the notes, tasks, examples, and assignments for each lesson. This is the working copy to give to students. I have purposely not included an answer key. I have found that by doing this, students check their answers with other students and it creates a discourse of mathematics in the classroom. Some of the tasks needed for the lessons are not included in this section since they are pull out pages needed for projects. Please refer back to the teacher resource for the necessary pages.

Please use the entire document, parts of the document, or even just a few activities. Feel free to contact me if you have any feedback regarding errors or additional activities that you think would be helpful for teachers.

Putting Theory into Practice Through the Lens of the Grade 9 Curriculum

1. **Outcome N9.1**

   Demonstrate (concretely, pictorially, and symbolically) understanding of powers with integral bases (excluding 0) and whole number exponents including:

   Representing powers
   Evaluating powers
   Powers with an exponent of zero
   Solving situational questions

   Lesson 1: Introduction to Exponents
   Lesson 2: Integers and Exponents
   Lesson 3: Powers of 10 and the Zero Exponent Law
   Lesson 4: Order of Operations
   Lesson 5: Exponent Laws

2. **Outcome N9.2**

   Demonstrate understanding of rational numbers including:

   Comparing and ordering
   Relating to other types of numbers
   Solving situational questions

   Lesson 1: What is a Rational Number?
   Lesson 2: What is a Rational Number (continued)?
   Lesson 3: Adding Rational Numbers
   Lesson 4: Multiplying Rational Numbers
   Lesson 5: Dividing Rational Numbers
   Lesson 6: Order of Operations
   Lesson 7: Working with Decimals (optional)

3. **Outcome N9.3**

   Extend understanding of square roots to include the square root of positive rational numbers.

   Lesson 1: What is a Square Root?
   Lesson 2: Square Roots of Fractions and Decimals
   Lesson 3: Square Roots of Non-Perfect Squares
   Lesson 4: Applications of Square Roots (Pythagorean Theorem)

4. Outcome P9.1

Demonstrate understanding of linear relations including:

Graphing
Analyzing
Interpolating and extrapolating
Solving situational questions

Lesson 1: Looking for Patterns
Lesson 2: Remembering How to Graph on a Cartesian Plane/Coordinate Plane
Lesson 3: Graphing Patterns
Lesson 4: The Number Transformer
Lesson 5: The Importance of Finding a Mathematical Equation & Key Terms
Lesson 6: Graphing and Solving Linear Relations
Lesson 7: Graphing $y = a$, $x = a$, and $ax + by = c$
Lesson 8: Matching Equations and Graphs
Lesson 9: Interpreting Graphs

5. Outcome P9.2

Model and solve situational questions using linear equations of the form:

$ax = b$
$\frac{x}{a} = b, \;\; a \neq 0$
$ax + b = c$
$\frac{x}{a} + b = c, \;\; a \neq 0$ Where a, b, c, d, e, and f are rational numbers
$ax = b + cx$
$a(x + b) = c$
$ax + b = cx + d$
$a(bx + c) = d(ex + f)$ $\frac{a}{x} = b, \; x \neq 0$

Lesson 1: What is an Equation?
Lesson 2: Solving Simple Equations
Lesson 3: Solving Equations with Variables on One Side
Lesson 4: Solving Equations with Variables on Both Sides
Lesson 5: Solving Equations Containing Parentheses
Lesson 6: Solving Equations Containing Fractions
Lesson 7: Solving Equations Containing Decimals
Lesson 8: Solving Situational Questions Using Equations

6. **Outcome P9.3**

Demonstrate understanding of single variable linear inequalities with rational coefficients including:

Solving inequalities
Verifying
Comparing
Graphing

Lesson 1: Review of Solving Equations
Lesson 2: What is an Inequality?
Lesson 3: Solving Inequalities by Using Addition and Subtraction
Lesson 4: Solving Inequalities by Using Multiplication and Division
Lesson 5: Solving Stories

7. **Outcome P9.4**

Demonstrate understanding of polynomials (limited to polynomials of degree less than or equal to 2) including:

Modeling
Generalizing strategies for addition, subtraction, multiplication, and division
Analyzing
Relating to context
Comparing for equivalency

Lesson 1: Terminology
Lesson 2: Terminology Continued & Like Terms
Lesson 3: Modelling Polynomials with Algebra Tiles & Collecting Like Terms
Lesson 4: Adding Polynomials
Lesson 5: Subtracting Polynomials
Lesson 6: Multiplying Polynomials
Lesson 7: Dividing Polynomials

8. **Research Notes**

Research Note #1: Math-In-Context
Research Note #2: Prior Knowledge
Research Note #3: Learner Generated Examples
Research Note #4: Inquiry
Research Note #5: Formative Assessment
Research Note #6: Representation
Research Note #7: Cooperative Learning

Feedback from Colleagues:

"This resource embodies an ambition of many classroom teachers to map an entire course curricula to rich learning opportunities. With research in mind, it combines a familiar lesson structure with opportunity for student-led creative control. The embedded balance of instructional practices situations the resource in the space between pedagogical poles while at the same time tethering all-too-distant theory with all-too-familiar practice. Such an attention to balance allows the classroom teacher to delicately pry notions of deep understanding from their instructional practice."

~Nathan Banting~
Mathematics Educator
Author of "MusingMathematically.blogspot.ca"
@NatBanting

I have used the Equations and Inequalities parts of the workbook so far, and am looking forward to the Linear Patterns and Relations section. I like the way that it scaffolds skills for the students, and explains for the teacher what skills the students do or do not have coming in from Grade 8, as well as explaining the theory behind the examples and explanations. Because the teacher guide, student examples, and student homework are all coming from the same place, there is a consistency that you do not get from other materials. As a first year teacher, this resource has been really helpful to me, especially since I want to include more formative assessment and cooperative learning in my classroom.

~Jenny Leake~
French Immersion Educator

OUTCOME: N9.1

Demonstrate understanding of powers with integral bases (excluding base 0) and whole number exponents including:

- representing using powers
- evaluating powers
- powers with an exponent of 0
- solving situational questions

Lesson 1: Introduction to Exponents
Lesson 2: Integers and Exponents
Lesson 3: Powers of 10 and the Zero Exponent Law
Lesson 4: Order of Operations
Lesson 5: Exponent Laws

Previous Experience:

Grade 8

Outcome N8.1

Demonstrate understanding of the square and principle square root of whole numbers concretely or pictorially and symbolically.

To The Teacher:

There are 5 lessons but it may take up to 10 class periods. Assignments, review of material and assessment will take additional class time. There are some duplicated questions from the Grade 9 Pearson and McGraw Hill Ryerson resources. I have noted them in the teacher package. Most of the assignments in the student workbook are original questions which leave the other resources for extra questions if necessary.

It is important to note that the previous grade curriculums only include squares of whole numbers. So for the majority of students, this is their first encounter with exponents other than the value of 2. Students have had experience with order of operations but exponents have not been included.

A very important component of this unit is the collaboration between teacher and student and student and student. Although there are skills needed to be learned, they should be done in such a way that students are working with the teacher and each other to construct some of the procedural aspects of exponents.

When students work in groups, 4 students in a group should be the maximum. You can either equip each student with a white board or one large sheet of paper with markers.

Lesson 1: Introduction to Powers **N9.1**

To The Teacher:

This lesson is a great way to introduce students to the fast action of powers. It starts by reading the story "One Grain of Rice" by Demi. It is not necessary to read the story and do the task, but it may help students understand how quickly things grow when they are continually doubled. It also leads to a discussion about the pattern that the numbers are following. Students may or may not be able to model the pattern with an expression. Do not give students the expression, come back to this after more discussion of powers.

I have included two other tasks to introduce powers if you prefer to use those instead of "One Grain of Rice". The tasks are "repeated addition vs. repeated multiplication" and the "paper folding activity" located at the end of the unit.

The focus of this lesson is for students to understand that exponents are repeated multiplication. There is no formal assignment for this lesson. However, you could assign the task of "Repeated Addition vs. Repeated Multiplication" and review it as a class the next day. Students will need to remember how to graph (which is an outcome in grade 8).

RN #4

Inquiry.

RN #1

Math-in-context

RN #2

Prior knowledge

Lesson Process:

1. Begin with "One Grain of Rice" activity. Read students the story but don't give away the ending. Once you get to page _____, have students predict the outcome. You can handout the worksheet if you want to give the kids more direction, or you can have them work through the problem without the guided sheet. **RN #4, RN #1**

2. **Student Workbook:** Ask students where they see exponents used. Examples of answers are; light years,, number of bacteria in a sneeze, size of cells. Exponents are used to represent something really large or small.

3. A key component of the lesson is for students to understand exponents as repeated multiplication. Have students look at the difference between 8^3 and 8×3.

4. Define the terminology needed for powers.
5. Example 1: Use the terminology and define the base, exponent, and power.
6. Example 2: Discuss with students what happens when the base and the exponent are swapped? Which do they predict will be larger, or will it be the same answer?
7. You may want to discuss with students how to use their calculators for exponents (if you allow them in your course).
8. Look at the common exponents of 2 and 3. Ask students if they know how to say 5^2 in math language (five squared). Ask them how to say 5^3 (five cubed). Ask students why do we say squared and cubed for exponents of 2 and 3 respectfully. Complete the remainder of the students' notes and hopefully they will see area as a square to the power of 2 and volume of a cube to the power of 3. You may need to discuss the fact that squares and cubes have equal side lengths. **RN #2**
9. Example 3: Have students write as a power
10. Example 4: Have students write as repeated multiplication
11. Example 5 & 6: Using students input, have them explain how to calculate the answers for these examples.
12. Assignment: You can assign #1-5 from the Lesson 1 & 2 assignment. You can also leave this until tomorrow since todays lesson is pretty lengthy and tomorrows is a bit shorter.

N9.1 Powers

Lesson 1: Introduction to Exponents **N9.1**

One Grain of Rice (adapted from illuminations)

In the book *One Grain of Rice* by Demi, the main character Rani cleverly tricks the raja into giving rice to the village. Use the story from the book to answer the questions below.

1. Estimate how many grains of rice you think Rani will have at the end of 30 days. _________
2. Use the chart below to record the number of grains of rice Rani would receive each day.

Day 1 ***1*** *Grain of rice*	*Day 2* ***2*** *Grains of rice*	*Day 3* *Grains of rice*	*Day 4* *Grains of rice*	*Day 5* *Grains of rice*	**Total days 1-5**
Day 6 *Grains of rice*	*Day 7* ***64*** *Grains of rice*	*Day 8* *Grains of rice*	*Day 9* *Grains of rice*	*Day 10* *Grains of rice*	**Total days 6-10**
Day 11 *Grains of rice*	*Day 12* *Grains of rice*	*Day 13* *Grains of rice*	*Day 14* *Grains of rice*	*Day 15* ***16,384*** *Grains of rice*	**Total days 11-15**
Day 16 *Grains of rice*	*Day 17* *Grains of rice*	*Day 18* *Grains of rice*	*Day 19* ***262,144*** *Grains of rice*	*Day 20* *Grains of rice*	**Total days 16-20**
Day 21 *Grains of rice*	*Day 22* *Grains of rice*	*Day 23* *Grains of rice*	*Day 24* *Grains of rice*	*Day 25* *Grains of rice*	**Total days 21-25**
Day 26 *Grains of rice*	*Day 27* *Grains of rice*	*Day 28* *Grains of rice*	*Day 29* *Grains of rice*	*Day 30* *Grains of rice*	**Total days 26-30**
					Total days 1-30

3. If the story continued how can you determine how many grains of rice she would receive on day 31?

4. How can you determine how many grains of rice she would receive on day 65?

Bonus: Can you come up with a mathematical equation or expression that models this pattern?

A. Terminology

$$8^3$$

What does this mean? ____

Why would we needs this?____

Where do we see exponents?______________________________________

What is the difference between?	
8^3	8×3

Base:____

Exponent:____

Power:____

Example 1: List the base, exponent, and power for each of the following:

a) 12^4		b) 3^7		c) 2^8		d) 4^3	
Base:	____	Base:	____	Base:	____	Base:	____
Exponent:	____	Exponent:	____	Exponent:	____	Exponent:	____
Power:	____	Power:	____	Power:	____	Power:	____

Example 2: What happens if we swap the base and the exponent, do we get the same answer? Expand the following powers (don't evaluate).

a)

8^3 and 3^8	

b)

6^4 and 4^6	

Q: Which would you predict to be larger?

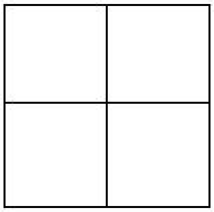
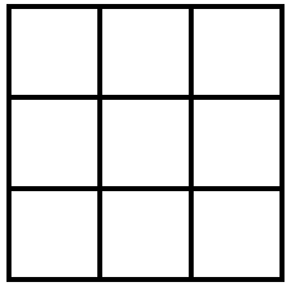
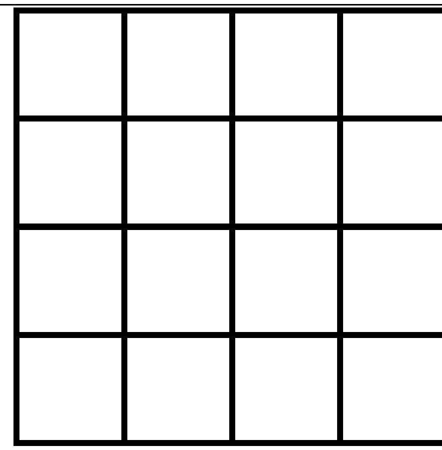
Q: Which would you predict to be larger?

Verify (check your answer with a calculator)

B. Common Exponents

A power with an integer base and exponent 2 is a **square number**.

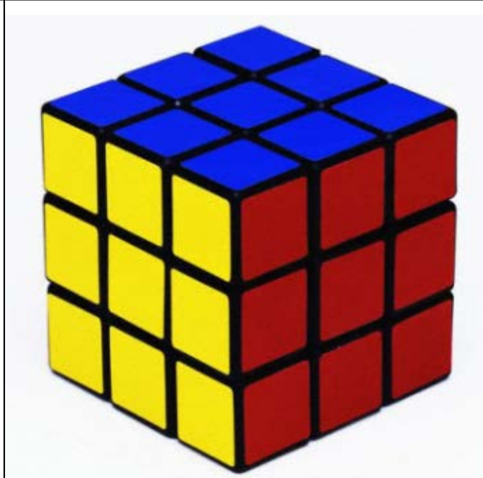
Why do you suppose they are called **SQUARES?**

2^2	3^2	4^2
		

Can you explain how the models given represent the powers 2^2, 3^2, and 4^2?

A power with an integer base and exponent 3 is a **cube number.**

Why do you suppose they are called **CUBES?**

2^3	3^3	4^3
		

Can you explain how the models given represent the powers 2^3, 3^3, and 4^3?

Example 3: Write as a power

a) $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$ b) 3×3 c) 6

Example 4: Write as repeated multiplication

a) 3^5 b) 2^7 c) 12^1 d) 11^5

Example 5: Calculate the volume of a cube that has side lengths of 6 cm.

Example 6: If the volume of a cube is 729, what are the side lengths?

Lesson 2: Integers and Powers **N9.1**

To the Teacher:

This lesson is important in establishing rules for integers and exponents. It is important to use repeated multiplication when establishing the concepts of brackets, negative signs, and exponents. Approach the lesson as if there were no known rules at all. By looking at the patterns of repeated multiplication, students should be able to construct an understanding of what happens to positive and negative bases when the exponent is odd or even. Some students might find it helpful if you circle the base that the exponent is effecting.

Example: $-(-3)^4$

I have made all the examples to have smaller exponents so repeated multiplication isn't too tedious.

RN #4

Inquiry

Lesson Process:

1. Use the lesson provided to work through the first four questions. Make sure to use repeated multiplication. Ask students to spot the pattern. They should see that;
   a. Negative bases to even exponents yield positive results
   b. Positive bases to even exponents yield positive results
   c. Negative bases to odd exponents yield negative results
   d. Positive bases to odd exponents yield positive results **RN #4**

2. Example 1: Discuss with students the importance of brackets and what they mean. While going through example 1 all questions should be answered using repeated multiplication. Have students circle the base that is effected by the exponent and expand the circle. Have them simplify the signs last.

3. Example 2: Have students predict if the solution will be positive or negative without using repeated multiplication. Have them evaluate the answer after they have predicted the sign.

4. At the end of the lesson ask students if we can get the correct answers without expanding the power using repeated multiplication.

5. Assignment: Lesson 1 & 2

Integers and Exponents

A. Find my Rule

Exponents are not limited to just positive bases, negative bases can have exponents too! See if you can spot a pattern?

A. $(-2)^1$

$(-2)^2$

$(-2)^3$

$(-2)^4$

$(-2)^5$

$(-2)^6$

B. $(3)^1$

$(3)^2$

$(3)^3$

$(3)^4$

$(3)^5$

$(3)^6$

C. $(-1)^1$

$(-1)^2$

$(-1)^3$

$(-1)^4$

$(-1)^5$

$(-1)^6$

D. $(1)^1$

$(1)^2$

$(1)^3$

$(1)^4$

$(1)^5$

$(1)^6$

What is the pattern?

Positive base to an odd exponent	
Positive base to an even exponent	
Negative base to an odd exponent	
Negative base to an even exponent	

B. The Importance of Brackets

Brackets don't always look like the ones above. There are many different ways that brackets are involved in a question.

Example 1: Evaluate

a) $(-3)^4$

b) $-(3)^4$

c) -3^4

d) (-3^4)

e) $-(-3)^4$

f) $(-3)^5$

g) $-(3)^5$

h) -3^5

i) $-(-3)^5$

Example 2: Predict whether each solution will be positive or negative, then evaluate **without** using repeated multiplication.

a) $(-2)^4$

b) $-(-5)^3$

c) $-(-5^3)$

d) $-(2^3)$

How do I use my calculator for larger powers?

Assignment: Lessons 1 & 2 **N9.1**

1. What is the base of each power?

a) -3^4 b) $(-3)^4$ c) $-(-8)^8$ d) 12^3

2. What is the exponent of each power?

a) -3^4 b) $(-3)^4$ c) $-(-8)^8$ d) 12^3

3. Use repeated multiplication to show why 6^5 is not the same as 5^6. Verify using a calculator which one is larger.

4. Complete the following table.

Power	Base	Exponent	Repeated Multiplication	Evaluate
3^3				
$(-10)^4$				
	-3	5		
			$1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1$	

5. Write each as a power, then evaluate.

a) 5×5 b) $6 \times 6 \times 6 \times 6 \times 6 \times 6$

c) $-(2 \times 2 \times 2 \times 2)$ d) $(-10) \times (-10) \times (-10)$

e) $(-3)(-3)(-3)(-3)$

f) $-(-7)(-7)(-7)$

g) $-(5)(5)(5)$

g) $(10)(10)(10)(10)(10)(10)$

6. For each power.

- Are the brackets needed?
- If you answer is yes, what purpose do the brackets serve?
- Evaluate

a) $(-3)^5$ b) $-(3)^5$ c) $-(-3)^5$ d) (-3^5)

7. Predict whether each solution will be positive or negative, then evaluate.

a) 4^3 b) 10^2 c) 2^4 d) 8^1

e) $(-3)^5$ f) -3^5 g) $(-3)^4$ h) -3^4

i) $-(-7)^2$ j) $-(-7)^3$ k) $(-5)^4$ l) -5^4

8. Write each number as a power with a base of 3.

a) 27 b) 2187 c) 81 d) 729

Lesson 3: Powers of 10 and the Zero Exponent Law **N9.1**

To the Teacher:

The purpose of this lesson is to establish the zero exponent rule. Another option to this lesson is to do the paper folding activity (included at the end of the unit). By looking at bases of 10, students should be able to continue the pattern and see that decimals less than 1 can be created using negative exponents and what happens when you have a power of 0. You can use the whiteboard activity to assess how the students are doing up to this point, it includes integers and exponents (using exponents 0 and larger). When using the whiteboard activity, randomly select the questions rather than going through alphabetically.

RN #4

Inquiry

RN #5

Formative Assessment

Lesson Process:

1. Begin the lesson by discussing the powers of 10. Start with “Ten” on the chart, moving up from there. Once you establish the standard form and powers for ten to Quintillion, discuss the pattern. Ask students

    a. Why do the exponents start going up by 3 after hundred thousand?
    b. What do we put in the “One” row?
    c. If we kept up with the pattern, how do we fill in the rest of the chart?
    d. Put in the SI prefix for the chart. This is a tie to their science classes and future WPA 10 classes. You can ask students for some of the more familiar prefixes, but may need to have students research the others (or research together).

2. Bring the conversation back to the exponent of 0. Conduct the calculator experiment. Have students try different bases to make sure that negative bases, fraction bases, exponent bases, etc. all have an answer of 1 when put to the power of 0. Here are some questions to ask students.
    a. Does a base of 0 work?
    b. Why can’t a base of 0 work?
    c. Why does an exponent of 0 create a 1? You may not get any answers from students. Have them research it and come back next day with some ideas. Or you can wait until division of powers coming up. **RN #4**

3. Go over terminology with students regarding terms.
4. Complete examples 1-3 with students.
5. For the remainder of the class conduct the whiteboard activity. **RN #5**
6. There is an optional exit question for students to complete. **RN #5**
7. Assignment: Lesson 3

Powers of 10 and the Zero Exponent Law

A. Powers of 10

Name	Standard Form	Power	SI Prefix
Quintillion			
Quadrillion			
Trillion			
Billion			
Million			
Hundred Thousand			
Ten Thousand			
Thousand			
Hundred			
Ten			
One			
One Tenth			
One Hundredth			
One Thousandth			
One Ten Thousandth			
One Hundred Thousandth			
One Millionth			
One Trillionth			
One Quadrillionth			
One Quintillionth			

Calculator Experiment:

What happens when we have an exponent of 0? Using your calculator to experiment with different bases. We will share them as a class. Be adventurous with the numbers you choose.

Conclusion:____

B. Zero Exponent Law

A power with an integer base and an exponent of 0, is equal to 1.

* The base cannot be 0!

* $b^0 = 1,\ b \neq 0$

Terminology Refresher:

Repeated Multiplication	Expand	Power	Exponential Form	Standard Form	Evaluate
$4 \times 4 \times 4$	$4 \times 4 \times 4$	4^3	4^3	64	64

Example 1) Evaluate each power.

a) 5^0 b) 45^0 c) $(-3)^0$ d) -2^0

e) $(-2)^0$ f) $-(-2)^0$ g) 2245^0 h) -1000^0

Example 2) Write each number as a power of 10

a) 1 b) 100,000 c) One Billion d) -1

Example 3) Expand the following powers

a) 10^5 b) 10^8

Assignment: Lesson 3 **N9.1**

1. Evaluate the following powers.

a) $(-5)^0$ b) -3^0 c) 4^0 d) 1399^0

e) $-(-2)^0$ f) -2^0 g) 10^3 h) 10^7

2. Write each number as a power of 10

a) 10,000 b) Quadrillion c) 1 d) 100

3. Complete this table for powers of 10

Exponent	Power	Standard Form
7	10^7	10,000,000
6		
5		
4		
3		
2		
1		
0		

4. Arrange the following powers from least to greatest value: $1^{22}, 3^4,\ 4^3,\ 2^5,\ 7^2$

Whiteboard Activity for Lesson 1, 2, & 3 **N9.1**

1. Will the solution be positive or negative?

a) $(-3)^{30}$ b) 6^{89} c) -4^{62} d) -2^{99}

e) $(-2)^{0}$ f) $-(-2)^{0}$ g) $(-3)^{31}$ h) $-(-5)^{33}$

2. Show the expansion and evaluate.

a) 5^3 b) 2^7 c) 2^4 d) 8^2

e) 10^3 f) -10^4 g) -3^2 h) -6^2

i) $(-3)^2$ j) $(-6)^2$ k) $(-3)^3$ l) $(-3)^4$

m) $(-2)^3$ n) $(-2)^4$ o) $-(-2)^3$ p) $-(-2)^4$

q) 10^4 r) 10^2 s) -10^3 t) $(-10)^2$

u) 5^0 v) -3^0 w) $(-4)^0$ x) $(-3)^0$

Lesson 3: Exit Question (N9.1) Name:________________

1. Write as a power.
   a. $3 \times 3 \times 3 \times 3 \times 3$
   b. $-2 \times -2 \times -2 \times -2$

2. Show the expansion and evaluate.
   a. $(-1)^4$
   b. $(4)^3$
   c. 2^0
   d. 10^5

Lesson 3: Exit Question (N9.1) Name:________________

1. Write as a power.
   a. $3 \times 3 \times 3 \times 3 \times 3$
   b. $-2 \times -2 \times -2 \times -2$

2. Show the expansion and evaluate.
   a. $(-1)^4$
   b. $(4)^3$
   c. 2^0
   d. 10^5

Lesson 3: Exit Question (N9.1) Name:________________

1. Write as a power.
   a. $3 \times 3 \times 3 \times 3 \times 3$
   b. $-2 \times -2 \times -2 \times -2$

2. Show the expansion and evaluate.
   a. $(-1)^4$
   b. $(4)^3$
   c. 2^0
   d. 10^5

Lesson 4: Order of Operations **N9.1**

To the Teacher:

This is a fun lesson because there are many important discussions that come out of it. Students have already learned the order of operations in their elementary mathematics classes (although they may not remember). It might be interesting to ask students "why there is an order of operations?" From there, have students come up with as many different answers as they can for Example 1. The question comes from a viral Facebook feed in which it claims 74% of people got it wrong. A great conversation about the importance of order could stem from that example. Here is a link to the original posting and the data the author pulled from it. He also posted a chart depicting the percentage of people that got each solution. It might be interesting to compare that data to the classroom data.

http://www.classroomprofessor.com/teaching-math/why-did-74pc-of-facebook-users-get-this-wrong/.

Example 3 is another great question because it is frustrating for students. It should reinforce the importance of showing their work. Example 4 is best done as a group activity and is fun because students need to use the order of operations to create the question.

RN #2

Prior Knowledge

RN #7

Cooperative Learning

Lesson Process:

1. Ask students "what is an operation?" Students should answer with addition, subtraction, multiplication, division, exponents. Ask students "are brackets actually an operation?"

2. Example 1: Ask students to find the answer to the expression $7 - 1 \times 0 + 3 \div 3$. Some of the possible answers could be $0, 1, 3, 6, 7, 8$. This question comes from a viral Facebook feed and I have included a link where the original poster looked at the data of responses. You could compare the data from the website to the data from the class.

3. Ask students if they know the order of operations. They may answer BEDMAS or PEDMAS or PEMDAS. All three will work! Another important question to ask students is why they think that the order of operations is important. It might be an interesting assignment for them to look up the history of the order of operations. **RN #2**

4. Example 2: Work through example 2 with the students. Question 'i' is a little different because the value for r needs to be substituted into the formula. Ask the students if they know what the letters stand for in the formula and if they know what the formula is for. (Volume of a sphere where r is the radius)

5. Example 3: This is a great example and should not be skipped. It is an order of operations question and 3 students' answers are given. The tricky part is that no work is shown, only the answer. Students have to find out which one is correct AND where the students with the incorrect answers went wrong. It is a very frustrating exercise because students have to try different combinations of incorrect procedures to find where the students went wrong. The purpose is for students to understand that in order to get proper feedback on their mistakes, they need to show their work.

6. Task: This is best done as a group exercise. Assign each group 4-5 numbers from the list. Have students create the numbers 0-20 using only four 4's and the operations. Example: $1 = \frac{(4+4)}{(4+4)}$. Have students share their answers on the board. Encourage them to find more than one way for each number. The goals should be for them to use the order of operations properly as well as understand that there is more than one answer for each number. **RN #7**

7. Assignment: Lesson 4

Order of Operations

A. Why do we Need an Order?

Example 1: Answer the following. How many different answers can we come up with?

$$7 - 1 \times 0 + 3 \div 3$$

Warning! If you calculate in the wrong order, you will get the wrong answer. What is the correct order?

B. What is the Order?

A long time ago people agreed to follow rules when doing calculations, and they are:

D A

B E

M S

Example 2:

a. $20 - (3 \times 2^3 - 5)$

b. $(5 + 2)^2 - 9 \times 3 + 2^3$

c. $4^2 + 2^6$

d. $(6 - 2)^2 - 3 \times 2$

e. $3(-4)^3$

f. $4^2 - 8 \div 2 - 2^3$

g. $-2(-15-4^2)+4(4+1)^3$

h. $5^2+(-5^2)$

i. $V=\frac{4}{3}\pi r^3$; the radius is 5

Example 3:

Three students got different answers when they evaluated this expression:

$$-5^2-2[27\div(-9)]^3$$

Arshpreet's answer was 729, Jake's answer was 79, and Cooper's answer was 191.

a) Show the correct solution

b) Show and explain how the students who got the wrong answer may have evaluated. Where did each student go wrong? Provide feedback to the student so they don't make the same mistake again.

Task: Using the order of operations and four 4's, create the numbers 1 to 20.

0	11
1	12
2	13
3	14
4	15
5	16
6	17
7	18
8	19
9	20
10	

Assignment: Lesson 4 **N9.1**

1. Evaluate

a) $4^2 + 2$ b) $4^2 - 2$ c) $(4 + 2)^2$ d) $(4 - 2)^2$

e) $(2 - 4)^2$ f) $(2 - 4)^3$ g) $4^2 - 3^2$ h) $3^2 - 4^2$

2. Evaluate

a) $3^2 \times 5$ b) 3×2^3 c) $(3 \times 2)^3$ d) $4 \div 2^3$

e) $(8 \div 2)^3$ f) $8^2 \div 4^2$ g) $(-15 \div 3)^2$ h) $(-15 \div 3)^3$

3. Evaluate

a) $(18 \div 3^2 + 1)^3 - 3^2$ b) $5^3 - 2(5 - 2)^2$ c) $(12^2 + 5^3)^0 - 2[(-3)^3]$

4. Three students A, B, C did a question and got three different answers. You are the teacher and you need to find out who is correct AND where the other two students made their errors. Describe their error.

Student A

$$(-4)^2 - 3[(-9) \div 3]^2$$
$$(-4)^2 - 3[-3]^2$$
$$(-4)^2 + (9)^2$$
$$16 + 81$$
$$97$$

Is Student A correct? Circle their mistake(s) and give them feedback on where they went wrong.

Student B

$$(-4)^2 - 3[(-9) \div 3]^2$$
$$(-4)^2 - 3[-3]^2$$
$$16 - 3(9)$$
$$16 - 27$$
$$-11$$

Is Student B correct? Circle their mistake(s) and give them feedback on where they went wrong.

Student C

$$(-4)^2 - 3[(-9) \div 3]^2$$
$$(-4)^2 - 3[-3]^2$$
$$8 - 3[6]$$
$$8 - 18$$
$$-10$$

Is Student C correct? Circle their mistake(s) and give them feedback on where they went wrong.

Lesson 5: Exponent Laws **N9.1**

To the Teacher:

This lesson is designed to be done constructively where students discover the exponent rules on their own. You can have groups of students working through one law and have them share their ideas with the class, or you can facilitate the entire class together through all of the laws. Students should be able to see the patterns and develop their own rules when working with exponents. It is integral that students have been using repeated multiplication to represent powers in the previous lessons. An understanding that powers represent repeated multiplication is a key component for this lesson to be effective. This lesson may need to be extended to two class periods.

RN #7

Cooperative Learning

RN #4

Inquiry

Lesson Process:

1. **Student Workbook**: Ask students "what does an exponent mean?" They should answer repeated multiplication or something along that line. Discuss that a calculator is useful for evaluating exponents, but for this activity we are going to be looking at patterns in order to establish some exponent rules (really important for algebra!).

2. Begin the task in the student workbook. You can either choose to group the students or have them work through it as a class. All of the first questions are done in the chart so students can follow and use the same notation. Please tell students that it is very important to use repeated multiplication. They may find the pattern before they get to the end, but they need to complete all the examples on the chart before they can use the rule (no shortcuts). **RN #7, RN #4**

3. If students are working in groups, then they need to present their findings to the class. They need to prove their rule by going through the examples on their worksheets. All students should have their charts completed by the end of the lesson.

4. It may be a good idea to have them see if the evaluated answer from column 1 matches the evaluated answer in column 2.

5. Have a class discussion about why a base to the power of 0 is 1. This was asked in **Lesson 3**. Students should now understand that anything to the power of 0 is representing $\frac{b}{b}$. Therefore the answer is 1.

6. At the end of the lesson discuss why the summary of our rules have letter bases instead of numbers. (Because we need to be able to describe "any" situation). Look through the summary of each rule.

7. Examples 1-5: Go through the examples as a class. Discuss what rule is needed for each question.

8. Assignment: Lesson 5

Exponent Laws

A. What is an Exponent?

Before we begin, we must remember what an exponent means. An exponent is a shorter way to write ____.

For instance, 3^6 **can be thought of as repeatedly multiplying 3 by itself, 6 times, or** $3 \times 3 \times 3 \times 3 \times 3 \times 3$. We will use this notation to rewrite various expressions involving exponents.

It is important to keep in mind that there are other ways to simplify these expressions (including the calculator). But we are doing this to help us establish rules that are going to apply to algebraic expressions as well (which a calculator wouldn't be able to help us). Follow the first example and keep the same notation throughout the exercise. You should be able to see a pattern develop.

1. Complete the table using the idea of repeated multiplication. Then in the final row, use the pattern to write the general exponent rule.

Expression	**Repeated Multiplication**	**Simplified**
$3^3 \cdot 3^4$ or $3^3 \times 3^4$	$(3 \cdot 3 \cdot 3) \cdot (3 \cdot 3 \cdot 3 \cdot 3)$	3^7
$2^6 \cdot 2^3$ or $2^6 \times 2^3$		
$8^7 \cdot 8^3$ or $8^7 \times 8^3$		
$6^5 \cdot 6^3$ or $6^5 \times 6^3$		
Make up your own!		
$a^3 \cdot a^2$ or $a^3 \times a^2$		
$a^m \cdot a^n$ or $a^m \times a^n$		

Does this rule work for $5^3 \cdot 8^2$?

Rule: When the bases are the same…..

2. Complete the table using the idea of repeated multiplication. Then in the final row, use the pattern to write the general exponent rule.

Important: These problems also use the fact that $\frac{5}{5} = 1$. Anything divided by itself is always 1. $\frac{5 \cdot 3}{3} = 5$ since the 3's on the top and bottom can be thought of $\frac{3}{3}$ which is just 1.

Expression	Repeated Multiplication	Simplified as a power
$\frac{2^4}{2^3}$ or $2^4 \div 2^3$	$\frac{2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2}$	2^1
$\frac{10^6}{10^3}$ or $10^6 \div 10^3$		
$\frac{4^8}{4^3}$ or $4^8 \div 4^3$		
$\frac{7^9}{7^2}$ or $7^9 \div 7^2$		
$\frac{a^7}{a^3}$ or $a^7 \div a^3$		
$\frac{a^m}{a^n}$ or $a^m \div a^n$		
What happens when… $\frac{5^3}{5^3}$ Or $5^3 \div 5^3$		
What happens when… $\frac{0^4}{0^3}$ Or $0^4 \div 0^3$		

What happens when… $\frac{5^6}{4^3}$ or $5^6 \div 4^3$		

Rule: When the bases are the same… EXCEPT

3. Complete the table using the idea of repeated multiplication. Then in the final row, use the pattern to write the general exponent rule.

You will need to expand what is inside the bracket first, then apply the outer exponent.

Expression	**Repeated Multiplication**	**Simplified**
$(4^2)^3$	$(4 \cdot 4)^3$ which is $(4 \cdot 4) \cdot (4 \cdot 4) \cdot (4 \cdot 4)$	4^6
$(6^3)^5$		
$(2^5)^3$		
$(8^4)^1$		
$(a^3)^4$		
$(a^m)^n$		

Rule:

4. Complete the table using the idea of repeated multiplication. Then in the final row, use the pattern to write the general exponent rule.

These problems also use the commutative property of multiplication that says the order in which I multiply two numbers does not matter. For example, $2 \times 3 = 3 \times 2$

Expression	**Repeated Multiplication**	**Simplified**
$(2 \cdot 3)^4$	$(2 \cdot 3) \cdot (2 \cdot 3) \cdot (2 \cdot 3) \cdot (2 \cdot 3)$ Which is.. $(2 \cdot 2 \cdot 2 \cdot 2) \cdot (3 \cdot 3 \cdot 3 \cdot 3)$	$2^4 \cdot 3^4$
$(5 \cdot 3)^3$		
$(4 \cdot 6)^2$		
$(7 \cdot 9)^5$		
$(a \cdot b)^3$		
$(a \cdot b)^m$		

Try BEDMAS with all the above and see if you get the same evaluated answer if you do the exponent rule. Are they the same?
Is $2^4 \cdot 3^4 = 6^4$?

Note: When we get into algebra $a \cdot b$ is just ab.

Rule:

5. Complete the table using the idea of repeated multiplication. Then in the final row, use the pattern to write the general exponent rule.

Use the idea from exponent rule #2 to complete the second column. To complete the third column, remember what you get when you divide a number by iself $\frac{5}{5}$ =?

Expression	**Work it out using exponent rule #2**	**Work it out using what you know about a number divided by itself.**
$\frac{6^3}{6^3}$ or $6^3 \div 6^3$	6^{3-3} 6^0 _______	$\frac{216}{216}$ Which is just 1
$\frac{10^4}{10^4}$ or $10^4 \div 10^4$		
$\frac{8^6}{8^6}$ or $8^6 \div 8^6$		
$\frac{3^5}{3^5}$ or $3^5 \div 3^5$		
$\frac{a^4}{a^4}$ or $a^4 \div a^4$		
$\frac{a^m}{a^m}$ or $a^m \div a^m$		

Rule: Anything to the power of 0 is ______.

Exponent Rules

- You can apply the exponent rules to help simplify expressions.

1. You can simplify a product of powers with the same base by adding exponent.

$(a^m)(a^n) = a^{m+n}$ Or $a^m \times a^n = a^{m+n}$	$(4^7)(4^3)$

2. You can simplify a quotient of powers with the same base by subtracting exponents.

$\frac{a^m}{a^n} = a^{m-n}, \qquad m > n$ OR $a^m \div a^n = a^{m-n}$	$\frac{4^7}{4^3}$

3. You can simplify a power that is raised to an exponent by multiplying the exponents.

$(a^m)^n = a^{mn}$	$(4^6)^3$

4. When a product is raised to an exponent you can rewrite each number in the product with the same exponent.

$(ab)^m = a^m b^m$	$(5 \cdot 3)^6$

5. When the exponent of a power is 0, the value of the power is 1 if the base is not 0.

$a^0 = 1, \qquad a \neq 0$	12^0

6. When a quotient is raised to an exponent, you can rewrite each number in the quotient with the same exponent.

$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	$\left(\frac{3}{4}\right)^4$

Example 1: Simplify and evaluate (Law 1 & 2)

Together	On Your Own
a) $(2^3)(2^8)$	b) $(-7)^9(-7)^5$
c) $(-3)^5 \div (-3)^2$	d) $(-7)^9 \div (-7)^5$
c) $12^7 \div 12^5 \div 12^2$	d) $3^2 \times 3^4 \times 3^5$
e) $(-10)^4[(-10)^6 \div (-10)^4]$	f) $-2^2(2^3 \div 2^1)$

Example 2: Write as a power and evaluate. (Laws 3&4)

a) $[(-3)^5]^2$

b) $-(4^2)^3$

c) $(9^2)^4$

d) $(5^3)^5$

e) $(8^2)^6$

Example 3: Why is the value of $[(-4)^3]^2$ positive and the value of $[(-4)^3]^3$ negative?

Example 4: For each expression below, evaluate in two different ways:

i. follow order of operations
ii. use the exponent laws first

a) $(5 \times 3)^4$

b) $[(-3) \times 5]^3$

c) $[15 \div (-3)]^2$

d) $-(2 \times 4)^2$

e) $\left(\frac{144}{12}\right)^3$

f) $\left(\frac{1}{4}\right)^3$

Example 5: Simplify and evaluate.

Together	On Your Own
a) $2(3)^4$	b) $-4(-2)^2$
c) $[(10^2)^4 \div (10^3)^2]$	d) $(7^4 \times 7^3)^0$
e) $2^3 - 2^0 \times 2^2 + 2^2$	f) $\frac{3^2 \times 3^8}{3^3 \times 3^6}$

Assignment: Lesson 5 **N9.1**

Write each product as a single power (if possible).

a) $3^5 \times 3^8$

b) $2^0 \times 2^0$

c) $10^3 \times 10^8$

d) $(-7)^6 \times (-7)^3$

e) $-6^3 \times (-6)^4$

f) $12^7 \times 12^{12}$

2. Write each quotient as a single power (if possible).

a) $3^8 \div 3^5$

b) $12^{15} \div 12^6$

c) $(-1)^5 \div (-1)^0$

d) $\frac{8^9}{8^5}$

e) $\frac{(-6)^8}{(-6)^2}$

f) $\frac{3^4}{3^4}$

3. Express as a single and evaluate.

a) $5^3 \times 5^7 \div 5^4$

b) $(-3)^{12} \div (-3)^3 \times (-3)^8$

c) $4^0 \times 4^8 \div 4^7$

d) $\frac{4^7 \times 4^3}{4^4 \times 4^2}$

4. Express as a single power (if possible) and then evaluate.

a) $2^3 \times 2^6 \div 2^9$

b) $(-5)^8 \div (-5)^4 \times (-5)^3$

c) $\frac{6^3 \times 6^5}{6^2 \times 6^4}$

d) $2^2 - 2^0 \times 2 + 2^3$

e) $(-2)^6 \div (-2)^5 \times (-2)^5 \div (-2)^3$

f) $(8^3 \times 8^7) \div (8^5 \div 8^2)$

5. Here is a student's work. Please correct their work and provide feedback on any errors.

a) $3^4 \times 3^2$

3^8

b) $5^3 \times 2^3$

10^6

c) $(-3)^8 \div (-3)^4$

$(-3)^4$

d) $\frac{4^2 \times 4^4}{4^2 \times 4^1}$

4^2

6. Write the following expression as a power raised to an exponent.

$$(3 \times 3) \times (3 \times 3) \times (3 \times 3) \times (3 \times 3)$$

7. Simplify and evaluate.

a) $(4^2 \times 4^3)^2 - (4^4 \div 4^2)^2$

b) $(2^3 \div 2^2)^3$

8. Find and correct any errors in each solution.

a) $(4^3 \times 2^2)^2 = (8^5)^2$
$= 8^{10}$
$= 1\ 073\ 741\ 824$

b) $[(-10)^3]^4 = (-10)^7$
$= -10\ 000\ 000$

c) $(2^2 + 2^3)^2 = (2^5)^2$
$= 2^{10}$
$= 1024$

Lesson 6: Solving Situation Questions **N9.1**

Students will be using their knowledge of exponents to solve situation questions (word problems). These questions can be dispersed into the previous assignments or be taught at the end of the unit.

RN #7

Cooperative learning

Learning Process:

1. Before the students start working on their questions, ask the students to read through the questions. Ask them if there is anything that they need to know (formulas, definitions of words etc.) before they can start the worksheet. Have students come up to the board to write those "road blocks" down. Depending on what has been covered before this unit, Pythagorean theorem, area of circle, area of square, surface area of a cube, volume of a cube, and what is an expression, could all be things that may need to be addressed before the students begin working.

2. Break students into groups and have them work through the questions together. Once groups are done, collect their sheets and have them disperse into other groups to help students or groups who are struggling. **RN #7**

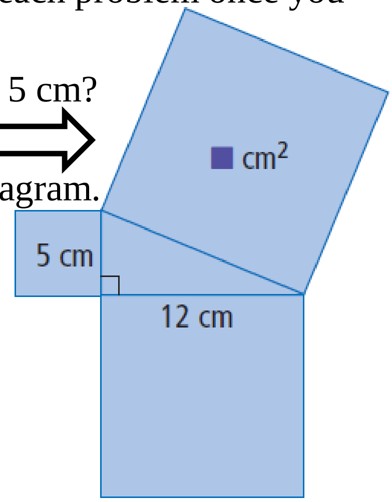
3. When all groups have completed the worksheet, discuss the answers together as a class. You could have all the groups write their final solution on the board. Discuss each question as a class and make sure that all groups agree on the solution before moving on.

4. Assignment: Lesson 6
   Some of the questions in the assignment are from McGraw Hill & Ryerson Grade 9 Textbook (p. 118)

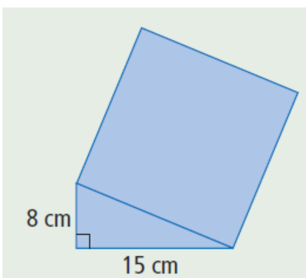
Assignment: Lesson 6 **N9.1**

1. The surface area, SA, of a sphere can be calculated using the formula $SA = 4 \times \pi \times r \times r$, where r is the radius. Rewrite the formula using powers and no multiplication signs. Identify the coefficient, variable, and exponent in your formula.

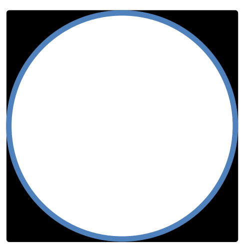
2. Write an exponential expression to solve each problem. Solve each problem once you have established an exponential expression.
    a. What is the surface area of a cube with an edge length of 5 cm?
    b. Find the missing side length of this right triangle.



3. Find the area of the square attached to the hypotenuse in this diagram.



4. A circle is inscribed in a square with a side length of 3 cm. What is the area of the shaded region?



5. What is the volume of a cube with an edge length of 7 cm? Write an exponential expression to solve the problem.

6. Which is larger, the area of a square with a side length of 14 cm or the surface area of a cube with an edge length of 6 cm? Show your work. Write an exponential expression to solve the problem;.

7. The number 10^{100} is known as a googol.
    a. Research where the term googol originated. Why do you think that the founders of Google$_{TM}$ used that name for their search engine?
    b. How many zeros would follow the 1if you wrote 10^{100} as a whole number?
    c. If you were able to continue writing zeros without stopping, how long would it take you to write a googol as a whole number? Explain why you believe your answer is correct. You will need to be prepared to share your answer and reasoning to the whole class.

Task: Repeated Addition vs. Repeated Multiplication: Which grows the fastest?

Students need to know: how to graph, add, multiply, interpret graphs

Prompt: Using only the number 2's. Show the repeated addition of 2's. Create a chart, or another visual form to show how the numbers grow.

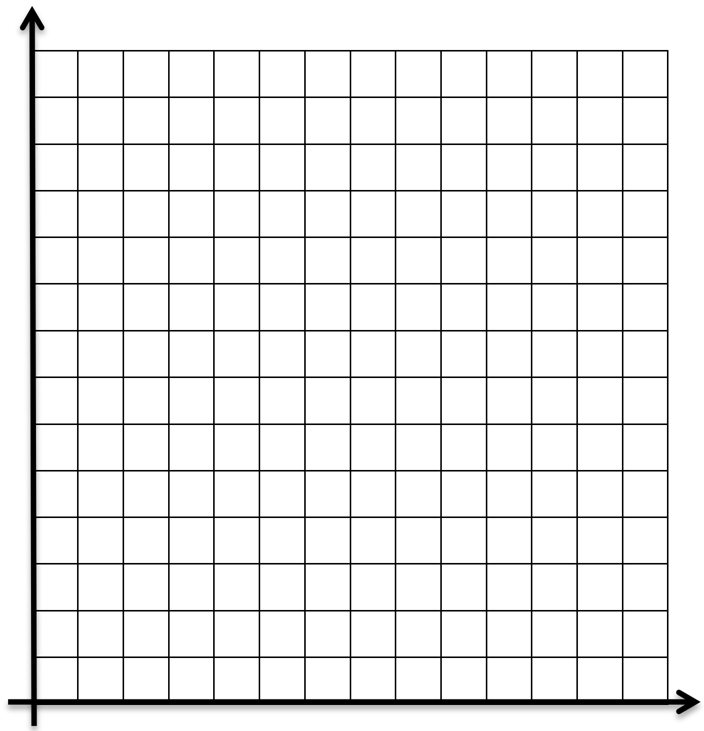
Prompt: Now that you have your values. Graph them on the Cartesian plane to see what the pattern looks like. Have the students describe the pattern.

Prompt: Using only the number 2's. Show the growth of repeated multiplication of 2's. Create a chart, or another visual form to show how the numbers grow.

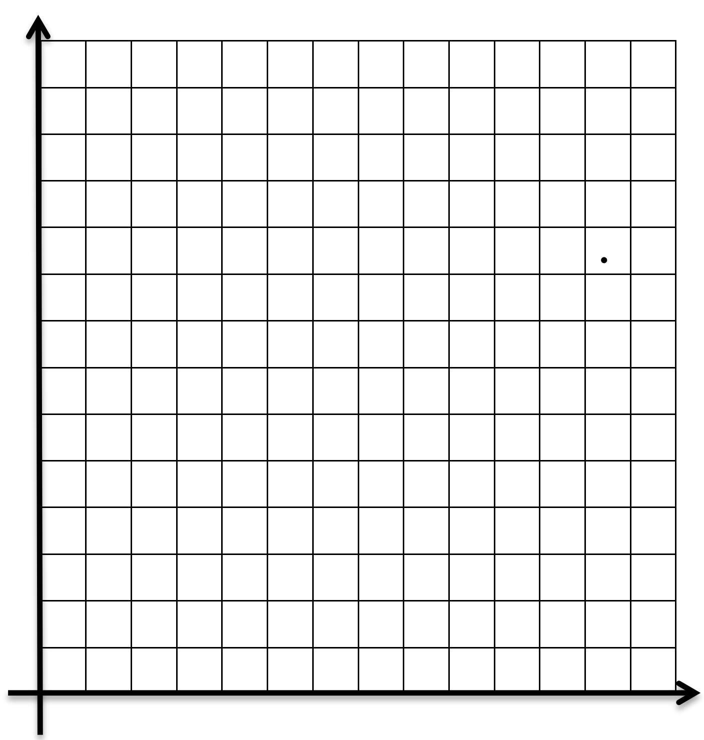
Prompt: Now that you have your values. Graph them on the Cartesian plane to see what the pattern looks like. Have the students describe the pattern.

Questions to ask students after graphing:
1. Which grows faster? Why is that?
2. Ask students if they know of any "real-life" situations that model each pattern. Could they assign a story to it? Do they know of anything that follows that growth pattern?

Repeated Addition:



Repeated Multiplication:



Inquiry Task

Paper folding Assignment (page 57: teaching secondary and middle school math)

Instructions:

1. Take a sheet of paper and fold it in half. Then take the half sheet and fold it in half again, and again, and again.
2. How many times can you continue to fold the paper in half before it becomes inpossible to fold again?
3. Record your data in the following chart.

Fold #	1	2	3	4	5
How many pages does it create?	2				
Fold #	6	7	8	9	10
How many pages does it create?					

4. Do you think that the number of folds you got for your paper is true for all pieces of paper or just the one you were folding?
5. What if you piece of paper was a 25-foot stretch of cash register tape? What if it was a sheet of newspaper?
6. What patterns do you see?

Teacher Notes

1. Have students arrange their data in chart format. Students should have trouble getting past 8 folds.
2. Ask students how they can figure out more than 8 folds. Ask them to find the pattern. Have them come up with the fact that it is powers of 2, where the exponent represents how many folds they have done. Hopefully this will show that the 2^0 represents 0 folds, therefore we have one layer of paper.
3. A standard phone book is about 900 pages, so attempting to fold the paper in half 12 times would be like folding the telephone book.

OUTCOME: N9.2

Demonstrate understanding of rational numbers including:

- comparing and ordering
- relating to other types of numbers
- solving situational questions

Previous Experience:

Grade 7

Outcome N7.2

Expand and demonstrate understanding of the addition, subtraction, multiplication, and division of decimals to greater numbers of decimal places, and the order of operations.

Outcome N7.3

Demonstrate an understanding of the relationships between positive decimals, positive fractions (including mixed numbers, proper fractions and improper fractions), and whole numbers.

Outcome N7.4

Expand and demonstrate an understanding of percent to include fractional per cents between 1% and 100%.

Outcome N7.5

Develop and demonstrate an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially, and symbolically (limited to positive sums and differences).

Outcome N7.6

Demonstrate an understanding of addition and subtraction of integers, concretely, pictorially, and symbolically.

Grade 8

Outcome N8.2

Expand and demonstrate understanding of per cents greater than or equal to 0% (including fractional and decimal per cents) concretely, pictorially, and symbolically.

Outcome N8.4

Demonstrate understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially, and symbolically.

To The Teacher:

This outcome is open to interpretation and quite vague. It takes reading the indicators to understand what skills and concepts the ministry wants the students to be working on. Negative rational numbers and order of operations with rational numbers are new for students. Although students have had experience with number lines, they haven't placed negative rational numbers on the number line. Less emphasis is placed on per cents and decimals in grade 9.

I dedicated a great deal of time modelling fractions when applying the operations to fractions. Because of the difficulty of representing negative fractions pictorially, I have only asked students to draw positive fractions since negative values are difficult to depict using paper and concrete objects.

Many students enter grade 9 with a limited ability to work with fractions even though they have spent a great deal of time working with them in elementary school. My goal with these lessons is to spend more time on developing what a rational number is and how it can be shown visually. In this way, students will have a stronger understanding of why it is necessary to find a common denominator when adding or subtracting fractions. As well, it will also help students to understand why the invert and multiply algorithm works when dividing fractions. Rational numbers include more than just fractions, which is why there is a lesson on decimals at the end of the unit as well as discussion about decimals and the number line at the beginning of the unit. It is your choice if you let your students use a calculator in the decimal sections. I have also included a supplemental worksheet on labelling number lines and converting mixed numbers to improper fractions.

It is important for students to work in groups when pictorially representing (modelling) fractions. They can discuss with each other the different ways to model as well as problem solve some of the tougher situations.

Lesson 1: What is a Rational Number? **N9.2**

To The Teacher:

This lesson is designed for students to experience fractions and remember skills from previous grades. Skittles, Smarties, or any candy that has different colors in the bag/box would work for the first activity. You don't need a magnetic sign for the "Remembering Skills Task", you can just write the words on the board. It is important for the students to supply almost all if not all of the information for the lesson.

RN #1

Mathematics-In-Context

RN #2

Prior Knowledge

RN #5

Formative Assessment

Lesson Process:

1. Begin with the Skittle task. This should be a review for students. However, we know that not all students arrive in grade 9 with a solid foundation of fractions. **RN #1**

2. The last two skittle discussion questions lead into the next portion of the lesson on how to convert between fractions, decimals, and percentages (see Remembering Skills Task).

3. Complete the "remembering skills task". Students should provide the steps to convert between fractions, decimals, and percentages. The teacher should facilitate the instruction and give hints, but not dictate the answers to students. When converting decimals to fractions, be sure to discuss the difference between terminating and repeating decimal. **RN #2**

4. At the end of the "remembering skills task" there is a quick check to see if the students understand how to convert between decimals, per cents, and fractions. You can correct it on your own or have the students pass it to a partner to correct. **RN #5**

Lesson 1: Skittle Task **N9.2**

1. Open a bag of skittles on the front desk. Have two students organize the skittles into color groups while another student records the data on the board.

2. Once the data is complete here are some questions to ask students to open a dialogue about fractions.
   a. How can we represent the data as a fraction?
   b. Arrange the fractions from smallest to greatest. How would you know how to do this if you didn't have the skittles right in front of us (only the fractions)?
   c. Have students explain what each fraction means. (12 out of the 35 skittles are green). What do the denominator and numerator represent?
   d. What color occurs least frequently? How do you know?
   e. What color occurs most frequently? How do you know?
   f. How can we represent the data as a decimal?
   g. How can we represent the data as a percentage? ie: what percentage of the package is red?
   h. Of the three ways to represent the data, which one(s) are the easiest to interpret?
   i. Of the three ways to represent the data, which one(s) are the hardest to interpret?
   j. Where do you see data represented as a fraction?
   k. Where do you see data represented as a percentage?
   l. Where do you see data represented as a decimal?

The circled questions above should open up a conversation on how to convert between fractions, decimals, and percentages. Students should provide the knowledge on how to do it. Discuss with students what they remember about this topic.

It will be interesting to open a dialogue on the last 5 questions from the list. In student's current lives, they will have all seen fractions and percentages on exam results. Some students will have very little experience with data as a decimal. An example for data represented as a decimal would be sports (especially baseball).

Candy Math

You Need

- One package of candy
- One large group paper
- One marker per member of group
- Knowledge from grade 8 ☺

Story

Have you ever bought a package of candy to find that your favourite flavour is barely represented? Let's look at some packages of candy to find out how many of each color there are in each one.

Your Task

1. Open a package of candy (don't eat it!) and count how many of each color you have.
2. Represent the data as fractions. Record the data on the board at the front of the room.
3. Arrange the fractions from smallest to largest.
4. What does each fraction mean? What do the denominator and numerator represent?
5. Represent the data as decimals.
6. Represent the data as percentages.

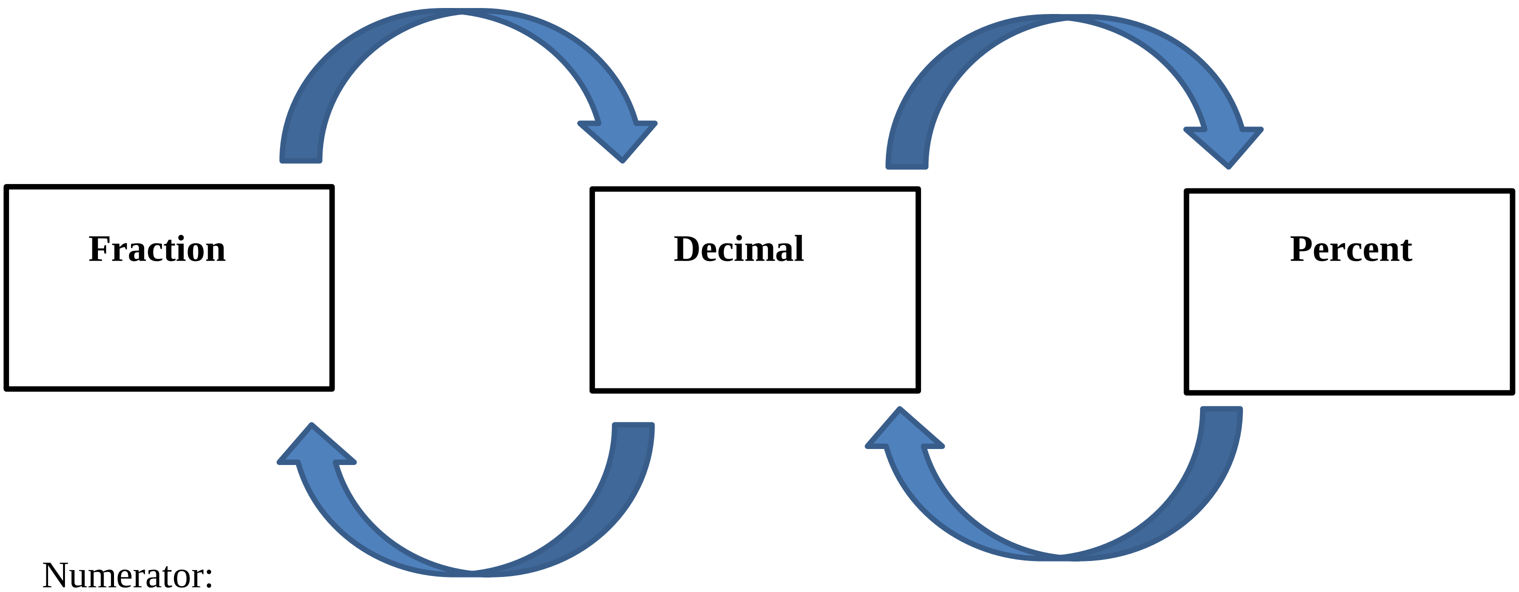
Challenge

Looking at all of the data from the entire class, how many candies were there in total? Calculate the percentages of each color represented for the entire class. How does it compare to your group's data?

Lesson 1: Remembering Skills Task **N9.2**

1. Put up magnetic signs (decimal, percent, fraction) onto the board.
2. Ask students to remember how to manipulate (convert) between all 3 (some of which was covered in the skittle task).
3. Students need to discuss **in groups** what they need to do in order to convert. It should look something like this.

Numerator ÷ denominator Multiply by 100



Numerator:
Move decimal ___ places to create whole number
Denominator (terminating decimal):
Place a 1 with ____ zeros behind it

Divide by 100

4. Have students copy down on a white sheet of paper and place in their binders for future use.
5. If time, give students fractions, decimals, and percentages and have them convert to another form.

Lesson 1: Quick Check (N9.2) Name:____________________________

Convert the following to a decimal:

a. $\frac{5}{8}$ b . $1\frac{3}{5}$ c. 48% d. 125% e. 0.04%

Convert the following to a percentage:

a. 1.75 b. $.445$ c. $\frac{32}{53}$ d. $\frac{6}{14}$ e. $\frac{3}{4}$

Convert the following to a fraction:

a. $.334$ b. $.02$ d. 55% e. 120% f. 0.08%

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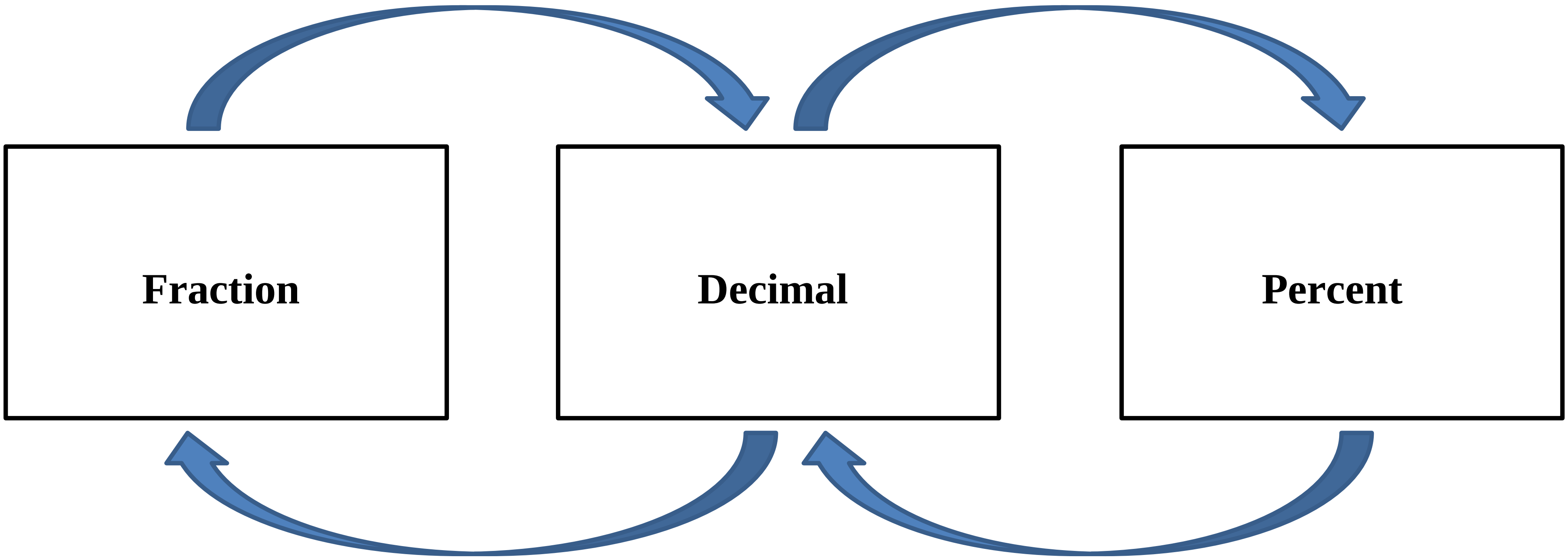
N9.2 Rational Numbers

Lesson 1: What is a Rational Number? **N9.2**

What is a Rational Number?



To The Teacher:

This is a very basic lesson. It should give you an idea of what skills and vocabulary students have by assessing their prior knowledge. When you ask students "what is a rational number?" and "what does equivalent mean?" they may have a tough time coming up with a definition, but they should be able to give examples of what they mean. From there, you can come up with a class definition for the terms. There is an emphasis on having students model rational numbers either on a number line or with a picture. It is a very simplistic lesson and depending of the skill level of the students, you may have plenty of time to start lesson 3. There is also a whiteboard activity that accompanies this lesson.

RN #2

Prior knowledge

RN #5

Formative assessment

RN #6

Representation

Lesson Process:

1. Ask students "what is a ratio?" Students should answer along the lines of "a comparison between two things." Ask students for some examples of what a ratio could look like. Possibilities are: 3:1 or $\frac{3}{1}$. **RN #2**

2. Ask students to provide a story for the ratio $2:5$. They may say "2 out of 5 people were girls" or "2 out of every 5 skittles are blue".

3. Ask students "what is a rational number?" The answer should be: any number that can be written as a ratio of integers OR any number that can be written as a fraction. Ask students for some examples of rational numbers. Make sure the list includes fractions, whole numbers, decimals, percentages, and negative values of all those types listed.

4. Whiteboard Activity: Depicting Rational Numbers Visually (in groups). Hand out whiteboards, markers, and eraser. **RN #5, RN #6**

a. Have students draw what a $\frac{1}{2}$ represents. Have students share their answers on the board. Hopefully circles, rectangles, discrete pictures, and number lines will be modelled. If not, ask students for other ways to show $\frac{1}{2}$.
b. Continue with #2 from the whiteboard questions. It might be a good idea to ask students to display a certain picture for each fraction. For example: use a number line to represent the first question. For the next question you could have them use a pie graph. For negative values, a number line is recommended. You may want to ask students why.
c. Students may need assistance, or a quick review, on how to depict fractions on a number line. Have students pay close attention to negative values and how they work on a number line.
d. Have students place 0.8, $\frac{8}{10}$, and $\frac{4}{5}$ on the same number line. This should lead to a discussion of **equivalence.**

5. **Equivalence Review (Student Workbook):** have students provide some examples of equivalent rational numbers. Include these in the student notes in the space provided. Having fractions compared to fractions as well as fractions compared to decimals are all great examples that students can provide.

6. Example 1: On the first example, have students use the pictures to record what each means as a fraction. The pictures are clearly equivalent (same amount shaded in each one). Hopefully students will make the connection as well.

7. Example 2: Students are then asked to shade the shapes according to the given fraction. This should lead to a discussion about what each number means in the fraction and how it translates to shading a picture. The importance of the numerator and the denominator.

8. Use the class notes to write in some information about fractions and what the numerator and denominator numbers mean. $a =$ numerator, how many parts we have. $b =$ denominator, how many total parts are represented.

$$\frac{a}{b}$$

9. Ask students: "How can we reduce fractions without changing the value?" Give students an example of $\frac{6}{10}$. How can we reduce that? The answer is to find a number that can divide into both 6 and 10. Have students check on a calculator that $6 \div 10$ is the same decimal as $3 \div 5$.

10. Example 3-5: Have students practice reducing fractions using examples #3, 4, 5. Note, example 5 is from the McGraw Hill & Ryerson Grade 9 textbook, page 47.

11. Ask students “How can we enlarge fractions without changing the value?” Give students the example $\frac{3}{5}$. How can we enlarge that? The answer is to choose ANY number and multiply it to both the 3 and the 5. Have students come up with a few different equivalent fractions and check them on a calculator to make sure they are equivalent.

12. Example 6: Have students practice enlarging fractions using example 6.

13. There is no assignment for this lesson.

What is a Rational Number?

A. Rational Number

Rational Number: ____

Provide some examples of rational numbers:

B. Background Essentials

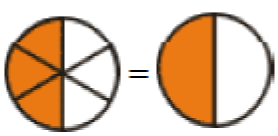
1. Equivalent:

____

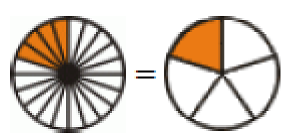
Provide some examples of rational numbers that are equivalent:

Example 1: Write down the simplification process

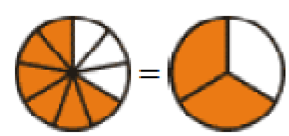
a.



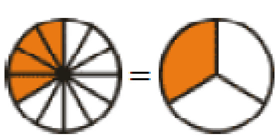
b.



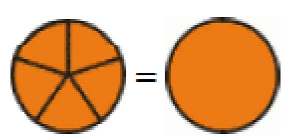
c.



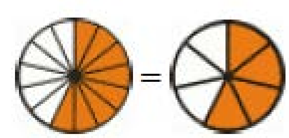
d.



e.



f.

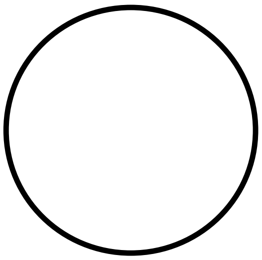


Example 2: Depict the following fractions using the following pictures.

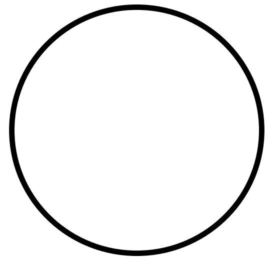
$\frac{1}{2}$

$\frac{6}{12}$

$\frac{2}{3}$



$\frac{8}{12}$



$\frac{3}{4}$

$\frac{9}{12}$

2. Fractions

   a. Numerator and Denominator

$$\frac{a}{b}$$

   b. Reducing Fractions/Enlarging Fractions

Reducing Fractions	**Enlarging Fractions**
• Some fractions can be reduced. They are another fraction in disguise. • Find a number that divides into both the numerator and denominator. It has to be the SAME for both. • It is customary to write fractions with the negative sign in the numerator or out in front of the fraction (if it is negative).	• You can enlarge fractions. • Pick any number and multiply it to both the numerator and denominator. It has to be the SAME for both. • It is customary to write fractions with the negative sign in the numerator or out in front of the fraction (if it is negative).

Example 3: Practice reducing fractions!

a. $\frac{5}{5}$ b. $\frac{15}{30}$ c. $\frac{36}{72}$ d. $\frac{125}{200}$

e. $-\left(-\frac{3}{10}\right)$ f. $\frac{10}{-40}$ g. $\frac{-3}{-4}$ h. $-\frac{-4}{9}$

Example 4: Some of the following fractions CANNOT be simplified, cross them out. Some of the following fractions CAN be simplified, simplify those.

$$\frac{2}{3} \quad \frac{2}{6} \quad \frac{6}{12} \quad \frac{6}{13} \quad \frac{7}{12} \quad \frac{11}{12} \quad \frac{11}{22} \quad \frac{9}{21} \quad \frac{9}{20} \quad \frac{6}{13} \quad \frac{14}{27} \quad \frac{14}{28} \quad \frac{14}{29} \quad \frac{8}{21} \quad \frac{8}{15} \quad \frac{8}{16} \quad \frac{8}{22}$$

Example 5: Identify equivalent rational numbers from the following list.

$$\frac{12}{4} \quad \frac{-8}{4} \quad \frac{-9}{-3} \quad -\frac{4}{2} \quad \frac{4}{-2} \quad \frac{12}{3} \quad -\left(\frac{4}{-1}\right) \quad -\left(\frac{-4}{-2}\right)$$

Example 6: For each rational number, write two fractions that represent the same number.

a. $\frac{1}{2}$ b. $\frac{8}{9}$ c. $\frac{-3}{4}$

Lesson 2: Whiteboard Activity **N9.2**

1. Represent the following using a visual

$$\frac{1}{2}$$

2. Represent the following rational numbers using a picture (circle, rectangle, discrete)

a. $\frac{6}{11}$ b. $\frac{3}{8}$ c. $\frac{8}{9}$ d. $1\frac{3}{4}$

e. $2\frac{7}{8}$ f. $\frac{5}{2}$ g. 0.6 h. $2\frac{3}{5}$

Lesson 3: Comparing and Ordering Rational Numbers **N9.2**

To The Teacher:

In the curriculum outcomes, it is clearly stated that students have to be able to compare and order rational numbers. Students should be able to use the skills learned/reviewed in the past few lessons to compare and order rational numbers. The knowledge of how to put a rational number on a number line and change rational numbers into decimals is crucial. This lesson is best done in groups. It might be a good idea to review how to depict numbers on a number line properly. Students may struggle with what the increments between the whole numbers should be. On the supplementary materials page at the end of the unit, there is a section on how to label fractional parts of a number line if you need. There is a whiteboard activity that goes well with that skill at the end of this lesson.

RN #7

Cooperative Learning

RN #5

Formative Assessment

Lesson Process:

1. **Student Workbook**: Example 1: Put $\frac{4}{9}$ and $\frac{3}{7}$ on the board. Ask students "how do we know which one is smaller and which one is larger?" Give students a minute to discuss different ideas. Ask students to provide their ideas to the class. The conclusion that students should come to is
   a. Give each fraction the same denominator so it is easier to compare:
   $$\frac{28}{63} \qquad \frac{27}{63}$$
   b. to turn each fraction into a decimal. Ask students "how do you know if .44444 is larger or smaller than .42857?" Some students may look it as money, thereby rounding it to two decimals: .44 is more money than .43. Ask students to share their ideas with the class.

2. Example 2: Work through with students. Have students compare the rational numbers as fractions. They may struggle with the repeating decimal ($0.\overline{3}$ =3/9). Hopefully students will see that comparing them as decimals is less work than converting them to fractions.

Have a discussion that $\frac{9}{10}$ and $-\frac{9}{10}$ are opposites on the number line. It is important to have a discussion on what increments to use between the whole numbers on the number line.

3. Have students complete the "on your own #1" in their groups. Choose a group(s) to come to the board and share their answers.
4. Example 3: This example is similar to the intro question. The difference is that both fractions are negative so students need to think hard when they decide which one is greater. The two different methods would be comparing as a decimal and comparing as a fraction (with same denominator).
5. On Your Own #2: Have students work through "on your own #2" individually and then have them discuss their findings with their group. They may choose to convert each fraction to a decimal and pick a decimal between the two values and then convert to a fraction. Or they may convert the question to a fraction and then pick a number between. If they choose the second option, they will need to change the denominator to something larger than 10 in order to find a fraction between them.
6. Choose a group(s) to come to the board and share their answers. **RN #7**
7. Example 4: Work though as a class or in their groups.
8. Example 5: Ask students to answer 5a in their groups. There are a couple of different methods that students can use.
   a. Some students will intuitively know that $\frac{1}{2}$ or $\frac{1}{3}$ are between the two fractions. Have them explain how they know to the class.
   b. Some students will convert to decimals and then convert that decimal to a fraction.
   c. Some students will give each of the fractions a common denominator and then pick a numerator that is between the two given numerators.

   Discuss with students the methods the method they chose.
9. Example 5: Have students complete 5b in their groups. Any of the methods above will work for this question. Make sure that students understand that the first common denominator (8) won't work. They need to change them both to 16 in order to find a numerator between the given numbers.
10. There is a whiteboard activity included. **RN #5**
11. Assignment: Lesson 2 & 3
    #2 and #3 are from McGraw-Hill and Ryerson Text Page 51.

Comparing Rational Numbers

Example 1: Which fraction is larger?

$$\frac{4}{9} \qquad \frac{3}{7}$$

Method 1:	Method 2:

Example 2: Compare and order the following rational numbers. Record the numbers on a number line.

$$-1.5 \qquad \frac{4}{5} \qquad \frac{9}{10} \qquad -0.\overline{3} \qquad -\frac{9}{10}$$

Solution:

Method 1:	Method 2:
Depict on a Number Line:	

On Your own #1:

Compare and order the following rational numbers from least to greatest. Record the numbers on a number line

$$1\frac{2}{5} \qquad -0.8 \qquad 2.1 \qquad -1 \qquad \frac{4}{5} \qquad -1.\overline{3}$$

______ ______ ______ ______ ______ ______

Example 3: Which fraction is greater, $-\frac{7}{8}$ or $-\frac{8}{10}$?

Method 1:	Method 2:

On Your Own #2:

Which fraction is smaller, $-\frac{5}{6}$ or $-\frac{9}{11}$?

Example 4: Identify a **fraction** between -0.5 and -0.6.

Example 5: For each pair of fractions, find one that is in between them. Make sure your answer is in fraction form, there is more than one answer.

a. $\frac{1}{6} < - < \frac{5}{9}$

b. $\frac{3}{8} < - < \frac{1}{2}$

Lesson 2 & 3 White Board Activity **N9.2**

1. Represent the following rational numbers using a number line.

a. $2\frac{1}{2}$ b. $-\frac{3}{4}$ c. $\frac{5}{6}$ d. $2\frac{7}{8}$

e. $\frac{9}{16}$ f. 1.2 g. $2\frac{9}{10}$ h. $1\frac{1}{3}$

i. $\frac{5}{8}$ j. $-\frac{2}{3}$ k. $-0.\overline{7}$ l. 0.35

p. $-2\frac{5}{9}$

2. Represent the following rational numbers **on the same number line.**

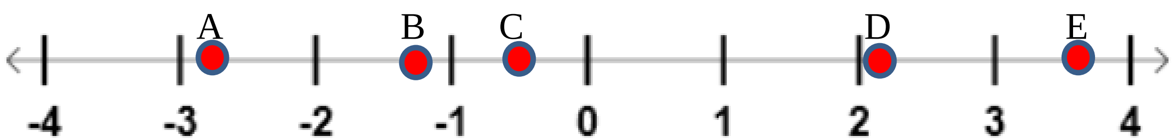
a. 0.8 b. $\frac{8}{10}$ c. $\frac{5}{4}$

Lesson 2 & 3: Assignment **N9.2**

1. Identify equal rational numbers in the list that follows. Write you answer in the same way as the example from the notes.

$$\frac{15}{3} \qquad \frac{-12}{4} \qquad -\frac{9}{3} \qquad -\left(\frac{-20}{4}\right) \qquad -\left(\frac{-6}{-2}\right) \qquad \frac{-10}{-2}$$

2. Match each rational number to a point on the number line.



a. $\frac{18}{5}$

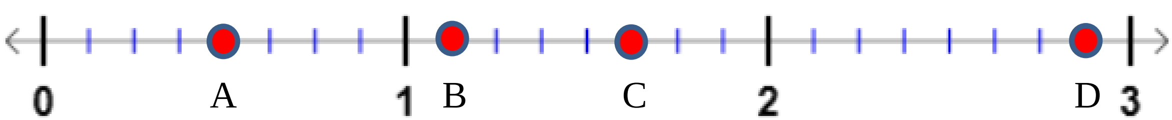
b. -0.5

c. $2\frac{1}{5}$

d. $-\frac{14}{5}$

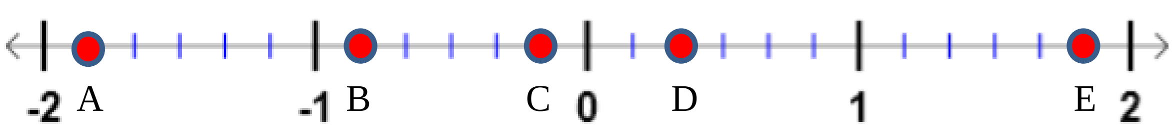
e. $-\frac{4}{3}$

3. Write the rational number represented by each letter on the number line, as a decimal.



A ________ B ________ C ________ D ________

4. Write the rational number represented by each letter on the number line, as a fraction. Ensure that all fractions are reduced.

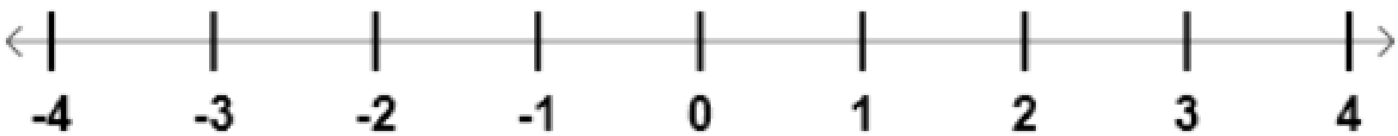


A ________ B ________ C ________ D ________ E ________

5. Place each number on the same number line. Be as accurate as possible. Label the dot.

a. -2.8 b. -1.7 c. $2\frac{2}{7}$ d. $-\frac{11}{3}$

e. $\frac{5}{6}$ f. 2.5 g. $\frac{2}{5}$ h. $-\frac{5}{8}$



6. Compare the following numbers and arrange in increasing order. Use the original numbers from the question in your answer.

$$\frac{3}{2}, -\frac{5}{4}, -1.5, -\frac{1}{3}, 0.9, 0.92, -0.\overline{6}$$

7. For each pair of fractions, find one that is in between them. Any fraction will do!

a. $\frac{2}{3} < - < \frac{3}{4}$ b. $\frac{1}{7} < - < \frac{1}{3}$

c. $\frac{4}{12} < - < \frac{4}{11}$ d. $\frac{1}{2} < - < \frac{7}{10}$

8. Identify a rational numbers (in the same form) between each pair of numbers.

a. 1.2, 1.3 (as a decimal) b. $\frac{3}{4}, \frac{4}{5}$ (as a fraction) c. $\frac{19}{21}, \frac{20}{21}$ (as a fraction)

Lesson 4: Fraction Addition & Subtraction **N9.2**

To The Teacher:

The first part of the lesson is to practice modelling fractions**.** They should have a strong understanding since they have practiced modelling fractions in the previous lesson. By modelling, students should come to realize that representing both fractions with the same denominator makes it easier to add and subtract. It is assumed that students will know how to change a mixed fraction into an improper fraction, so if you think they need some review, there are questions included in the supplemental materials.

RN #7

Cooperative Learning

RN #6

Representation

Lesson Process:

1. Students sit in groups. Hand out whiteboards, markers and erasers. **RN #7**
2. Ask students to draw 3 circles (representing pumpkin pies). **RN #6**
   a. Have them cut the first pie into quarters. How many people would that feed?
   b. Have them cut the second pie into eighths. How many people would that feed? Are the pieces smaller or larger than the first pie?
   c. Have them cut the third pie into sixteenths. How many people would that feed? Are the pieces smaller or larger than the other two pies?
   d. Have students represent $\frac{1}{2}$ of each of the pies mentioned above (quarters, eighths, and sixteenths) pie. What are the fractions created for each? The key idea is to represent that $\frac{2}{4}$, $\frac{4}{8}$, and $\frac{8}{16}$ are equivalent and mean the same thing.
3. Ask students to model the two fractions $\frac{1}{2}$ and $2\frac{1}{2}$ with a picture (circle or rectangle). Ask them to use the models and create another model that shows $2\frac{1}{2}+\frac{1}{2}$ and $2\frac{1}{2}-\frac{1}{2}$. After each one have a discussion with students about how each model works.
4. Ask students to model $\frac{1}{2}$ and $\frac{1}{4}$ with a picture (circle or rectangle). Discus with students how to add $\frac{1}{2}+\frac{1}{4}$. It isn't as easy to do. Some students will know intuitively $\frac{3}{4}$. Have them explain their understanding to the class. Ask students to model $\frac{1}{2}+\frac{1}{4}$. Let them

know that it is okay to manipulate their fractions. Students may or may not realize to change the $\frac{1}{2}$ into quarters.

5. **Student Workbook**: Ask students to model $\frac{5}{8}+\frac{3}{4}$. Students should model $\frac{5}{8}$ by drawing a picture, dividing it into eighths, and shading 5 parts. Students should then model $\frac{3}{4}$ by drawing a picture, dividing it into fourths, and shading in 3 parts. In order for students to add them, they will need to divide $\frac{3}{4}$ into eighths, which will make it $\frac{6}{8}$. When they add them together they will have $\frac{11}{8}$, which is $1\frac{3}{8}$.
6. Discuss with students how we used equivalent fractions to get both of them with the same denominator.
7. Ask a student to come up to the board and answer the question $\frac{5}{8}+\frac{3}{4}$ numerically. Since students come from different elementary schools, there may be different procedures that they follow. Ask students if they have a different procedure they would like to share.
8. Example 1: Model two different ways to work with mixed fractions (see teacher answered examples for clarification). Pay close attention to the negative and positive signs in the questions. Ask students what we should do (clean up the signs, add the opposite, etc.).
9. Example 2: Ask students where they see decimals in their day to day lives. The most obvious answers should be money, temperatures, percentages etc. Discuss with students the importance of estimation. When working through the examples, estimate each answer before evaluating. Write the estimated answer in the margin. It is your choice if you would like to use a calculator or have the students calculate each question by hand. Check the finalized answer with the estimated one to see if the estimation was close.
10. Assignment: Lesson 4

Adding & Subtracting Rational Numbers

A. Adding and Subtracting Fractions

1. Modelling: model the following expression pictorially

$$\frac{5}{8}+\frac{3}{4}$$

2. Mathematically: add the following expression numerically

$$\frac{5}{8}+\frac{3}{4}$$

We need the denominators of each fraction to be the same in order to add. Although you can add some fractions intuitively, there are others that require manipulation in order to add or subtract. LCD (lowest common denominator). There are times when the question can't be modelled because you aren't able to represent a negative fraction pictorially.

Example 1: Evaluate, give your answer in lowest terms.

a) $-\frac{5}{6}+\left(-\frac{3}{4}\right)$

b) $\frac{4}{5}-\frac{7}{8}$

c) $2\frac{4}{5}-\frac{1}{4}$

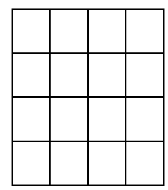
d) $-\frac{5}{11}+\frac{1}{8}$

c) $-3\frac{2}{3}+2\frac{3}{4}$

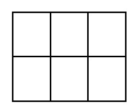
d) $-\frac{7}{8}-(-2)$

e) Assume that the entire figure shown represents 1 unit. Shade the appropriately to show the addition of :

$$\frac{5}{8}+\frac{1}{16}$$



$$\frac{1}{2}+\frac{1}{3}$$



B. Adding and Subtracting Decimals

A strong skill to have is estimation. When working with money it is always handy to estimate the cost of a number of items or a restaurant bill to make sure you have enough money before you order.

Example 2: Estimate and calculate.

a) $3.22+(-5.75)$

b) $-4.65-(-8.97)$

Lesson 4: Assignment **N9.2**

1. Evaluate, leaving your answer in lowest terms.

a. $\frac{6}{9}+\frac{2}{9}$ b. $\frac{1}{4}+\frac{1}{4}$ c. $\frac{3}{4}-\frac{7}{4}$ d. $-\frac{1}{4}-\frac{3}{4}$ e. $\frac{7}{15}-\frac{2}{15}$

2. State the lowest common denominator that would be used to evaluate the following questions. **Do not evaluate!**

a. $\frac{2}{3}+\frac{1}{2}$ b. $\frac{5}{8}+\frac{2}{3}$ c. $-\frac{3}{4}-\frac{1}{2}$ d. $\frac{4}{5}-\frac{3}{4}$ e. $\frac{7}{15}-\frac{3}{5}$

3. Evaluate, leaving your answer in lowest terms.

a. $\frac{4}{9}-\frac{4}{6}$ b. $-\frac{9}{20}-\frac{3}{20}$ c. $-\frac{2}{9}+\frac{5}{4}$

d. $\frac{3}{25}+\frac{7}{10}$ e. $\frac{3}{8}-\frac{4}{5}$ f. $\frac{5}{6}+\frac{2}{3}-\frac{1}{2}$

g. $-\frac{3}{4}+\left(-\frac{7}{8}\right)$ h. $-\frac{1}{2}+\frac{2}{3}$ i. $\frac{9}{4}-\frac{4}{5}$

j. $2\frac{3}{5} + 1\frac{1}{2}$

k. $\frac{1}{9} - \frac{1}{6}$

l. $-\frac{1}{6} - \frac{1}{4}$

m. $\frac{5}{6} - \left(-\frac{1}{8}\right)$

n. $-\frac{1}{10} - \left(-\frac{1}{12}\right)$

o. $\frac{1}{6} + \frac{3}{8}$

p. $-\frac{5}{6} + \frac{5}{8}$

q. $\frac{3}{10} - \frac{1}{4}$

4. Estimate and then evaluate.

a. $-5.3 + 2.4$

b. $6.55 + (-3.62)$

c. $-0.228 + (-14.8)$

d. $-32.55 + 14.76$

e. $50.25 + 120.56$

f. $-9.45 + 3.89$

5. Jessie practices running, and he is told to use 10 minutes for a warm-up and 10 minutes for a cool down. In total he only has 1 hour to complete the warm up, workout, and cool down.

a. What fraction of the total time is the warm up?

b. What fraction of the total time is the workout?

c. What fraction of the total time is the cool down?

5. Here is a monthly budget for a student. Write a mathematical expression that shows the calculation. What does your final answer mean?

Item	Income	Expense
Money from work	\$825.75	
Rent		\$625
Food		\$200
Car Payment		\$175.87
Insurance		\$144.23
Student Loan	\$350	
Gas		\$200

a. Expression and answer:

b. What does that number mean?

6. Below is a chart with some Saskatoon temperatures from the past year.

Date	Temperature on that Day	Average Temp for that Day	Difference between column 2 and column 1	What does it mean?
January 31st, 2013	−36.2°C	−19.7°C		
March 19th, 2013	−27.1°C	−8.8°C		
August 25th, 2013	33.8°C	23.3°C		
January 15th, 2013	4.4°C	−10.9°C		

Lesson 5: Fraction Multiplication **N9.2**

To The Teacher:

Students are going to be coming to your math classes with the assumption that multiplication means that the answer will be bigger than the numbers you started with. That is not always the case with fractions. Also many see multiplication as repeated addition. It works for 3×6 and even $5 \times \frac{1}{2}$. But what happens when we try to explain to students $\frac{1}{2} \times 5$ or $\frac{1}{2} \times \frac{5}{8}$? How do we model repeated addition in this case? This has led to a heated debate started by Keith Devlin. I strongly suggest you read it. Rather than thinking of it as repeated multiplication, think of it as scaling. It will really help students in their future math classes if they do. So if you have 3 and you are making it 6 times as a long. Or you have 5 and you are making it half as long. And in the other case, you have half, and you are making it 5 eighths the size. It is really important that students work in groups for this activity. Talk about putting multiplication into words (3 of 5, 5 of a half, half of 5, half of five-eighths).

RN #6

Representation

RN #9

Nix the Trix

RN #4

Inquiry

RN #3

Leaner Generated Examples

Indicator(s):

Lesson Process:

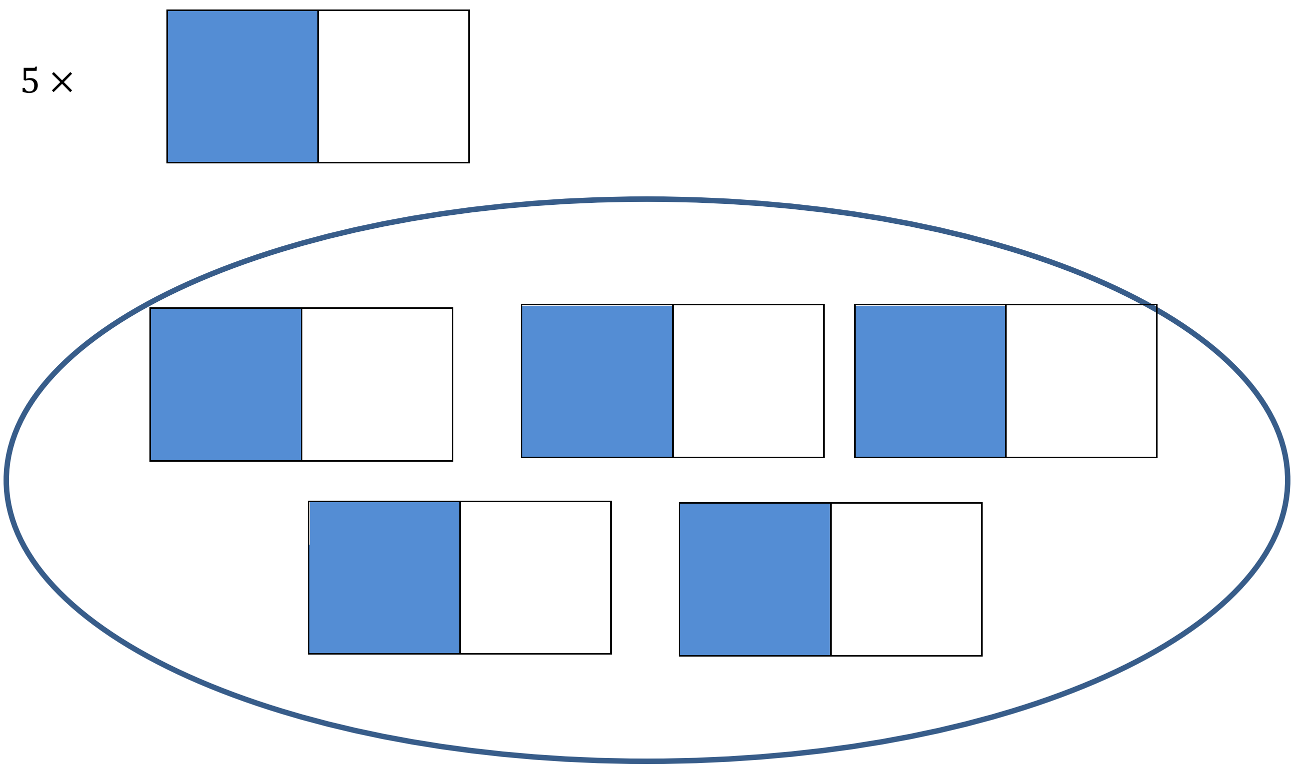
1. Ask students “what is multiplication?” This is often a tough question for students to answer. You might get the answer “a way to show repeated addition.” Please see **RN #9** for discussion on the topic**.** Because the idea of repeated addition falls apart when you have two fractions multiplied together, it might be best to talk about multiplication as a way to scale things. The symbol " $\times$ " represents “of”.

2. Example 1: Look at the question of 2×3. Ask students to translate it using words and represent it pictorially. So if you have 2×3, that translates to "2 of 3". Thus your 3 things are now twice as big. Pictorially it would look like: **RN #6**

2 groups of 3 things

3. You may want to do the majority of the lesson on the whiteboards. In this way, students can feel free to make mistakes because the whiteboard is not permanent. After the class agrees on a model, have them draw it in their student notebook.

4. Example 2a: Ask students to represent $5 \times \frac{1}{2}$ in words as well as pictorially. They can write "5 groups of ½" or "5 of ½". Pictorially it could look something like this:



Ask students what the answer to this question is. If students answer "5" they are forgetting that each portion represents a half. So we have 5 halves, what does 5 halves look like as a number $\left(\frac{5}{2}\right)$? How many wholes is that? The answer is two and a half $\left(2\frac{1}{2}\right)$. Record the answer in the student workbook (as well as in the example 3 table).

5. Example 2b-f: Complete the remainder of example 2 the same as 2a.

6. You may want to ask students why $4 \times \frac{2}{3}$ and $\frac{2}{3} \times 4$ are the same answer. This can lead to a discussion about the commutative property of multiplication.

7. Once example 2 is complete, have students fill in the chart with the answers they found in example 2. Ask students if they can see any patterns emerge. Some may notice that the order of multiplication doesn't matter (discussed above), others may see that the numerators multiplied together and denominators remained the same. Have students write their proposed algorithm in the space provided. (What is the pattern). **RN #4**

8. Example 3: Have students work on $\frac{1}{2} \times \frac{3}{4}$ using whiteboards. They must be able to write what it means in words as well as display it pictorially with a model. Once the class comes up with an agreed upon model, write it in their notes. There are more questions in example 3 to work through using the same procedure.

   b. $\frac{3}{4} \times \frac{1}{2}$ c. $\frac{2}{3} \times \frac{3}{4}$ d. $\frac{3}{4} \times \frac{2}{3}$ e. $\frac{3}{5} \times \frac{7}{4}$ f. $\frac{7}{4} \times \frac{3}{5}$

9. Once all the questions have been completed, have students transfer their answers into the chart. Asks students if they can see a pattern. Have the students write their proposed algorithm in the space provided. **RN #4**

10. You may want to ask students if the algorithm we generated from $\frac{a}{b} \times c$ and $c \times \frac{a}{b}$ is different from $\frac{a}{b} \times \frac{c}{d}$. They should realize that the c from the first algorithm is really $\frac{c}{1}$ so it is the same algorithm as $\frac{a}{b} \times \frac{c}{d}$, just that $d = 1$.

11. Ask students to create their own fraction multiplication question and share with a friend. Is there any type of question that we haven't encountered? Some students my try multiplying two mixed numbers together. **RN #12**

12. Example 4: After the algorithm of $\frac{numerator \times numerator}{denominator \times denominator}$ is established. Introduce negative fractions. Complete example 4 as a class.

13. Example 5: Lead a discussion on the importance of reducing before multiplying. Complete example 5 with students to show how the reducing before multiplying will save time.

14. Example 6: Complete with students.

15. Assignment: Lesson 5
    Question #3 and #4 are from McGraw-Hill & Ryerson Grade 9 Textbook Page 68

Multiplying Rational Numbers

1. What is multiplication?____

Example 1: What is 2×3? Write out what this expression means in words as well as pictorially.

In Words:	Pictorially/Model:

2. Multiplication of an Integer and a Fraction: $a\left(\frac{b}{c}\right)$ or $\left(\frac{b}{c}\right)c$

Example 2: Write out what each of the following expressions means in words as well as pictorially

a) $5\left(\frac{1}{2}\right)$ or $5 \times \frac{1}{2}$

In Words:	Pictorially/Model:

b) $4\left(\frac{2}{3}\right)$ or $4 \times \frac{2}{3}$

In Words:	Pictorially/Model:

c) $(3)\frac{7}{8}$ or $3 \times \frac{7}{8}$

In Words:	Pictorially/Model:

d) $\left(\frac{1}{2}\right) \times 5$

In Words:	Pictorially/Model:

e) $\left(\frac{2}{3}\right) \times 4$

In Words:	Pictorially/Model:

f) $\left(\frac{7}{8}\right) \times 3$

In Words:	Pictorially/Model:

Do you notice a pattern?

$5 \times \frac{1}{2}$	$4 \times \frac{2}{3}$	$3 \times \frac{7}{8}$	Algorithm:
$\frac{1}{2} \times 5$	$\frac{2}{3} \times 4$	$\frac{7}{8} \times 3$	

3. Multiplication of a Fraction and a Fraction: $\left(\frac{a}{b}\right)\left(\frac{c}{d}\right)$

Example 3: Write out what each of the following expressions means in words as well as pictorially

a) $\frac{1}{2} \times \frac{3}{4}$

In Words:	Pictorially/Model:

b) $\left(\frac{3}{4}\right)\left(\frac{1}{2}\right)$

In Words:	Pictorially:

c) $\left(\frac{2}{3}\right)\left(\frac{3}{4}\right)$

In Words:	Pictorially:

d) $\left(\frac{3}{4}\right)\left(\frac{2}{3}\right)$

In Words:	Pictorially:

e) $\frac{3}{5} \times \frac{7}{4}$

In Words:	Pictorially:

f) $\frac{7}{4} \times \frac{3}{5}$

In Words:	Pictorially:

Do you notice a pattern?

$\frac{1}{2} \times \frac{3}{4}$	$\left(\frac{3}{4}\right)\left(\frac{1}{2}\right)$	$\left(\frac{2}{3}\right)\left(\frac{3}{4}\right)$	$\left(\frac{3}{4}\right)\left(\frac{2}{3}\right)$
$\frac{3}{5} \times \frac{7}{4}$	$\frac{3}{5} \times \frac{7}{4}$	Algorithm:	

Example 4: Calculate using the algorithm. All answers need to be in reduced form.

a. $\left(\frac{2}{5}\right)\left(\frac{9}{4}\right)$

b. $\left(\frac{5}{6}\right)\left(-1\frac{1}{2}\right)$

c. $(-10)\left(\frac{3}{7}\right)$

d. $\left(-\frac{1}{4}\right)\left(-\frac{1}{2}\right)$

4. Reducing BEFORE Multiplying (only do this if you are wanting to save time)

Example 5: Find the product of $\left(\frac{15}{14}\right)\left(\frac{7}{65}\right)$. Make sure you reduce your answer.

Reduce AFTER Multiplying	Reduce BEFORE Multiplying

Example 6: Find each of the following products, writing your answers in lowest terms. Be sure to reduce first, then multiply.

a. $\left(\frac{5}{16}\right)\left(-\frac{8}{20}\right)$

b. $\left(\frac{12}{56}\right)\left(\frac{16}{24}\right)$

c. $\left(3\,\frac{3}{5}\right)\left(-2\,\frac{7}{9}\right)$

d. $\left(5\,\frac{1}{3}\right)\left(-2\,\frac{1}{4}\right)$

Lesson 5: Assignment **N9.2**

1. Calculate the following using a model.

a. $\frac{3}{4} \times \frac{7}{8}$

b. $\frac{5}{8} \times \frac{1}{2}$

c. $1\frac{3}{4} \times \frac{2}{3}$

2. Calculate. Write all answers in reduced form.

a. $\frac{9}{11} \times \frac{5}{6}$

b. $\frac{2}{5} \times \frac{1}{2}$

c. $\left(-\frac{6}{13}\right)\left(-\frac{1}{4}\right)$

d. $\left(\frac{8}{9}\right)\left(\frac{1}{6}\right)$

e. $\frac{2}{7} \times \frac{7}{4}$

f. $-\frac{5}{6} \times \frac{6}{5}$

g. $\left(\frac{9}{10}\right)\left(\frac{13}{15}\right)$

h. $\frac{9}{2} \times 5$

i. $\frac{10}{11} \times 1\frac{1}{2}$

j. $\left(\frac{7}{4}\right)\left(-1\frac{2}{3}\right)$

l. $\left(\frac{7}{2}\right)\left(\frac{6}{5}\right)\left(\frac{11}{6}\right)$

m. $\left(\frac{12}{5}\right)\left(\frac{15}{11}\right)(11)$

n. $2\frac{1}{3} \times 1\frac{1}{8}$

o. $\left(\frac{1}{12}\right)\left(4\frac{1}{2}\right)\left(\frac{5}{9}\right)$

p. $\left(\frac{2}{7}\right)\left(\frac{1}{4}\right)\left(1\frac{2}{3}\right)$

3. In everyday speech, *in a jiffy* means in a very short time. In science, a specific value sometimes assigned to a jiffy is $\frac{1}{100}s$. You just told someone you would be there "*in a jiffy*" but it really took you 5 minutes. How many jiffies was that?

4. Li and Ray shared a vegetarian pizza and a Hawaiian pizza of the same size. The vegetarian pizza was cut into eight equal slices. The Hawaiian pizza was cut into six equal slices. Li ate two slices of the vegetarian pizza and one slice of the Hawaiian pizza. Ray ate two slices of the Hawaiian pizza and one slice of the vegetarian pizza.

a) Who ate more pizza?

b) How much more did that person eat?

c) How much pizza was left over?

Lesson 6: Dividing Fractions **N9.2**

To The Teacher:

Division of fractions is difficult for students to model. Even teachers have a hard time modelling expressions such as $\frac{2}{3} \div \frac{7}{8}$ **RN #9.** The statement does not mean $\frac{2}{3}$ divided into $\frac{7}{8}$. By using the phrase "how many b's are in a" from the expression $a \div b$, I have attempted at trying to get students to discover the invert and multiply rule by drawing pictures of the expression. Rectangles are much easier to use than circles for pictures. I strongly recommend practicing this lesson before you teach it to students. I have had the awkward moment when we were modelling fractions and we decided to try division and I got completely lost. Honestly, I needed some time to really understand how the model represented the expression without reverting back to the "invert and multiply" algorithm. Depending on students understanding of division, most of them will identify with the quotitive perspective when dividing fractions. I made the choice to focus on the quotitive perspective for this lesson. It is important to show how having a common denominator works for division of fractions too. Once students see how the modelled answer follows the "invert and multiply" algorithm, it is important to have the conversation that dividing by 2 is the same thing as multiplying by $\frac{1}{2}$. There are two ways to answer division of fractions questions:

1. Model the problem
2. Find a common denominator
3. Invert and multiply.

The "invert and multiply" algorithm is most often used when dividing fractions. Modelling the problem takes too long, and finding a common denominator entails one more step than the "invert and multiply" algorithm.

RN #6

Representation

RN #1

Math-In-Context

RN #9

Nix the Trix

RN #10

Quotitive Division vs. Partitive Division

Lesson Process:

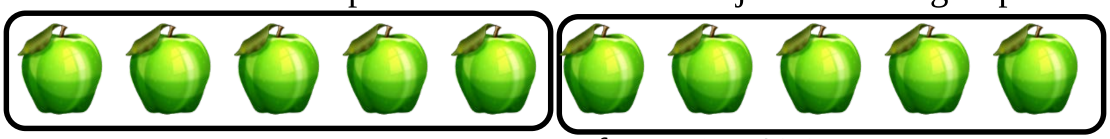
1. Example 1: Ask students to create a story and model for the expression $5 \div 10$ and $10 \div 5$. Ask them if they think their answer is going to be smaller or larger than the numbers in the story? Depending on how the student views the expression, you may get two different views. **RN #6, RN #1**

2. There are two ways to think about division: quotitive and partitive **RN #10**

$10 \div 5$

Quotitive: $10 \div 5$

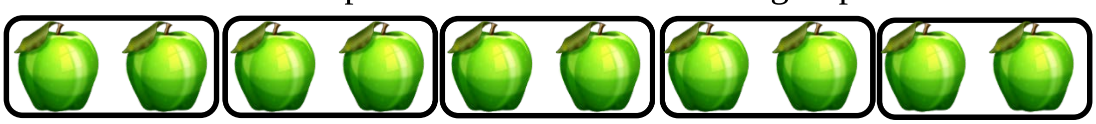
10 represents the total number of objects
5 represents the number of objects in each group



How many groups of 5 are in 10? 2

Partitive: $10 \div 5$

10 represents the total number of objects
5 represents the total number of groups



If I partition 10 into 5 groups, how many are in each group? 2

3. Before you begin division of fractions, ask students to answer $\frac{3}{4} \div \frac{1}{3}$. Students will either not remember how to answer, or they will invert and multiply. Have a student share their answer of $\frac{9}{4}$ or $2\frac{1}{4}$ and get them to share with the students how they found that. Then ask students if they are able to model how $\frac{3}{4} \div \frac{1}{3}$ has an answer of $\frac{9}{4}$ or $2\frac{1}{4}$. This is actually really difficult to do, and would make a great bonus question on a grade 12 exam. Do not show students how to model the question at this time (save that for later).

4. Investigate what will happen when we divide a fraction by a whole number. Ask students if they think the answer will be smaller or larger than what we started with.

5. Example 2: I have provided circles representing pizzas or pies. The shaded region represents how much is left. I then ask them to divide the shaded item up between people. Have them write an expression showing the math as well as a sentence

explaining their findings. Afterwards, gather the data and see if students can spot a pattern with the numbers. Create an algorithm. **RN #6, RN #1**

6. Example 3: Have the students try the algorithm by themselves. Have them model question d.

7. Example 4: Investigate what will happen when we divide a whole number by a fraction. Ask students if they think the answer will be smaller or larger than what we started with. Work through example 3 with the students by modelling the expression and answer. I enjoy working with rectangles more than circles. They are a little easier to partition. Gather the data and see if they can spot a pattern with the numbers. Create an algorithm. **RN #6, RN #1,**

8. Example 5: Have students try the algorithm by themselves. Have them model question d. You may want to point out that the questions in example 5 are just written in opposite order of example 3. Make sure that students realize that division is not commutative. Therefore $\frac{1}{5} \div 2$ is not the same as $2 \div \frac{1}{5}$.

9. Investigate what will happen when we divide a fraction by a fraction. Ask students if they think the answer will be smaller or larger than what we started with when the first fraction is larger than the second.

10. Example 6: Work through example 6 with students. A story that might work for the expression $\frac{1}{2} \div \frac{1}{4}$ is "how many ¼ cups are there in ½ of a cup?" **RN #1**

11. Using the same question as example 6b, show what happens when you create a common denominator before modelling.

12. Example 7: Have student work together to model and answer example 7. Have students share their answers with the class. **RN #6**

13. Continue the investigation of a fraction divided by a fraction. What happens when we switch it around and it is smaller fraction divided by larger fraction? Will the answer be larger or smaller than what we started with?

14. Example 8: Complete with the students and gather data to find the pattern and the algorithm for division of fractions. It is important to ask students "Did we just found 4 different algorithms for division of fractions? How are we going to keep it all straight?" Hopefully students will see that we have only found one algorithm.

    $\frac{a}{b} \div c$ is really $\frac{a}{b} \div \frac{c}{1}$, therefore equal to $\frac{a}{b} \times \frac{1}{c}$

$a \div \frac{c}{d}$ is really $\frac{a}{1} \div \frac{c}{d}$, therefore equal to $\frac{a}{1} \times \frac{d}{c}$

$\frac{a}{b} \div \frac{c}{d}$ is equal to $\frac{a}{b} \times \frac{d}{c}$

They are all invert and multiply!

15. It is extremely important to explain to students why inverting and multiplying work. Write the following two expressions on the board. Why are these expressions the same?

$$16 \div 2 \quad and \quad 16 \times \frac{1}{2}$$

Division by 2 and multiplication of $\frac{1}{2}$ yield the same result!

16. Example 9: This example is completely new to all students. They have not worked with negative fractions before. Students do not have to model these questions.

17. Example 10: This is a word problem involving fractions. Have students try on their own and discuss as a class.

18. Assignment: Lesson 6

Lesson 6: Dividing Fractions N9.2

1. What is division? ____

Example 1: Write a story that would work for the following expressions. Create a model that demonstrates how that division works.

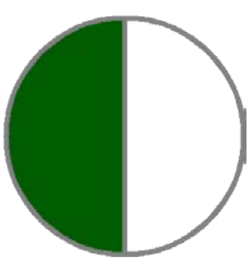
$5 \div 10$	$10 \div 5$
Story:	Story:
Model:	Model:

2. Division of a Fraction by a Whole Number $\frac{a}{b} \div c$

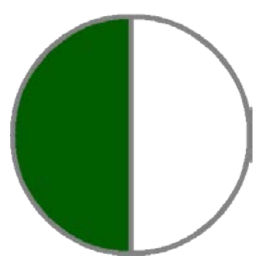
When you divide by a whole number, think of it as dividing something between people. The following pictures show how much pie is left (shaded region). If you are dividing equally between people, how much does each person get? **Write a division expression for each**.

Example 2:

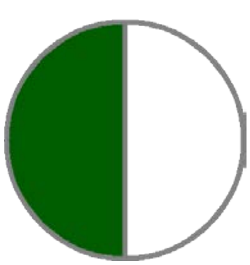
a. Divide between two people



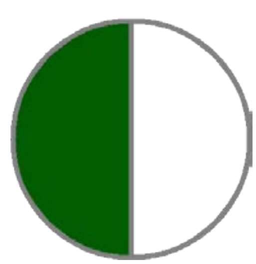
b. Divide between three people



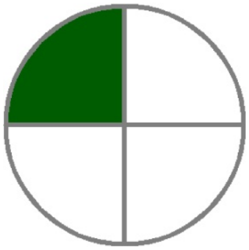
c. Divide between four people



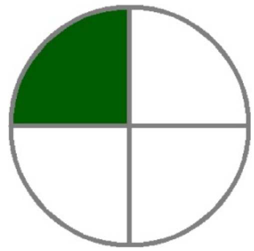
d. Divide between 5 people



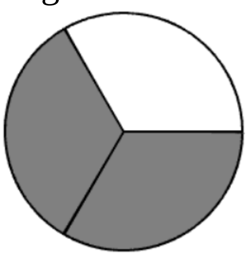
e. Divide between two people



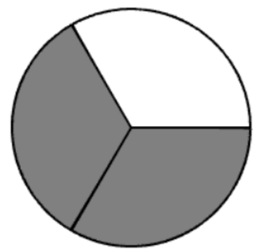
f. Divide between three people



g. Divide between two people



h. Divide between four people



Can you spot a pattern?

$\frac{1}{2} \div 2$	$\frac{1}{2} \div 3$	$\frac{1}{2} \div 4$	$\frac{1}{2} \div 5$	Algorithm:
$\frac{1}{4} \div 2$	$\frac{1}{4} \div 3$	$\frac{2}{3} \div 2$	$\frac{2}{3} \div 4$	

Example 3: Based on your observations, try the following. Draw a model for question d.

a. $\frac{1}{5} \div 2$ b. $\frac{1}{3} \div 3$ c. $\frac{2}{3} \div 2$ d. $\frac{4}{5} \div 4$

3. Division of a Whole Number by a Fraction:

$$c \div \frac{a}{b}$$

When you have a fraction as the second value, it is hard to imagine a fraction of a person. You can also think of division as "how many of the second number fits into the first?"

Example 4: Using the idea of rectangles or pies, calculate the following

a. $1 \div \frac{1}{4}$

b. $2 \div \frac{1}{3}$

c. $3 \div \frac{1}{2}$

d. $2 \div \frac{1}{4}$

e. $2 \div \frac{2}{5}$

Can you spot a pattern?

Example 5: Based on your observations, try the following. Check by drawing a picture that represents the question and answer.

$2 \div \frac{1}{5}$

b. $3 \div \frac{1}{3}$

c. $2 \div \frac{2}{3}$

d. $4 \div \frac{4}{5}$

4. Division of a Fraction by a Fraction

A. Bigger Value ÷ Smaller Value

Example 6: Write a story that would work for the following expressions. Create a model that demonstrates the division statement.

a. $\frac{1}{2} \div \frac{1}{4}$

Story:	Model:

b. $\frac{7}{8} \div \frac{2}{3}$

Story:	Model:

Finding a common denominator is a very useful skill for division of fractions. If you can put both fractions into the same increments, it makes it a lot easier to model. Let’s try the last example with a common denominator.

$$\frac{7}{8} \div \frac{2}{3}$$

Model:

Example 7: Model the following expressions with rectangles and evaluate.

a. $\frac{3}{4} \div \frac{1}{8}$ b. $\frac{2}{3} \div \frac{1}{2}$ c. $\frac{5}{4} \div \frac{2}{3}$

B. Smaller Value ÷ Larger Value

Example 8: What happens when we reverse the numbers? Use the idea of rectangles or pies to calculate the following.

a. $\frac{1}{8} \div \frac{3}{4}$ b. $\frac{2}{3} \div \frac{5}{4}$ c. $\frac{1}{2} \div \frac{2}{3}$

Can you spot a pattern?

$\frac{3}{4} \div \frac{1}{8}$	$\frac{2}{3} \div \frac{5}{4}$	$\frac{2}{3} \div \frac{1}{2}$
$\frac{1}{8} \div \frac{3}{4}$	$\frac{2}{3} \div \frac{5}{4}$	$\frac{1}{2} \div \frac{2}{3}$

What is the pattern?

Algorithm

Some division statements are hard to model (numbers are just too big or they have negative values). So an algorithm is used. This way you can compute the answer without having to draw a model.

Example 9: Let's add in some mixed numbers and negative signs. You will not be able to model negative fractions, so use the division algorithm.

a. $-1\frac{2}{3} \div \frac{5}{6}$

b. $-\frac{2}{3} \div \left(-2\frac{2}{5}\right)$

Example 10: Sara has $2\frac{1}{2}$ cups of chocolate chips to make cookies. The recipe uses $\frac{1}{3}$ cup of chips in each batch. How many batches of cookies can Sara make?

a) Model the problem.

b) Write an expression and show how to solve the problem.

Lesson 6: Assignment **N9.2**

1. For the following division statements
   a. Write out what each statement means in words
   b. Model and answer the statement WITHOUT using the division algorithm

a. $2 \div \frac{2}{3}$

b. $\frac{2}{3} \div 2$

c. $\frac{3}{4} \div \frac{1}{2}$

d. $\frac{1}{2} \div \frac{3}{4}$

2. Simplify, leaving your answers in lowest terms and as improper fractions, if applicable. You may invert and multiply OR find a common denominator and divide.

a. $\frac{3}{5} \div \frac{7}{8}$

b. $\frac{1}{8} \div \frac{1}{2}$

c. $\frac{9}{10} \div \frac{7}{5}$

d. $\frac{7}{10} \div \left(-\frac{4}{9}\right)$

e. $-\frac{5}{8} \div \frac{3}{4}$

f. $-\frac{1}{5} \div \left(-\frac{8}{15}\right)$

g. $-\frac{2}{3} \div \frac{5}{7}$

h. $\left(-1\frac{7}{8}\right) \div \left(-1\frac{1}{3}\right)$

i. $2\frac{2}{3} \div \frac{3}{4}$

j. $-5 \div \frac{2}{3}$

k. $\frac{2}{3} \div 5$

l. $24 \div \frac{3}{2}$

3. Sanjay works for the SPCA and has to buy food for the dogs. She bought $6\frac{1}{2}$ pounds of dog food. She feeds each dog about one-third of a pound. How many dogs can she feed with one bag?

   a) Model the problem.

   b) Write an expression and show how to solve the problem.

Lesson 7: Order of Operations with Fractions **N9.2**

To The Teacher:

There are not many notes for this lesson because it was mostly covered in lessons 1 to 6. I have included 3 examples to work through with students. Have students pay close attention to the signs of the rational numbers since this is new for them. A review of what the order of operations may be needed. I did not include any exponents in this lesson.

Lesson Process:

1. Review the order of operations with students. Depending on the order you have taught the course, they may not have had a chance to review yet. It was included in lesson 4 of N9.1 Exponents.

2. If students have already completed the exponent unit, you may want them to try some questions involving exponents. A great idea would be to have them generate a question in their groups. Gather the questions together and have the groups solve them.

3. Assignment: Lesson 7

B E D M A S

Example 1: Simplify $$-\frac{9}{4} \times \left(-\frac{10}{21}\right) \div \left(\frac{45}{7}\right)$$

Example 2: Simplify $$\frac{2}{3} + \left(-\frac{1}{4}\right) - \left(\frac{-5}{6}\right)$$

Example 3: Simplify $$\left(-\frac{5}{6} + \frac{2}{3}\right) \times \left(\frac{3}{4}\right) \div \left(-\frac{5}{6}\right)$$

Lesson 7: Assignment **N9.2**

1. Simplify

   a. $-\frac{2}{3}+\frac{1}{4}-\left(\frac{-5}{6}\right)$

   b. $\frac{3}{2}-\left(\frac{3}{8}\right)-\frac{3}{4}$

   c. $-\frac{7}{2}+1\frac{1}{3}-\left(-\frac{5}{6}\right)$

   d. $\frac{5}{9}+\frac{2}{3}+\left(-\frac{7}{6}\right)$

2. Simplify

   a. $\left(\frac{4}{9}\right)\times\left(\frac{-21}{-32}\right)\times\left(\frac{-3}{14}\right)$

   b. $\left(\frac{3}{4}\right)\left(\frac{8}{5}\right)\left(\frac{20}{9}\right)$

   c. $\left(-\frac{4}{9}\right)\div\left(\frac{5}{6}\right)\times\frac{3}{10}$

   d. $\left(\frac{15}{8}\right)\div\left(\frac{25}{16}\right)\div\left(-\frac{6}{5}\right)$

3. Simplify

a. $\left(\frac{5}{6}+\frac{2}{3}\right)\times\frac{4}{9}$

b. $\frac{7}{8}\left[\frac{4}{3}-\left(-\frac{5}{18}\right)\right]$

c. $\frac{4}{5}\times\left[\frac{3}{8}+\left(-\frac{4}{7}\right)\right]$

d. $\left(-\frac{5}{6}+\frac{2}{3}\right)\times\frac{3}{4}\div\left(-\frac{5}{6}\right)$

e. $\frac{3}{5}+\left(-\frac{2}{3}\right)\times\left[-\frac{3}{4}\div\left(-\frac{1}{2}\right)\right]$

f. $\left[\frac{7}{12}\div(-14)\right]-\frac{3}{8}\times\frac{5}{3}$

4. Which student correctly answered the following question? Find the student's mistake and give him feedback on his/her error.

$$\frac{3}{4} - \left(-\frac{1}{2}\right)\left(\frac{5}{8}\right) \div \left[\left(-\frac{1}{4}\right)\left(-\frac{1}{4}\right)\right]$$

Student #1

$$\frac{3}{4} - \left(-\frac{1}{2}\right)\left(\frac{5}{8}\right) \div \left[\left(-\frac{1}{4}\right)\left(-\frac{1}{4}\right)\right]$$

$$\frac{3}{4} - \left(\frac{-5}{16}\right) \div \frac{1}{16}$$

$$\frac{3}{4} + \frac{5}{16} \div \frac{1}{16}$$

$$\frac{17}{16} \div \frac{1}{16}$$

$$17$$

Student #2

$$\frac{3}{4} - \left(-\frac{1}{2}\right)\left(\frac{5}{8}\right) \div \left[\left(-\frac{1}{4}\right)\left(-\frac{1}{4}\right)\right]$$

$$\frac{3}{4} - \left(\frac{-5}{16}\right) \div \frac{1}{16}$$

$$\frac{3}{4} - \left(\frac{-5}{16}\right) \times \frac{16}{1}$$

$$\frac{3}{4} - (-5)$$

$$\frac{3}{4} + 5$$

$$\frac{23}{4}$$

Lesson 8: Working With Decimals **N9.2**

To The Teacher:

This lesson is optional. There is no mention in the curriculum guide specifically mentioning decimals. It is your decision if you feel comfortable having students work with a calculator or not. I have made space in the lesson for pencil work as well as calculator work.

Lesson Process:

1. Work together as a group and answer the examples. The first column is for pencil work, the second is for the calculator work.

2. Assignment: Lesson 8

Working With Decimals

A. Addition & Subtraction

What do you remember?

$0.28 + 3.2 + 2.339$		
$0.6 - 0.22$		

B. Multiplication

What do you remember?

$(0.03)(2.5)$		

C. Division

What do you remember?

$\frac{0.64}{0.002}$		

D. Order of Operations with Decimals

Example 1: $(2.3)(-.003) - (2.1)^2$

Example 2: $4 \div 0.2 - 8(0.25)$

Example 3: Find the error in the student's work. Give the student feedback on their error.

$$0.24 \div (-0.6) \div 2(12)$$
$$-0.4 \div 2(12)$$
$$-0.2(12)$$
$$-2.4$$

Correctly answer the question here:

Lesson 8: Assignment **N9.2**

1. Evaluate. Do not use a calculator.

   a. $2.2 - 4.32 - 6.5 + 3.45$

   b. $2.4 - (-5.5) \times (0.3)$

   c. $(-12.3) \div (-0.3) - 2.5 \div (-0.5)$

   d. $2.5 - 6.2 + 7.8 \div 0.2$

2. Evaluate using a calculator.

   a. $[-3.8 + (-0.9)] \times [7.2 - 4.7]$

   b. $\frac{79.12}{9.2}(-2.18 + 5.27)$

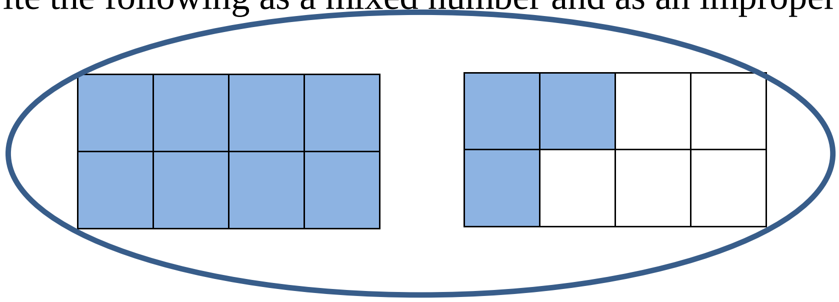
   c. $(-4.91) \times (-3.78) + \left(\frac{50.827}{-6.85}\right)$

   d. $(5.4)(-0.07) - (1.2)^2 \div (-0.3)$

Mixed Numbers

Mixed numbers have two parts. A part that tells you the wholes and a fractional part that is less than one whole. You can rewrite any mixed number as in improper fraction.

Example 1: Write the following as a mixed number and as an improper fraction:



As a mixed number: As an improper fraction:

Example 2: Draw pictures to illustrate these mixed numbers. Then write them as an improper fraction.

a. $1\frac{1}{2}$ b. $2\frac{2}{3}$ c. $1\frac{5}{6}$

Example 3: Draw enough circles or rectangles so you can make the picture, and write the fractions as mixed numbers.

a. $\frac{5}{4}$ b. $\frac{14}{6}$ c. $\frac{8}{5}$

The Shortcut (do you see it?)

To convert a mixed number into an improper fraction you can first multiply the whole to what kind of parts you have (denominator) and then add that to the numerator. That gives you total amount of whatever parts you have. The denominator stays the same

$$4\frac{3}{7}$$

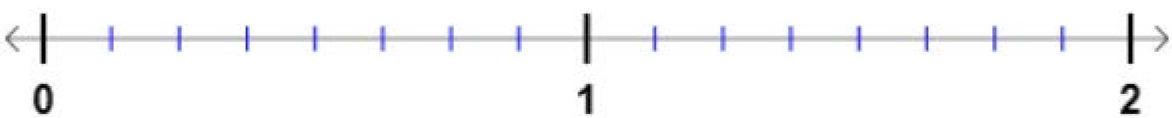
To convert an improper fraction into a mixed number, you need to find out how many times the denominator can fit perfectly into the numerator. That gives you the whole. The remainder becomes the numerator of the fraction. The denominator stays the same.

$$\frac{31}{7}$$

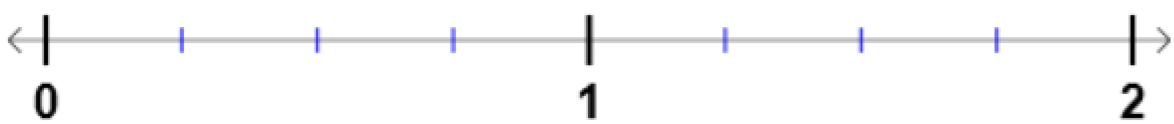
How to Label Fractional Parts of a Number Line

Example 3: Label the tick marks on the number lines. Between 1 and 2 label the fractional portions.

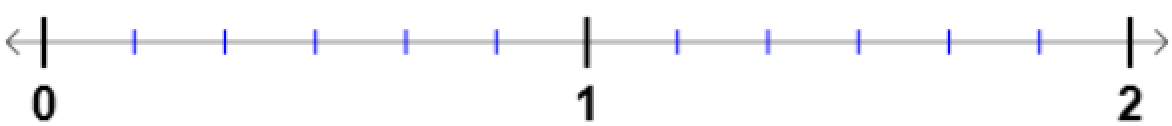
a.



b.



c.



Example 4: Shade each fraction on a number line and write in improper form (make sure you have the correct tick marks!)

a. $2\frac{5}{8}$ b. $2\frac{7}{12}$ c. $1\frac{2}{3}$

OUTCOME N9.3

Extend understanding of square roots to include the square root of positive rational numbers.

Previous Experience:

Grade 8

Outcome N8.1

Demonstrate understanding of the square and principle square root of whole numbers concretely or pictorially and symbolically.

Outcome SS8.1

Demonstrate understanding of the Pythagorean Theorem concretely or pictorially and symbolically and by solving problems.

Lesson 1: What is a Square Root? **N9.3**

To The Teacher:

Students should have knowledge about square roots from grade 8. This first lesson is to close gaps, or backfill, for students who have maybe forgotten the information. An important concept for these four lessons is linking the idea of "square roots" to the area and dimensions of a square. This is something that may have been discussed in grade 8.

RN #2

Prior knowledge

RN #5

Formative assessment

RN #1

Math-in-context

RN #7

Cooperative Learning

RN #6

Representation

Lesson Process:

1. Students should be in their learning groups for this lesson. Students will need whiteboards. **RN #7**

2. Begin with the whiteboard activity to assess students understanding of squares and how they are connected to actual squares (4 sides equal length). A conceptual error that may appear is multiplying the exponent of 2 with the base i.e. $3^2 \neq 6$. This is important to address before the lesson officially begins. Students may or may not be able to draw a visual of why 8^2 is 64 depending on their understanding and remembering of square roots from grade 8. If a student does display it correctly, have them explain their diagram to the class. **RN #2, RN #5**

3. **Student Workbook -** Example 1: Continuing from the idea of the last whiteboard question, have students explain how the diagram in their notes is related to $\sqrt{9}$. Have students brainstorm ideas, key words, and connections that they can see and remember. Some ideas are: 9 squares, area, square root, side lengths 3, perfect square, the side

lengths are the square root of 9, the number under the root represents area, the answer to $\sqrt{9}$ is 3 which is represented by the side lengths, $3^2 = 9$, etc. **RN #1**

4. Example 2: Have student draw their own diagram that represents $\sqrt{16}$. This is different from the whiteboard question because it is asking them to determine the side lengths of the square. However, the diagram is going to follow the same idea as the whiteboard question. **RN #6**

5. Example 3: Have students draw $\sqrt{12}$. Students should recognize that 12 is not a perfect square and can't be drawn exactly on the grid. However, it can be done as an approximation. This might stump students. Have them work in their learning groups for 5 minutes and see if they can come up with anything. **RN #13**

6. It is important to establish the terminology needed for this unit. Ask students for as much input as possible when completing this portion of the student notebook. It will be interesting to see how many students remember the Pythagorean theorem. I prefer to write the theorem as $leg^2 + leg^2 = hyp^2$.

7. Example 4: Students should fill out the chart in their learning groups without calculators. This example should be a review of grade 8 material (except for the last two rational numbers). Have students discuss in their groups what they think the square of the rational numbers and perfect squares look like.

8. Example 5: The relationship between surface area and square roots was established in grade 8 (and again in the introduction to the lesson). However, it is important to review the topic once again to make sure all students are on board before we progress through the lesson. Have students find the area of the square in example 5. Not only do they need to show their answer numerically, they need to pictorially represent their solution as well.

9. Example 6: Hand out rulers to students and have them construct a square with side lengths of 6 cm for example 6. Have them calculate and pictorially show what the surface area of the square is.

10. Example 7: Have students numerically find $\sqrt{36}$. You may want to translate the question. I have an area of 36, what is the square root (side lengths) of this square?

11. Ask students the question "How are square roots and the surface area of a square related?" Have students use the whiteboard and collaborate in their learning groups to come up with an answer. Have each group share their ideas with the class. Choose one, and have all students write it down in their student notebook.

12. Example 8: In their learning groups, have students generate their own perfect squares for example 8. Have them share with the class. Encourage them to be creative when choosing their root (decimals, fractions, integers)

13. Have students complete the exit question before dismissal. There is no assignment for this lesson.

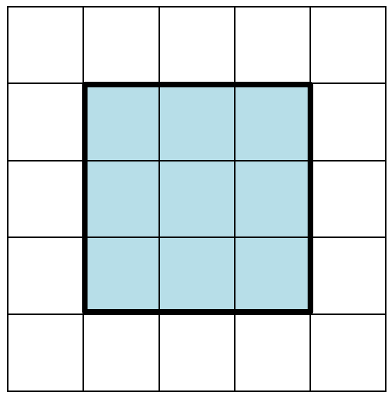
N9.3 Square Roots

Lesson 1: What is a Square Root?

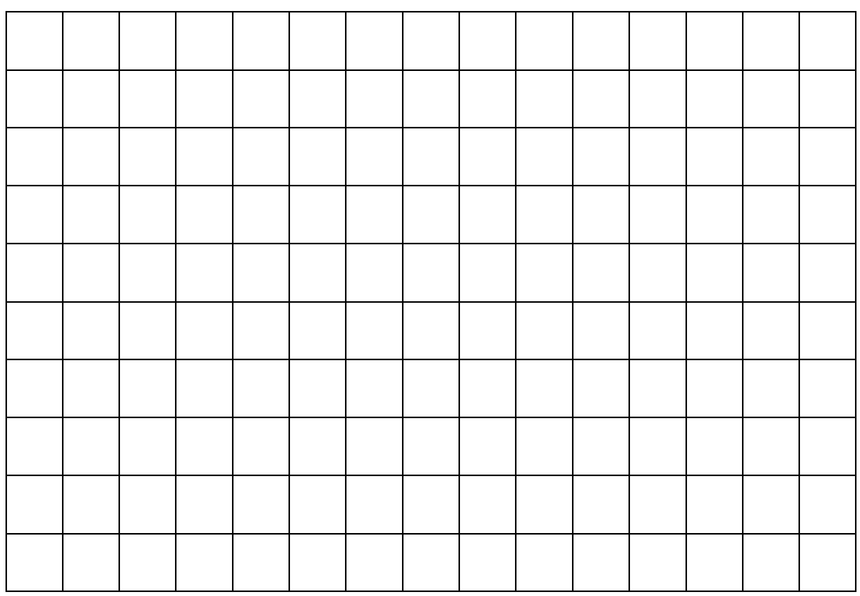
Terminology & Review

A. Square Roots & Terminology

Example 1: Explain how the shaded are in the diagram represents $\sqrt{9}$. What else can you tell me about the diagram?



Example 2: Draw a diagram that represents $\sqrt{16}$



Example 3: On the grid above, draw a diagram that represents an approximation of $\sqrt{12}$

1. Perfect Square: ____

____

2. Square Root: ____

____

3. Non-perfect Square: ____

____

4. Surface Area: ____

____

5. Pythagorean Theorem: ____

____

6. Rational Number: ____

____

B. Square Roots of Perfect Squares

Example 4: Complete the following chart.

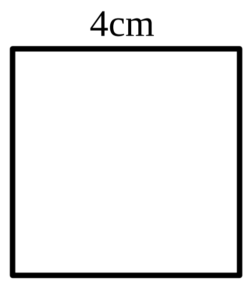
Square Root	Square of the Number	Perfect Square
1	1^2	1
2	2^2	
3		
4		
5		
6		36
7		
8		
9		
10		
11		
12		
$\frac{5}{8}$		
1.2		

The last two are new questions for you!

D. Square Roots and Surface Area

Square roots and the surface area of a square are related. Let's investigate!

Example 5: Find the area of the **square** below. Show how you got your answer numerically AND pictorially.



Example 6: Construct a square with side lengths of 6 cm. Calculate the surface area. Display your answer numerically AND pictorially.

Example 7: Calculate $\sqrt{36}$

How are square roots and the surface area of a square related?

Example 8: Provide 4 examples of perfect squares not already listed in the chart from example 3. How did you create them (you don't need to draw them)?

Exit Question (N9.3): Name:________________________

1. Use a ruler and draw a square with side lengths 4 cm.
   a. How is the area of the square related to the statement 4^2
   b. In your diagram show where the "16" comes from.

2. Calculate $\sqrt{256}$

Exit Question (N9.3): Name:________________________

1. Use a ruler and draw a square with side lengths 4 cm.
   a. How is the area of the square related to the statement 4^2
   b. In your diagram show where the "16" comes from.

2. Calculate $\sqrt{256}$

Lesson 1: Whiteboard Activity **N9.3**

What is the value of

a. 1^2
b. 2^2
c. 3^2
d. 4^2
e. 8^2

2. Using the back of the whiteboard, show a visual of why 8^2 is 64.

Lesson 2: Square Roots of Fractions and Decimals N9.3

To The Teacher:

Finding the square root of rational numbers is new for grade 9 students. Continuing with the method of modelling and the strategy of cooperative learning, students will gain a better understanding of square roots of rational numbers. It is important to establish with students that a perfect square can have a root that is a decimal or fraction. However, the decimal must be a terminating decimal. You may need to establish with students the difference between terminating and non-terminating decimals.

RN #5

Formative Assessment

RN #6

Representation

RN #3

Learner Generated Examples

RN #11

Conceptual Attainment

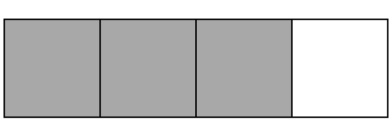
RN #1

Math-In-Context

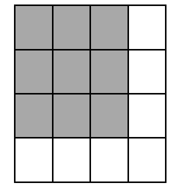
Lesson Process:

1. Have students sitting in their learning groups for this lesson. Students will need whiteboards. **RN #5**

2. Have students represent what $\frac{3}{4}$ looks like using a rectangular model. Ask, what does this mean in words? (3 of the 4 squares are shaded). Students may want to use the whiteboards and then transfer their answers to the notebook once they know they are correct. **RN #6, RN #1**



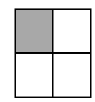
3. Ask students to now make a square using the rectangle as the length and the width. They may need help with this as it is a bit confusing for them to understand that 3 of the 4 are shaded for the width, and 3 of the 4 are shaded for the length.



4. Discuss how their pictorial answer translates to a numerical answer. (9 out of 16 are shaded). **RN #1**

5. Example 2: Have the students model example 2 using the whiteboards. Ask students how they translate the picture into numbers. Have them transfer their work to their notebooks.

6. Example 3: Have the students model example 3 using the whiteboards. This is an interesting question and it may prove to be difficult for some students to model because of the numerator of 1. Discuss as a class and have them translate their answer numerically. Students must then transfer their whiteboard work to their notebooks.



7. Ask students, how can we get the numerical result without drawing a picture. Hopefully students will see the pattern and be able to articulate that you square the numerator and denominator.

8. Example 4: Have students work in their learning groups to discuss which fractions from the list in example 4 are perfect squares. Have students share their answers with the class.

9. Ask students, how can we determine if a fraction is a perfect square? Students should realize that if the top and bottom (numerator and denominator) are BOTH perfect squares, then the fraction is a perfect square.

10. Example 5: Have students practice calculating square roots of fractions in example 5. There is one fraction that is not a perfect square.

11. Ask students, how can we build a perfect square with decimals? Discuss with students that you can create squares using numbers such as 3.5. Not every square in the world is made with whole numbers. Ask a student to come to the board and draw a square with dimensions 3.5×3.5. Ensure students understand that what was drawn was still a perfect square. Because students have only worked with whole numbers with respect to square roots, they are going to need time to rework the definition of square roots to include rational numbers.

12. Example 6: Have students use a calculator for example 6. Reiterate that the numbers we just created are ALL perfect squares. Ask students to take the square root of the numbers they just wrote down and they should find that they are the same as the question. Have students create their own perfect squares for example 7 in their learning groups. **RN #3**

13. Conceptual learning task: It is important to discuss when something is not a perfect square, and when it is. See the conceptual learning task included at the end of the lesson. The students have space in their workbook for this task. Place the roots given in the correct category and have students discuss why some of the numbers are perfect squares while other numbers that are similar are not perfect squares. Students should see that when the square root of the number terminates, then the square is perfect. When the square root of the number keeps going on and on, then the square is not perfect. **RN #11**

14. Example 8: Students need to realize that not all decimals are perfect squares even though they might look like it. In example 8 I have included a list of numbers that all use the number 4. If students don't see the pattern, ask them to go back and count the number of decimal places that each perfect square has. The decimal has to have an "even" number of decimal places. Ask students, why is that? Students may not be able to come up with a reason why. You may want to ask about place value.

    When you only have 1 decimal place, it is in the tenths position. Is 10 a perfect square (no)?

    When you have 2 decimal places, it is the hundredth position. Is 100 a perfect square (yes)?

    Etc.

15. Example 9: Have students answer and discuss as a class.

16. Before the end of class, have students complete the exit question.

17. Assignment: Lesson 2

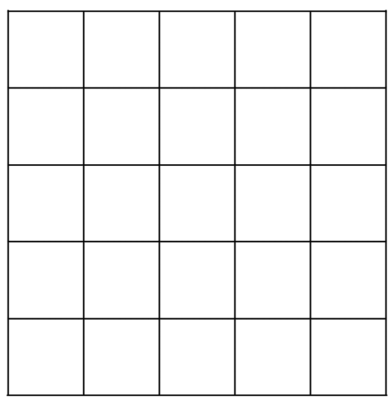
Square Roots of Fractions and Decimals

A. Square Roots of Fractions

Some fractions can also be perfect squares. Up until now, you have worked primarily with positive whole numbers (positive integers). If we can represent the area using squares, then it is a perfect square.

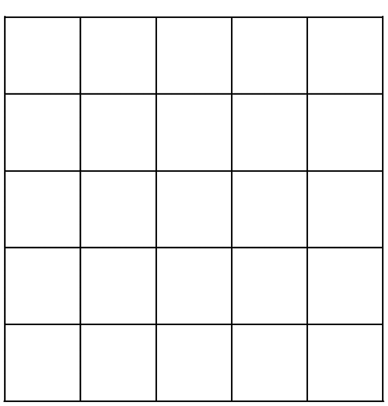
Example 1: a. Using a rectangular model, draw what $\frac{3}{4}$ looks like.

b. Using the grid below, model what $\left(\frac{3}{4}\right)^2$ looks like as a square.

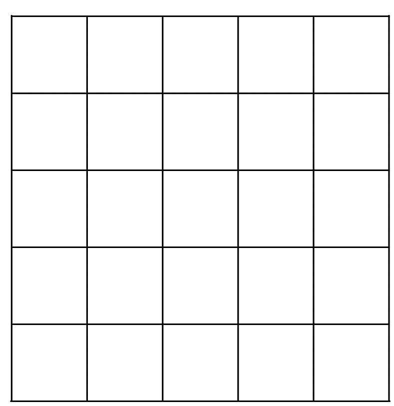


c. What is $\left(\frac{3}{4}\right)^2$? How is that represented in the above diagram?

Example 2: Model $\left(\frac{2}{3}\right)^2$



Example 3: Model $\left(\frac{1}{2}\right)^2$



When squaring fractions, how can we get the result without having to model the fraction?

$$\left(\frac{a}{b}\right)^2 =$$

Now that you know how to SQUARE a fraction, let's figure out how to determine if a fraction is a PERFECT SQUARE.

Example 4: Circle the following fractions that ARE perfect squares

$\frac{4}{9}$ $\frac{2}{3}$ $\frac{9}{16}$ $\frac{1}{2}$ $\frac{6}{10}$ $\frac{8}{50}$ $\frac{36}{49}$ $\frac{24}{16}$ $\frac{1}{25}$ $\frac{144}{225}$ $\frac{36}{99}$

How do we determine if a fraction is a perfect square?

Example 5: Calculate (if possible)

a. $\sqrt{\frac{16}{49}}$ b. $\sqrt{\frac{12}{25}}$ c. $\sqrt{\frac{1}{81}}$ d. $\sqrt{\frac{16}{144}}$

B. Square Roots and Decimals

Did you know that decimals can be perfect squares too? How can we build a perfect square with decimals? Let's square a few decimals and find out!

Example 6: Calculate the following

a. $(0.2)^2$ b. $(1.6)^2$ c. $(1.2)^2$ d. $(.005)^2$

Example 7: Create your own

a. b. c. d.

Conceptual Learning Task:

Perfect Square:	Non-Perfect Square:

How do we recognize a decimal that may be a **perfect** square?

Example 8: Find the square root of the following

a. $\sqrt{400}$

b. $\sqrt{40}$

c. $\sqrt{4}$

d. $\sqrt{0.4}$

e. $\sqrt{0.04}$

f. $\sqrt{0.004}$

g. $\sqrt{0.0004}$

All of the above questions contain a 4 (which is considered to be a perfect square). Why aren't all of the questions above perfect squares then?

____

____

How are we going to be able to recognize a decimal that is a perfect square without a calculator?

____

____

Example 9: Which decimal is a perfect square, 8.1 or 0.81?

Conceptual Learning Task:

Place the following numbers on the board under the following categories and have students calculate their square roots.

Perfect Squares:

$$\sqrt{36}, \sqrt{100}, \sqrt{0.25}, \sqrt{1.44}, \sqrt{0.0064}, \sqrt{8.41}, \sqrt{30.8025}$$

Non-Perfect Squares:

$$\sqrt{3.6}, \sqrt{5}, \sqrt{111}, \sqrt{2}, \sqrt{2.5}, \sqrt{5.5}, \sqrt{234.33}, \sqrt{14.4}, \sqrt{8.1}$$

Have students list the most common characteristic of the roots from the perfect squares compared to the roots of the non-perfect squares.

Assignment: Lesson 2 **N9.3**

1. Which decimals are perfect squares? You ARE NOT allowed to use a calculator! Circle the ones that are perfect squares.

a. 0.18 b. 0.4 c. 3.6 d. 0.36 e. 1.25 f. 2.25 g. 0.225

2. Which fractions are perfect squares? You ARE NOT allowed to use a calculator! Circle the ones that are perfect squares.

a. $\sqrt{\frac{1}{63}}$ b. $\sqrt{\frac{25}{49}}$ c. $\sqrt{\frac{8}{81}}$ d. $\sqrt{\frac{16}{25}}$ e. $\sqrt{\frac{144}{225}}$ f. $\sqrt{\frac{72}{50}}$ g. $\sqrt{\frac{100}{169}}$

3. Calculate the side length of each square from its given area.

a. $900\ m^2$ b. $0.09\ cm^2$

4. Which of the following ARE perfect squares?
   a. For those that are, find the square root.
   b. Explain why the others are NOT perfect squares.

900 90 9 0.9 0.09 0.009 0.0009

5. Calculate the square root of the following WITHOUT A CALCULATOR.

a. $\sqrt{\frac{36}{100}}$ b. $\sqrt{\frac{49}{144}}$ c. $\sqrt{\frac{1}{81}}$ d. $\sqrt{\frac{9}{25}}$

6. Calculate the square root of the following WITHOUT A CALCULATOR.

a. 0.01 b. 0.25 c. 1.69 d. 0.04

Lesson 2: Exit Question (N9.3) Name:________________

1. Explain what makes 0.81 a perfect square and 0.081 not a perfect square.

2. Calculate $\sqrt{\frac{1}{36}}$

Lesson 2: Exit Question (N9.3) Name:________________

1. Explain what makes 0.36 a perfect square and 0.036 not a perfect square.

2. Calculate $\sqrt{\frac{1}{36}}$

Lesson 2: Exit Question (N9.3) Name:________________

1. Explain what makes 0.36 a perfect square and 0.036 not a perfect square.

2. Calculate $\sqrt{\frac{1}{36}}$

To The Teacher:

This lesson builds off of the ideas from lesson 2. Students have concluded that non-perfect squares will have non-terminating decimals and perfect squares will have terminating decimals. Students are now going to practice estimating the root of non-perfect squares by using benchmarks (no calculators). I have included space where they check how close they are by using a calculator. It is important to stress to students that it is okay if their answers are off by a little bit. Students used benchmarks in grade 8 to estimate square roots of whole numbers that were non-perfect squares, so they should have an understanding of benchmarks. What is new is estimating the square root of non-perfect squares involving rational numbers.

RN #7

Cooperative Learning

Lesson Process:

1. Have students in their learning groups for this lesson. Hand out whiteboards. **RN #7**

2. **Student Workbook:** Building off of last lesson, ask students to provide some examples of numbers that are perfect squares in the categories of whole numbers, fractions, and decimals. Continue that idea into non-perfect squares. Place those answers in their student workbooks.

3. Because all of the non-perfect squares resulted in non-repeating decimals, ask students what they think about $0.\overline{3}$. Is it perfect or non-perfect? Even though it repeats with a pattern, it is still considered non-perfect. If you look at its equivalent fraction, the 3 in the denominator is not a perfect square (even though the numerator is).

4. Ask students what they believe is a benchmark. Students used benchmarks in grade 8 so they should be familiar with the terminology.

5. Example 1: Estimating square roots using benchmarks on whole numbers is from grade 8. Ask students how they are going to estimate the value of $\sqrt{30}$. Students should remember that they need to find the perfect squares on either side of 30 (25 and 36). Therefore, the square root is going to be between 5 and 6. You may want to create the following.

$\sqrt{25}$	$\sqrt{36}$
5	6

Have students estimate where $\sqrt{30}$ fits between (closer to $\sqrt{25}$ or $\sqrt{36}$) and then have them estimate the decimal.

6. Example 2: Repeat the above procedure for example 2.

7. Example 3: Extend students' knowledge of benchmarks to include benchmars for decimals and fractions. Have students find on the given number line where the roots of each number would be.

8. Example 4: For $\sqrt{13.7}$ what are the perfect squares that sandwich 13.7?

$$\begin{array}{cc} \sqrt{9} & \sqrt{16} \\ \hline 3 & 4 \end{array}$$

Students should estimate where on the line $\sqrt{13.7}$ would be in comparison to $\sqrt{9}$ and $\sqrt{16}$. Check with a calculator to see how close they are.

9. Example 5: Have students repeat the same process to estimate $\sqrt{1.52}$. Check with a calculator.

$$\begin{array}{cc} \sqrt{1} & \sqrt{4} \\ \hline 1 & 2 \end{array}$$

10. Example 6 & 7: Using the idea of benchmarks, estimate the value of $\sqrt{\frac{10}{47}}$. Have students check their estimate with a calculator.

11. Example 8 & 9:Using the idea of benchmarks, estimate the value of $\sqrt{\frac{30}{12}}$. Have students check their estimate with a calculator.

12. Assignment: Lesson 3

Square Roots of Non-Perfect Squares

A. What is a Non-Perfect Square Root?

Perfect Squares		
Whole Numbers	Fractions	Decimals
Non-Perfect Squares		
Whole Numbers	Fractions	Decimals

What about repeating decimals, are they perfect or non-perfect squares? i.e. $0.\overline{3}$

____

____

B. Estimating Non-Perfect Squares of Whole Numbers, Decimals, and Fractions

i. Whole Numbers

Example 1: Without using a calculator, estimate the value of $\sqrt{30}$

Example 2: Without using a calculator, estimate the value of $\sqrt{18}$

> We need to use benchmarks to find estimates of non-perfect squares. The benchmarks will be two perfect squares that sandwich your number.

Example 3: Between which two perfect squares would you place each value? Don't use a calculator.

Root	$\sqrt{0}$	$\sqrt{1}$	$\sqrt{4}$	$\sqrt{9}$	$\sqrt{16}$	$\sqrt{25}$	$\sqrt{36}$	$\sqrt{49}$
Square	0	1	2	3	4	5	6	7

a. $\sqrt{6}$ b. $\sqrt{14}$ c. $\sqrt{27}$ d. $\sqrt{45}$

ii. Decimals

Let's use the idea of benchmarks from above and extend it to decimals.

Example 4: Without using a calculator, estimate the value of $\sqrt{13.7}$

Example 5: Without using a calculator, estimate the value of $\sqrt{1.52}$

iii. Fractions

Let's use the idea of benchmarks again to help us with fractions.

Example 6: Without using a calculator, **estimate** the value of $\sqrt{\frac{10}{47}}$

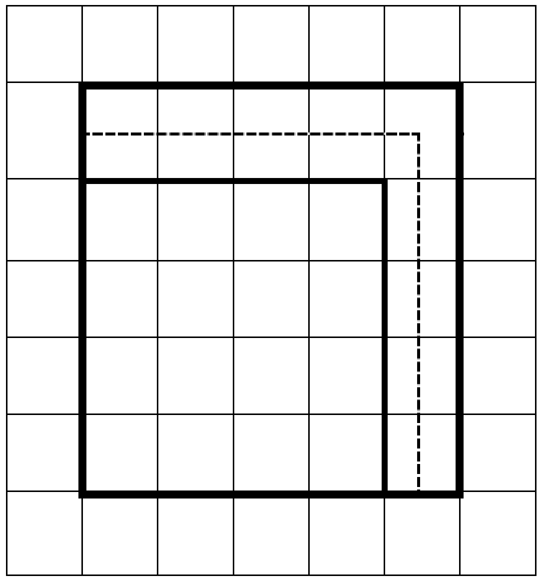
Example 7: Using a calculator, **calculate** the value of $\sqrt{\frac{10}{47}}$

Example 8: Without using a calculator, **estimate** the value of $\sqrt{\frac{30}{12}}$

Example 9: Using a calculator, **calculate** the value of $\sqrt{\frac{30}{12}}$

Assignment: Lesson 3 **N9.3**

1. Use the diagram to identify a rational number with a square root between 4 and 5 (dashed line). Using the diagram, what is the square of that number? Mark on the diagram how you found it. Check with a calculator.



2. Identify the rational number which has a square root of

   a. 0.22 b. 0.5 c. $\frac{5}{8}$ d. $\frac{1}{2}$ e. $\frac{1}{6}$

3. Show using benchmarks how you would estimate the square of 4.3.

4. Show using benchmarks how you would estimate the square root of 4.3.

5. What makes question #3 and #4 different because they look the same? Did I accidently type out the same question twice, what is going on here?

6. What would be the area of a square that has side lengths of $5.2m$?

7. Estimate each square root to one decimal place. Then, calculate it to the specified number of decimal places.

		Estimate to 1 decimal place (nearest tenth)	Calculate to 1 decimal place (nearest tenth)
a.	$\sqrt{42}$		
b.	$\sqrt{2.5}$		
c.	$\sqrt{0.96}$		
d.	$\sqrt{0.82}$		

8. If the area of the square is $1.96\ m^2$, what are the side lengths? Round your answer to two decimal places (nearest hundredth).

9. Use any strategy (besides a calculator) to estimate the value of each square root. You must explain and write out the strategy you used.

a. $\sqrt{6.8}$ b. $\sqrt{\frac{8}{32}}$ c. $\sqrt{\frac{60}{27}}$

10. Calculate the value of each square root to 2 decimal places (nearest hundredth)

a. $\sqrt{6.8}$ b. $\sqrt{\frac{8}{32}}$ c. $\sqrt{\frac{60}{27}}$

Lesson 4: Applications of Square Roots **N9.3**

To The Teacher:

Students should have an understanding of the Pythagorean Theorem. It is a stand-alone outcome in grade 8. I strongly recommend using $hyp^2 = leg^2 + leg^2$. Often students get wrapped up in which one is a and which one is b when using $c^2 = a^2 + b^2$. When you use hyp and leg instead of variables, it makes it more concrete for students. This way they can see that it doesn't matter which leg gets put in where.

RN #7

Cooperative Learning

RN #2

Prior Knowledge

Materials Needed (4): Scissors, Blank Paper, Ruler, Pythagorean Theorem Cut-out Page,

Lesson Process:

1. Have students in their learning groups for this lesson. Students will need scissors and rulers for this lesson. **RN #7**

2. Ask students what they know about a right triangle. Have them brainstorm what words come to mind with right triangles (hypotenuse, legs, Pythagorean Theorem, 90° angle, etc.). **RN #2**

3. It is important to note that the right triangle in the student workbook has side lengths of $3\ cm$, $4\ cm$, and $5\ cm$.

4. Discuss Pythagorean Theorem. Students may provide $c^2 = a^2 + b^2$, but you may want to stress $hyp^2 = leg^2 + leg^2$. Ask them what does this mean? The key is for them to understand, if our hypotenuse is $5\ cm$, what does that mean when we SQUARE $5\ cm$? How can we draw 5^2? Students should be able to answer back that we can draw a square with side lengths of 5 cm. Have students construct a 5×5 square on a blank sheet of paper and cut it out. Continue the same idea for the legs of the triangle ($3\ cm$ and $4\ cm$).

5. This next activity comes from Raymond Smullyan's book "5000 B.C. and other Philosophical Fantasies". Have the students put the cut out squares onto the corresponding sides of the triangle in their workbook. Ask students the following question:

"Suppose the squares were made of beaten (flat) gold. If you were offered either the one large square or the two small squares (put together), which would you choose?"

It is important that you make note of the squares being flat. Remember, the formula is squared, thus area of the squares (not volume) is what we are focusing on. Have students discuss in their groups which would be their choice and have them write their consensus on the board. Some students may realize that they represent the same amount and that either choice is valid. Have each group defend their choice and reach a consensus as a group. Some students may be surprised that the two amounts are in fact equal even though they have used the Pythagorean Theorem before.

6. Handout the "Pythagorean Theorem: Dissection Activity" cut out page. The purpose of the activity is for students to see that if they cut up the two smaller squares, the pieces will fit into the larger square perfectly. The website from which I found the worksheet is located on the bottom of the activity page.
7. **Alternate Activity to #6:** Handout the "Pythagorean Theorem Task" page. Instead of cutting out along pre-determined lines, like #6, the students can try cutting the squares without any guidance. I strongly recommend the students colour the squares with different coloured markers or pencil crayons. This activity will take longer than #6 and you may want to have more copies available for students that need a do-over.
8. It is important to have a discussion with students regarding the purpose of doing the activity. Students need to articulate what the Pythagorean Theorem means in words rather than regurgitating "a squared plus b squared equals c squared".
9. Example 1: The hypotenuse is the unknown. Students may have an easier time with this example because the hypotenuse is already isolated. Students may or may not know what to do when you end up with $hyp^2 = 325$. Ask students if they know what number squared gives them 325. Is there any way that we can find it? If students are still struggling, ask them what number squared equals 25? What process did they go through to find that? How do we undo squaring of a number? Students will need to realize that they must take the square root of both sides in order to find the root.
10. Example 2: The hypotenuse is known. Students are going to need to remember how to solve simple equations from grade 8 since this unit typically comes before solving equations in grade 9.
11. Work with the students to complete example 3 and 4.
12. Assignment: Lesson 4

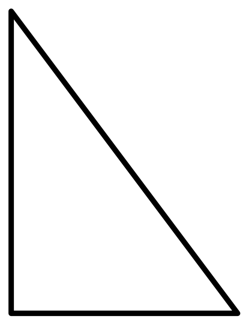
What does this mean?

Applications of Square Roots

A. Pythagorean Theorem

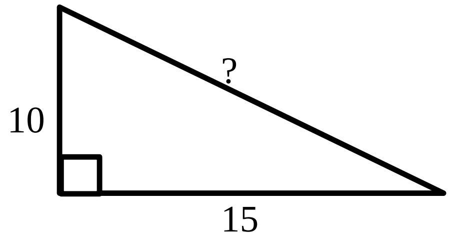
Pythagorean Theorem is a rule which states that, for any right triangle, the **area of the square** on the hypotenuse is equal to the **sum of the area of the squares** on the other two sides (legs).

What does that look like?

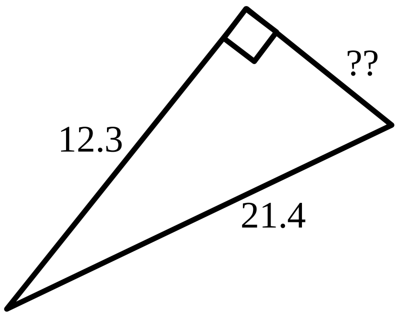


Pythagorean Theorem:______________________

Example 1: Use the Pythagorean Theorem to calculate the missing side of the right triangle.



Example 2: Use the Pythagorean Theorem to calculate the missing side of the right triangle.



B. When Would I use Pythagorean Theorem?

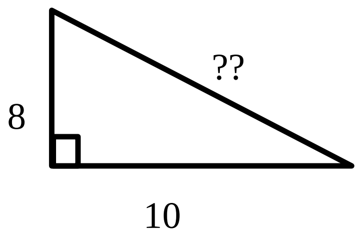
Example 3: Your TV just died and you need to purchase a new one. The spot on your entertainment system will fit a TV that has a width of 55 inches and a height of 33 inches. What is the maximum size of TV you can buy? Note: All TV's are listed by their diagonal size. Ex: A 32" TV has a diagonal of 32".

Example 4: A 20 foot ladder is leaning against a wall. If the base of the ladder is 2.5 feet away from the wall, how far does the ladder reach up the wall?

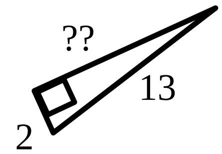
Assignment: Lesson 4 **N9.3**

1. Calculate the missing sides from each of the triangles. Show all your work, including the Pythagorean Theorem.

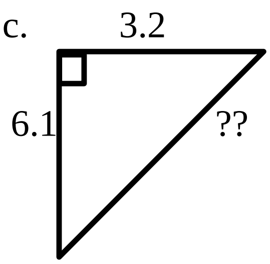
a.



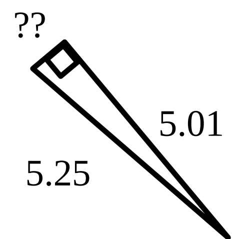
b.



c.



d.



2. If a 22 foot ladder is 3 feet from the base of the house. How far does the ladder reach up to the top of the house?

3. If you need to buy a ladder to reach up 18 feet and it needs to be placed at least 2.5 feet from the base of the house, what size of ladder do you need?

4. The length of the hypotenuse on an isosceles right triangle is 15. What are the lengths of the legs?

5. You are wanting to put a flat screen TV above your fireplace at home. You measured the dimensions to be:

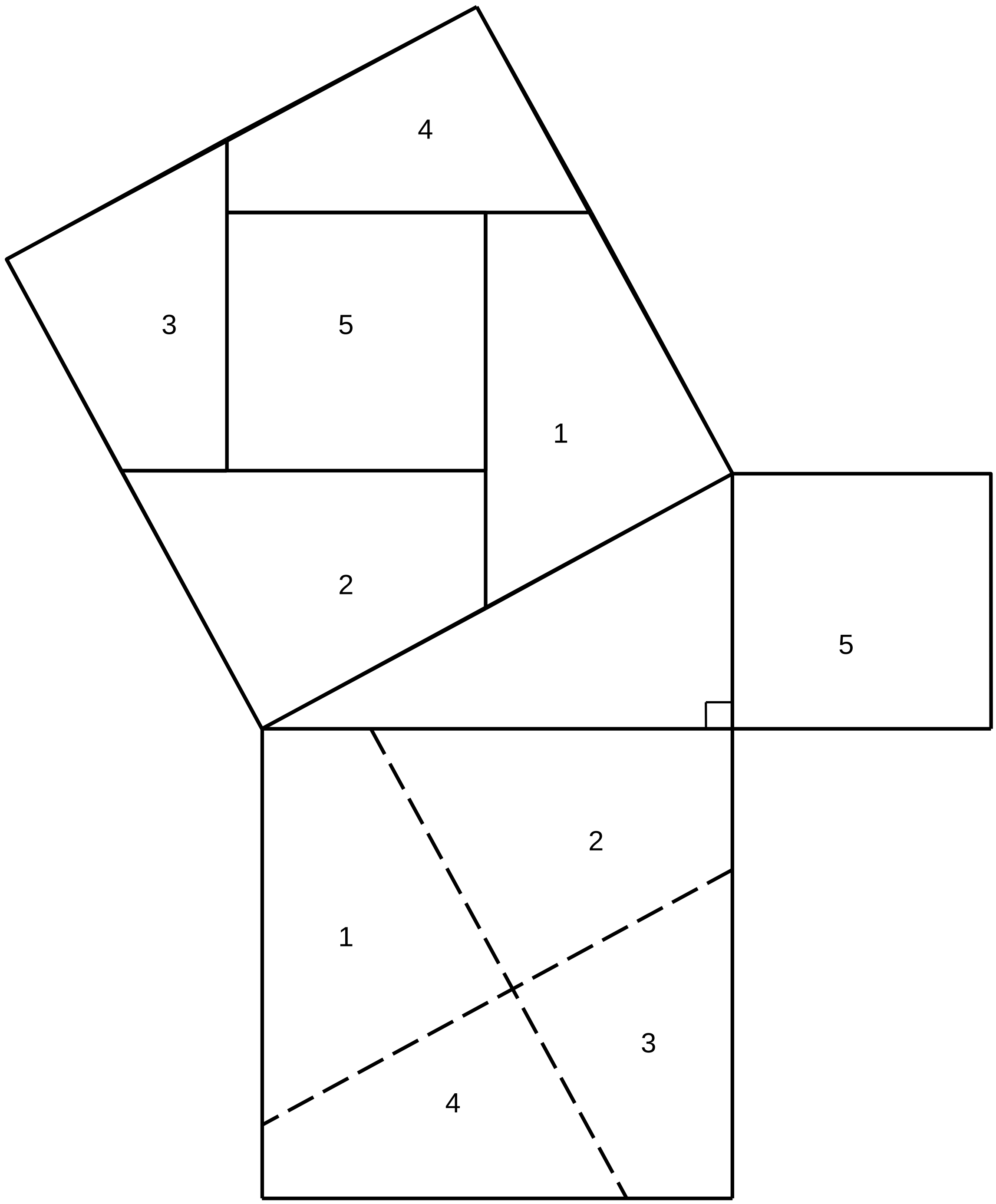
   Width: 44.70 inches

   Height: 26.00 inches

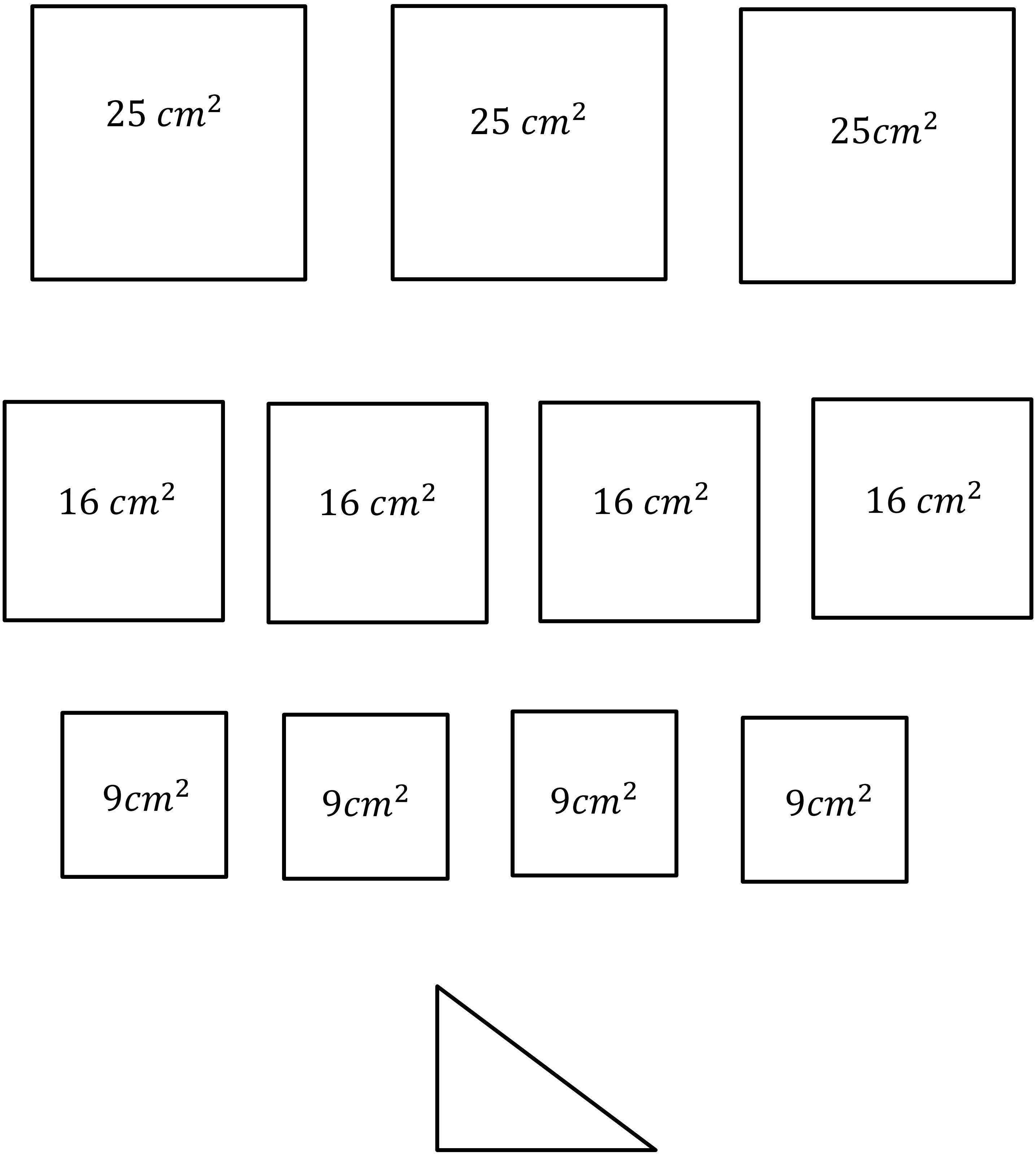
   a. What is the maximum size of television you can get? Round your answer to two decimal places.

   b. Most TV's list themselves as 30" or 60". They don't have decimal places. Without decimals, what is the largest size of TV you can fit in space? Does it make sense to round your answer up or down?

Pythagorean Theorem



Pythagorean Theorem Task:



OUTCOME P9.1

Demonstrate understanding of linear relations including:

- Graphing
- Analyzing
- Interpolating and extrapolating
- Solving situational questions

Lesson 1: Looking for Patterns

Lesson 2: Remembering How to Graph on a Cartesian Plane/Coordinate Plane

Lesson 3: Graphing Patterns

Lesson 4: The Number Transformer

Lesson 5: The Importance of Finding a Mathematical Equation & Key Terms

Lesson 6: Graphing and Solving Linear Relations

Lesson 7: Graphing $y = a$, $x = a$, and $ax + by = c$

Lesson 8: Matching Equations and Graphs

Lesson 9: Interpreting Graphs

Previous Experience:

Grade 7

Outcome P7.1

Demonstrate an understanding of the relationships between oral and written patterns, graphs and linear relations.

Grade 8

Outcome P8.1

Demonstrate understanding of linear relations concretely, pictorially (including graphs), physically, and symbolically.

To The Teacher:

Students should have previous experience with graphing linear relations from grade 8. Their vocabulary should consist of; ordered pairs, table of values, equations and linear relations. As with any of the units that require students to have previous knowledge, students may forget, have gaps in their learning, or have a good recollection of the content. I have created the unit under the assumption that students will need to go back and review some of the information. The first page of the unit consists of a vocabulary list. Have students fill it out as you move through the unit and encounter the words in the list.

Because it is more difficult to create assignments, I have relied on a few of the resources you may have in your classroom. You will find that I have given 2 options for assignments from two of the approved Saskatchewan grade 9 mathematics resources; Math Makes Sense 9 (Pearson) and MathLinks 9 (McGraw-Hill Ryerson). There are also plenty of free resources and worksheets online. Some good sites are kutasoftware.com, mathworksheets4kids.com, math-drills.com, and worksheets.com

Boaler (2008) has put forth that there are two versions of math in student's (and adults) lives. One they use in the classroom, and one they use outside of the classroom (p. 5). My goal is to try to use this unit to tie both of those versions together.

A lot of the activities in this unit are done in groups working through tasks. I have included an activity at the end of the unit that is a lot of fun and would make a good wrap up activity if you have extra time.

P9.3 Linear Inequalities

Key Terms:

1. Relation ____

____

2. Linear Relation ____

____

3. Table of Values ____

____

4. Input Values ____

____

5. Dependent Variable ____

____

6. Output Values ____

____

7. Independent Variable ____

____

8. Linear Graph ____

____

9. Linear Equation ____

____

10. Discrete Data ____

____

11. Interpolation ____

____

12. Extrapolation ____

____

Lesson 1: Looking for Patterns **P9.1**

To The Teacher:

Using two tasks, students will look for patterns and try to describe the patterns using math. The tasks are fairly prescriptive but it does give students a chance to find patterns without using any "rules and procedures". I have found that the resources Math Makes Sense (Pearson) and MathLinks 9 (McGraw-Hill Ryerson) jump too quickly into requiring students provide the linear equation for a pattern. My goal in this lesson is to let students rely on their intuition, rather than established rules and procedures, to try to describe the pattern

RN #7

Cooperative Learning

Lesson Process:

1. Students are going to need the following for each group.
   - Scissors
   - Large sheet of paper
   - Glue stick
   - Markers
   - Task #1 cut-out page (2 pages, do not double side)
   - Task #2 cut-out page (2 pages, do not double side)

2. Have students sit in learning groups. Hand out scissors, large sheet of paper, glue stick, markers and the first page of task #1 (cut-outs).

3. Without giving prompts or hints, have students work through the first 3 questions on the first page of task #1. You will need to choose the figure numbers for the cut-out task. I recommend using a smaller number than 10 and a number larger than 300. Once students are done, have them present their methods to the class.

4. Using one of the other methods presented in class, have them complete the fourth and fifth (if there were more than 3 methods) question from their task.

5. Students can glue the graph on the paper, but we won't go back and graph the data until lesson 3.

6. Going back to the student workbook, have them fill in their notes on page 1. As a class, have a discussion on how to determine the number of shaded squares will be in any figure.

7. On page 2 of their student workbook is a chart and graph. As a class, fill in the chart but leave the graph blank until lesson 3.
8. With students still sitting in their learning groups, hand out the first and second page of task #2 cut-outs. Have them complete both pages together as a group (except the graph). They can glue the graph on the page, but they won't graph the data until lesson 3. Once students are done, have students share their findings.
9. Take the findings from the task and have students complete the corresponding page in their student workbook (leaving the graph blank).
10. Assignment: Lesson 1

Looking for Patterns

Why are patterns and trends important in mathematics?

____

____

Task #1: Without counting 1 by 1, how many shaded squares are in figure 10 (10 x 10 square)?

My Calculations:
Some of my classmates ideas:

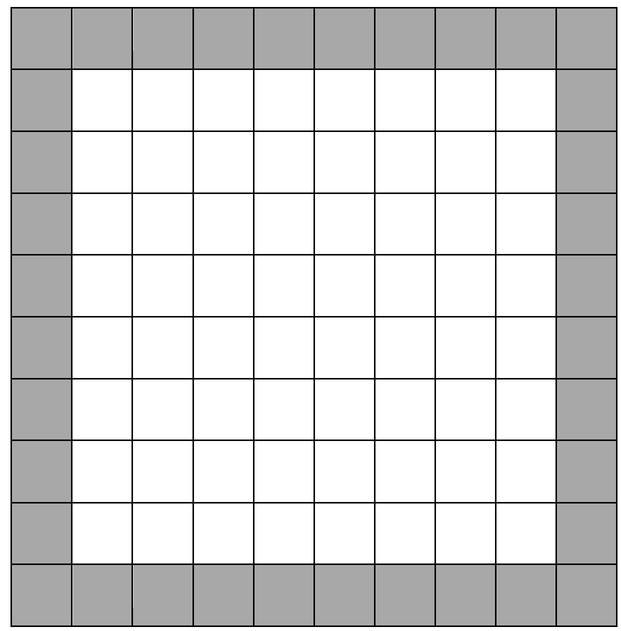


Figure 10

Assume that if figure 10 is a 10×10 grid, then figure 3 would be a 3×3 grid etc.

Using your own calculation idea from above, calculate how many shaded squares would be in

Figure ____	Figure ____

Using a classmate's calculation idea from above, calculate how many shaded squares would be in

Figure ____	Figure ____

How can we determine the number of shaded squares will be in any figure? You need to be able to justify your thinking.

Task #1 Cut-outs

Without counting 1 by 1, how many shaded squares are in figure 10 (10 x 10 square)? Show how you calculated your answer.

Using your calculations from above, how many shaded squares would be in figure ___?

Using your calculations from above, how many shaded squares would be in figure ___?

Using a classmate's calculation from the board, how many shaded squares would be in figure ___?

Using a classmate's calculation from the board, how many shaded squares would be in figure ___?

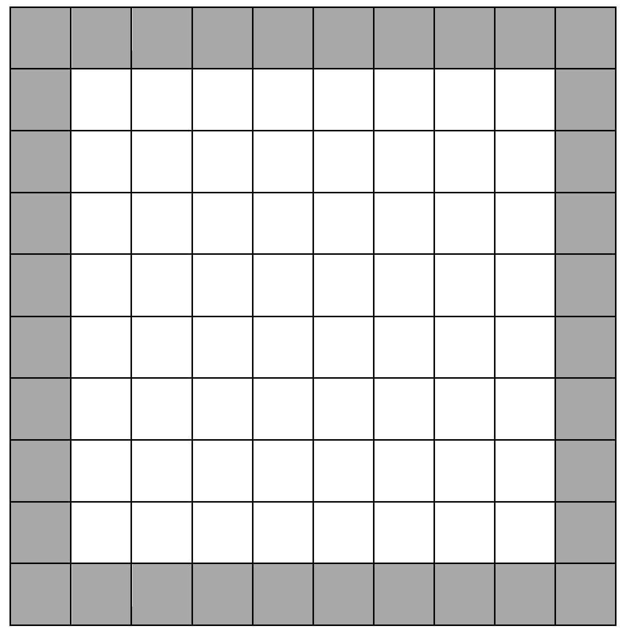
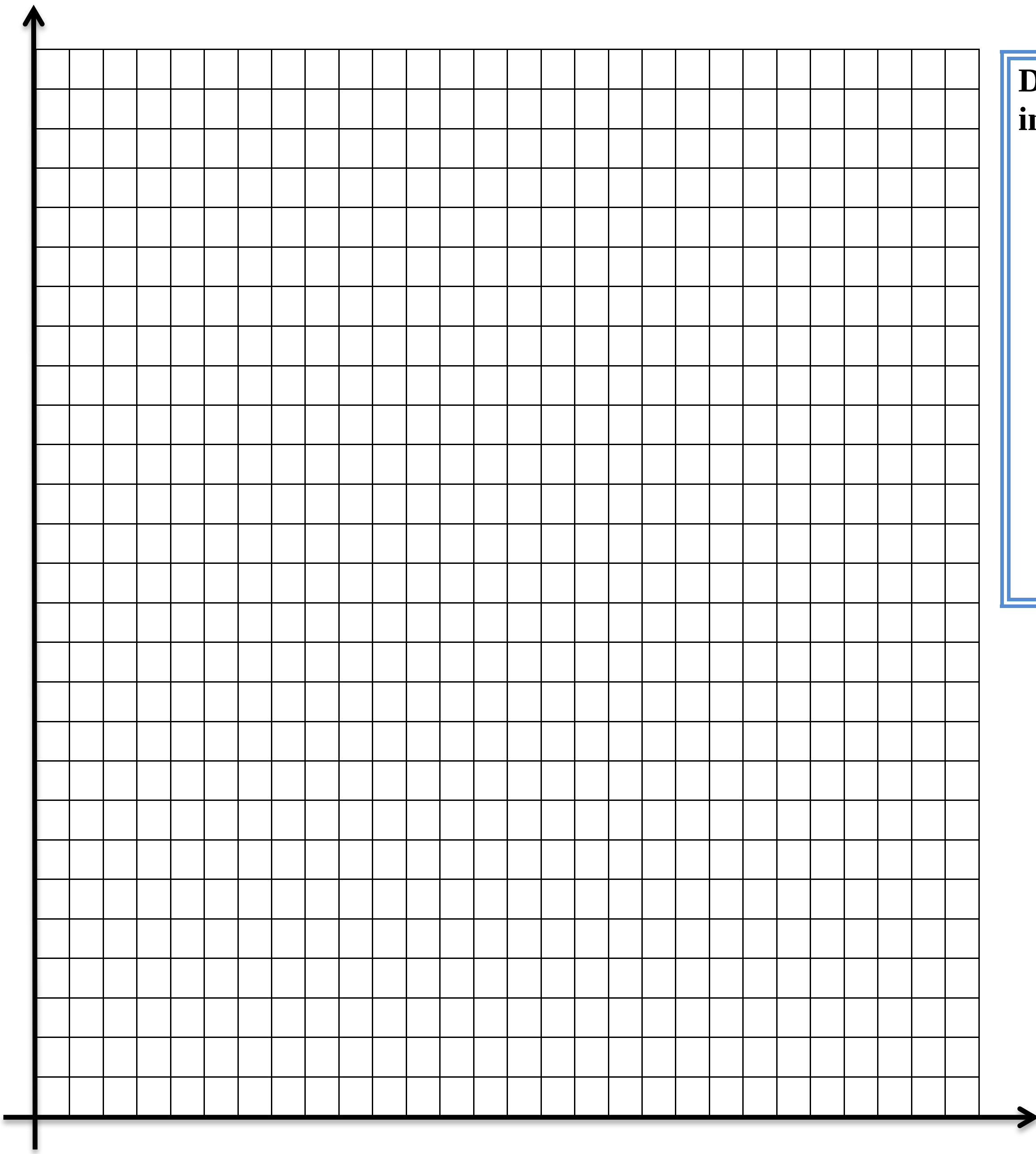


Figure #	Figure 1	Figure 2	Figure 3	Figure 4	Figure 5	Figure 6
Number of Shaded Squares						
Figure #	Figure 7	Figure 8	Figure 9	Figure 10	Figure 11	Figure 12
Number of Shaded Squares						



Describe the pattern in words…

Task #2: Draw the next L figure in the pattern.

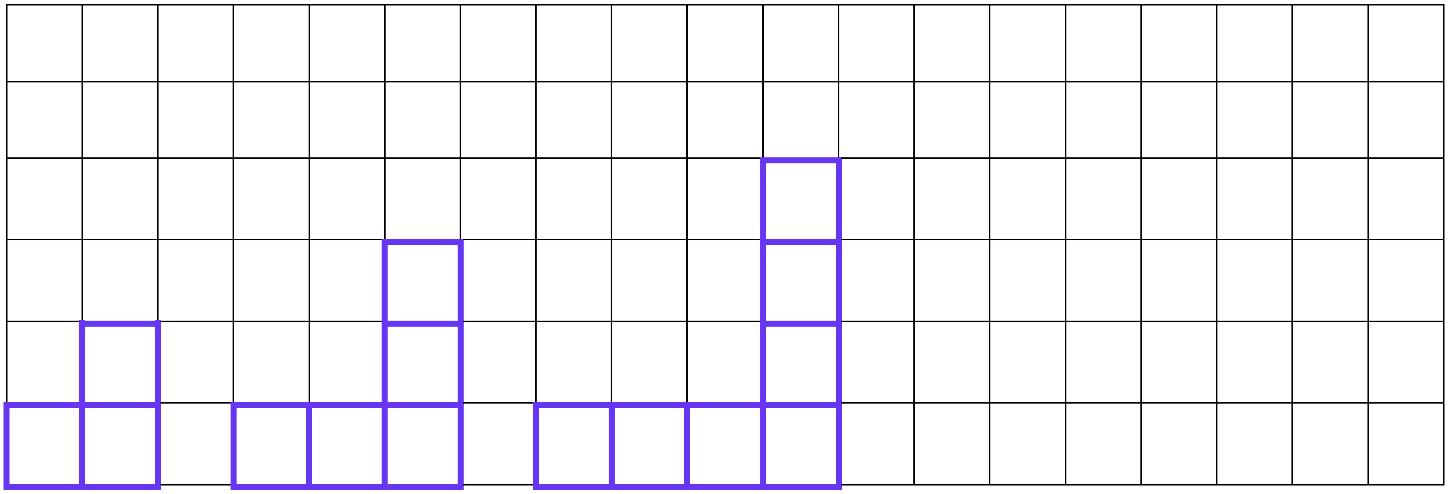


Figure 1 **Figure 2** **Figure 3** **Figure 4?**

How did you know what Figure 4 was going to look like?

How does it look like the pattern is growing?

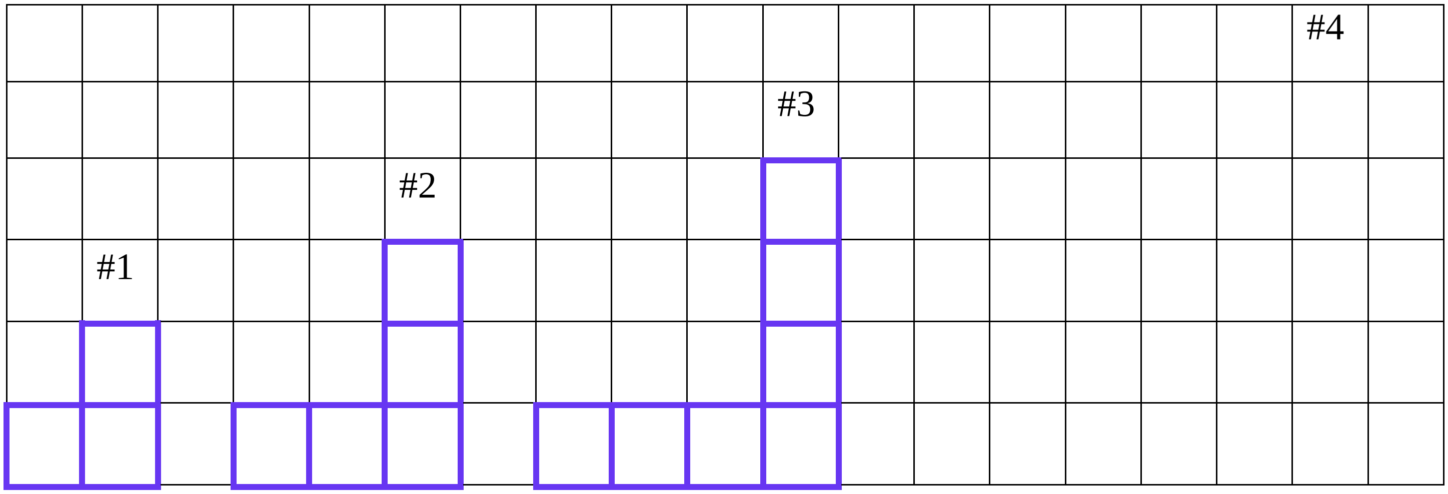
Count how many squares are represented by each figure.

Figure 1	Figure 2	Figure 3	Figure 4

If we weren't allowed to count by 1's, what are some of the calculations that could be used to find the number of squares in each figure?

How can we determine the number of shaded squares will be in any figure? You need to be able to justify your thinking.

Task #2 Cut-outs



a. How did you know what Figure 4 was going to look like?

b. How does it look like the pattern is growing?

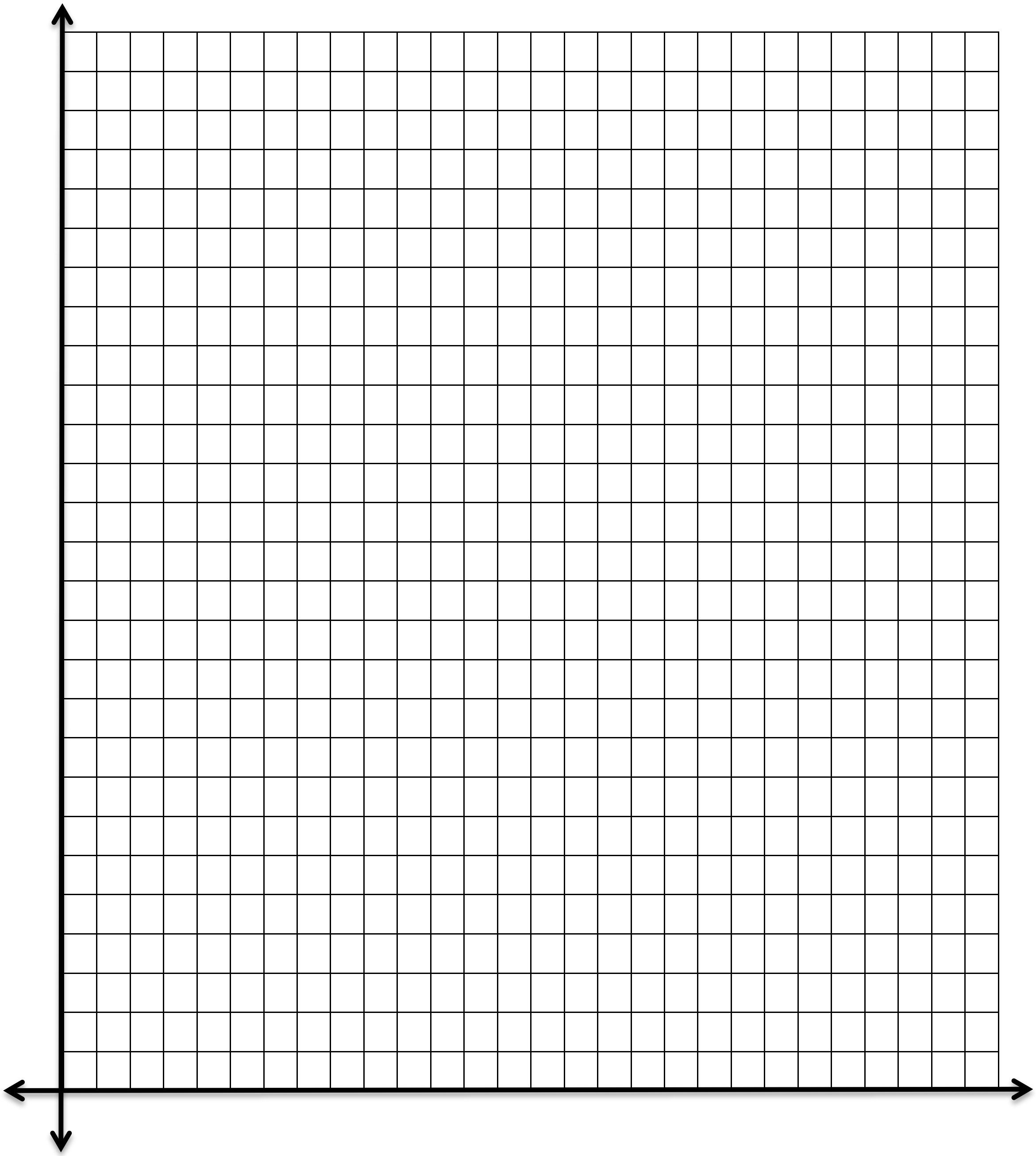
c. Count how many squares are represented by each figure.

Figure 1	Figure 2	Figure 3	Figure 4

d. If we weren't allowed to count by 1's, what are some of the calculations that could be used to find the number of squares in each figure?

e. How can we determine the number of shaded squares will be in any figure? You need to be able to justify your thinking.

Figure #	Figure 1	Figure 2	Figure 3	Figure 4	Figure 5	Figure 6
Number of Squares						
Figure #	Figure 7	Figure 8	Figure 9	Figure 10	Figure 11	Figure 12
Number of Squares						



Describe the pattern in words…

Lesson 1: Assignment **P9.1**

1. The square represents tables, the circles represents seats at the table.

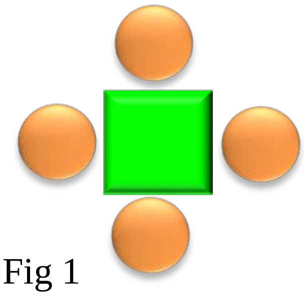


Fig 1

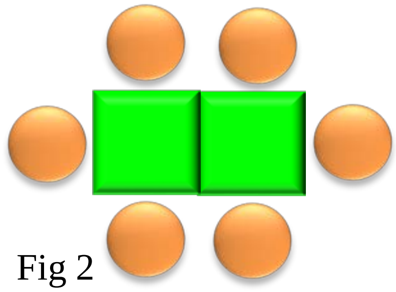


Fig 2

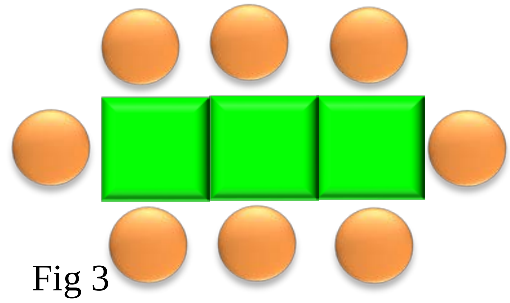


Fig 3

   a. Draw the next figure in the pattern (fig. 4)

   b. If you couldn't count by 1's, is there a technique to find out how many seats there are in each of the figures?

   c. How many seats would be in figure 103?

   d. Would you be able to fit exactly 17 seats in the pattern?

   e. What figure number has exactly 2694 seats? Show how you achieved your answer.

Lesson 2: Remembering How to Graph on a Cartesian Plane/Coordinate Plane P9.1

To The Teacher:

Students have graphed on a Cartesian plane in grade 8. This is a good lesson to ask students what they remember and build on their previous knowledge. Have students supply as much of the information as possible. This is a short lesson without an assignment. You may want to start lesson 3 immediately after this lesson.

RN #2

Prior Knowledge

Lesson Process:

1. At the start of the lesson (with their books closed) have students brainstorm terms that they remember from graphing in grade 8.

2. Using student's knowledge, define the terms quadrant, horizontal axis, vertical axis, coordinate axes, origin, coordinates, ordered pair.

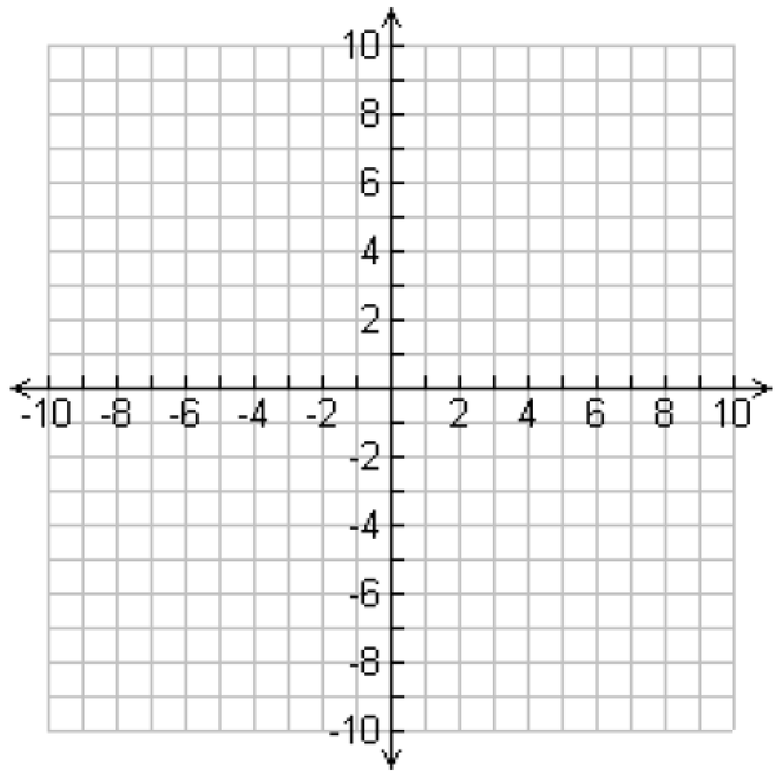
3. Have students plot the points in the Cartesian plane.

4. Have students answer questions 9-11.
   The Cartesian plane for question 9 came from math-aids.com

How to Graph on a Cartesian Plane/Coordinate Plane

Often, mathematicians graph their data on Cartesian Planes to determine patterns. Let's review how to graph ordered pairs before we apply it to linear relations.

A. Terminology



1. Quadrant:

2. Horizontal Axis:

3. Vertical Axis:

4. Coordinate Axes:

5. Origin:

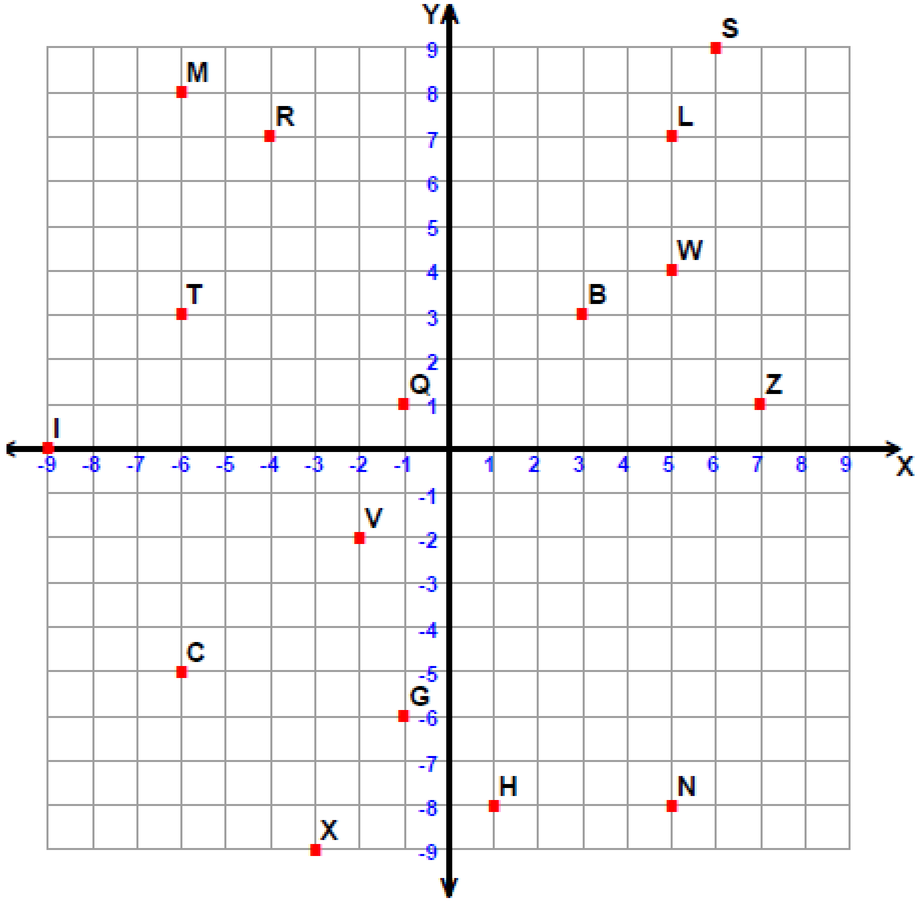
6. Coordinates:

7. Ordered Pair:

8. Plot the following points on the Cartesian plane above.

a) $A(5,-3)$ b) $B(0,9)$ c) $C(3,-10)$ d) $D(-3,0)$

e) $E(10,8)$ f) $F(-3,3)$ g) $G(-5,-5)$ h) $H(0,-4)$



9. Find what point is located at each ordered pair.

1) $(7,1)$ ________	3) $(-6,-5)$ _______	5) $(-1,1)$________	7) $(5,7)$ ________
2) $(-6,3)$ ________	4) $(1,-8)$ ________	6) $(-3,-9)$ _______	8) $(-9,0)$ ________

10. Write the ordered pair for each given point.

9) **N** ________	11) **R** ________	13) **G** ________	15) **W** _________
10) **V** ________	12) **S** ________	14) **B** ________	16) **M** ________

11. Plot the following points on the coordinate grid.

17) $\boldsymbol{E}$ $(-4,3)$	19) $\boldsymbol{F}$ $(-3,5)$	21) $\boldsymbol{P}$ $(8,4)$	23) **A** $(8,8)$
18) $\boldsymbol{Y}$ $(6,-1)$	20) $\boldsymbol{D}$ $(-3,-2)$	22) $\boldsymbol{O}$ $(-6,-9)$	24) **J** $(-2,1)$

To The Teacher:

Using the knowledge learned in lesson 1 and lesson 2, students are required to graph tables of values on the Cartesian plane. Students should connect that the patterns create a straight line when they graph their table of values. The majority of the lesson is task-based where students are in their learning groups working through a guided task. The lesson begins by going back to task #1 and task #2 from lesson 1. Students are to use their graphing skills to graph the information they found on a graph. This lesson is designed to transition students from intuitively talking about patterns to being able to transcribe the table of values and graph into a linear equation. Some of the important questions to focus on are “what stays the same?” and “what changes (remains constant)?”

RN #7

Collaborative/Cooperative Learning

RN #5

Formative Assessment

Lesson Process:

1. Have students sit in their learning groups. Hand out their large group sheets from lesson 1.
2. Going back to the tasks in lesson 1, have students graph the data from task #1 in the graph on their sheet as well in their workbook. Have them describe the patterns in words. Discuss as a class.
3. Going back to the task #2 from lesson 1, have students graph on their large sheet as well as in their workbook. Have them describe the patterns in words. Discuss as a class.
4. Starting fresh with task #3 (in the student workbooks), have students work in their learning groups to complete all aspects on the page.
5. Discuss task #3 as a class.
6. With students sitting in their learning groups, have students work through task #4.
7. Discuss task #4 as a class.
8. With students sitting in their learning groups, have students work through task #5. Task #5 has more questions at the end than the previous tasks.

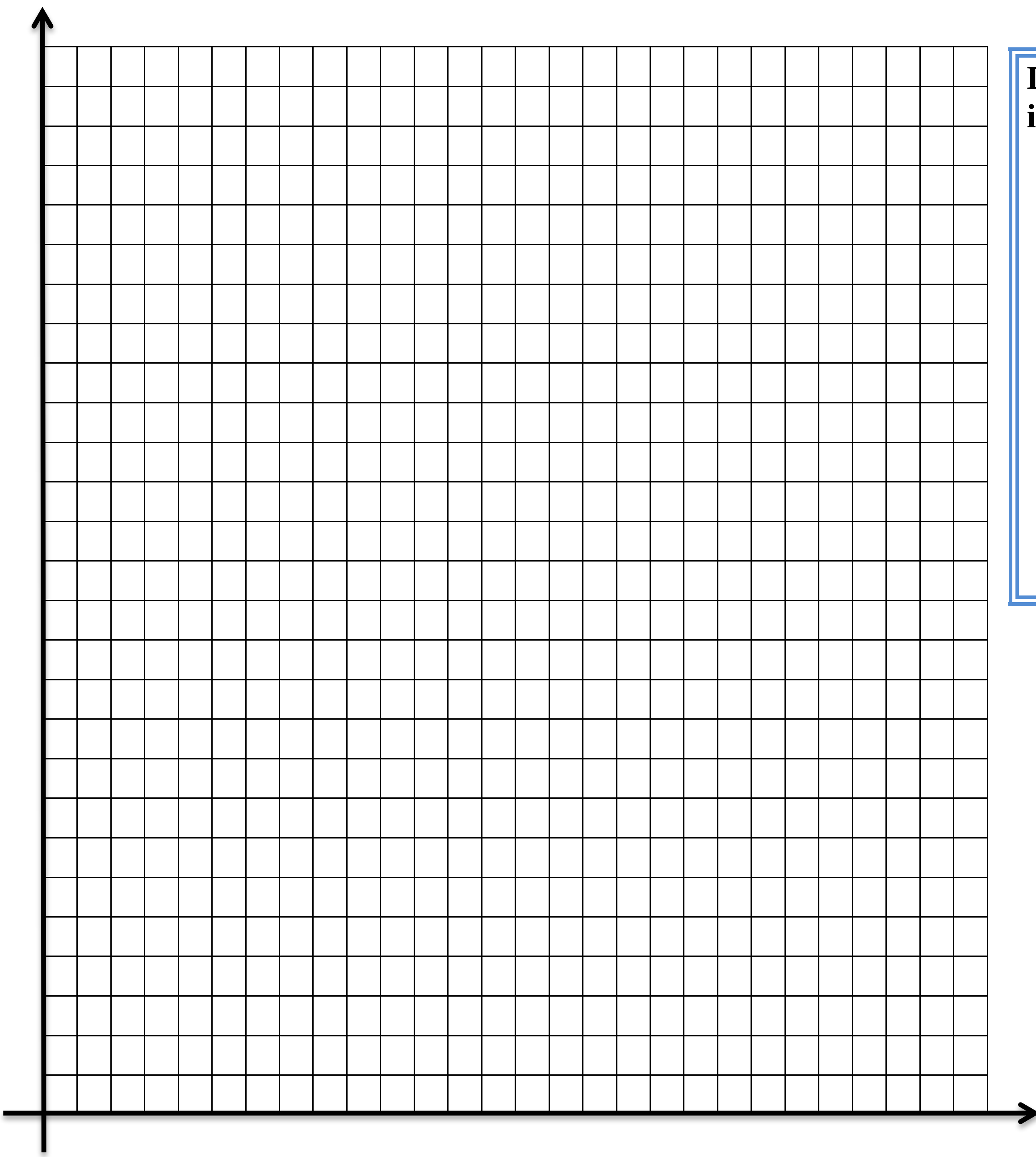
9. Discuss task #5 as a class.
10. Exit Task **RN #5**
11. Assignment: Lesson 3

Lesson 3: Graphing Patterns **P9.1**

Let's go back and plot the data from our first two patterns that we encountered last class.

Task #1: 10 × 10 Square

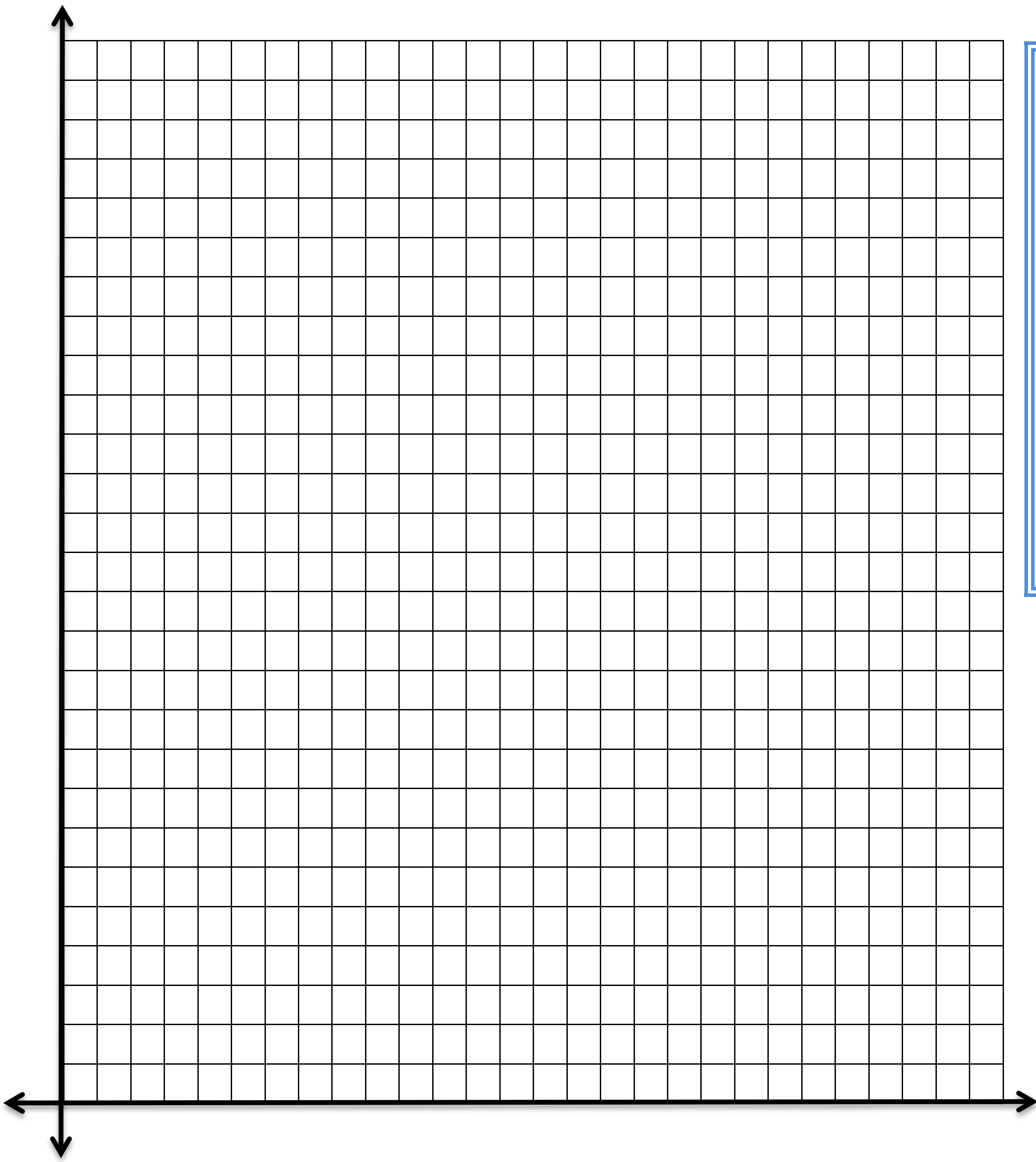
Figure #	Figure 1	Figure 2	Figure 3	Figure 4	Figure 5	Figure 6
Number of Shaded Squares						
Figure #	Figure 7	Figure 8	Figure 9	Figure 10	Figure 11	Figure 12
Number of Shaded Squares						



Describe the pattern in words…

Task #2: L Figures

Figure #	Figure 1	Figure 2	Figure 3	Figure 4	Figure 5	Figure 6
Number of Squares						
Figure #	Figure 7	Figure 8	Figure 9	Figure 10	Figure 11	Figure 12
Number of Squares						

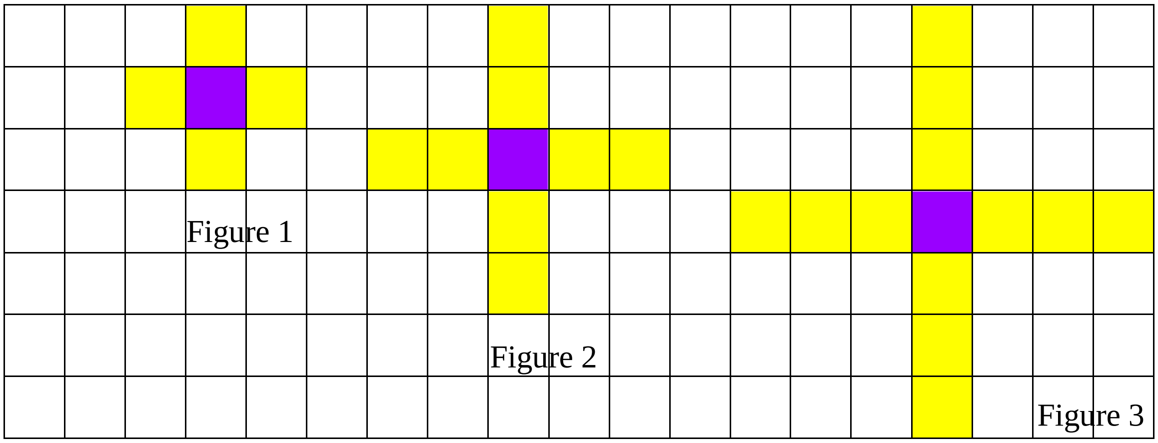


Describe the pattern in words…

What do we notice about the way the numbers look on the graph? What kind of pattern is this?

Graphing Patterns

Task #3: Lower Case t's



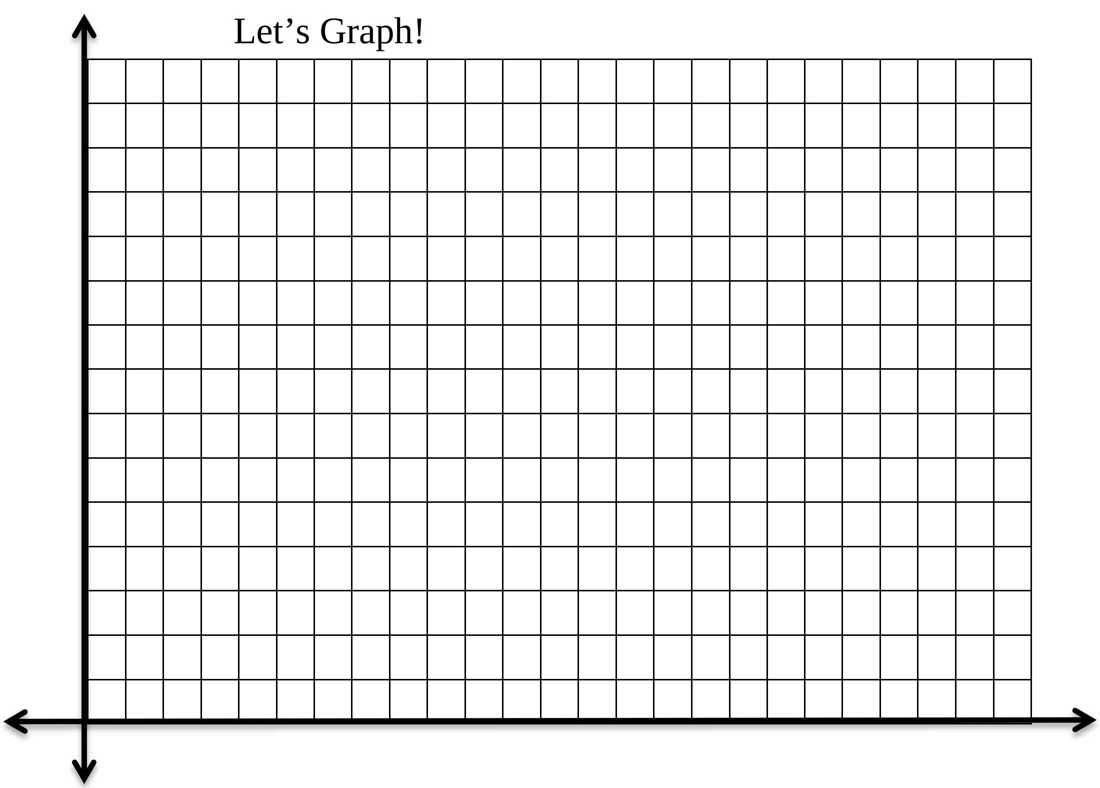
How many squares would figure 4 need? Draw a sketch of what you think it will look like.

What part is staying the same (constant) and what part is changing (multiplier)?

Let's make a chart

Figure #	Figure 1	Figure 2	Figure 3	Figure 4	Figure 5	Figure 6
Number of Squares						
Figure #	Figure 7	Figure 8	Figure 9	Figure 10	Figure 11	Figure 12
Number of Squares						

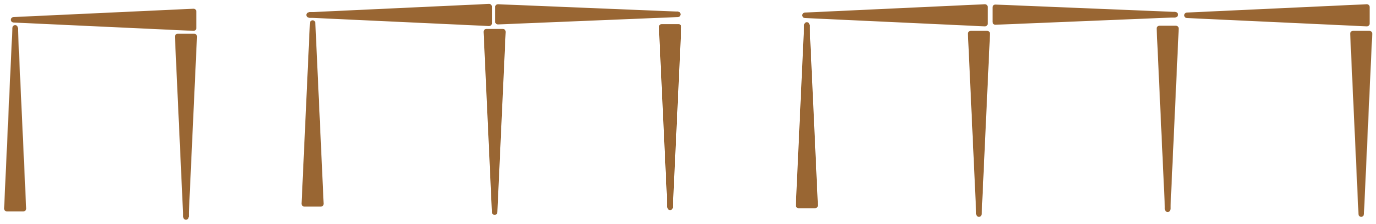
Let's Graph!



Describe the pattern in words…

Task #4: Toothpicks

Figure 1 Figure 2 Figure 3



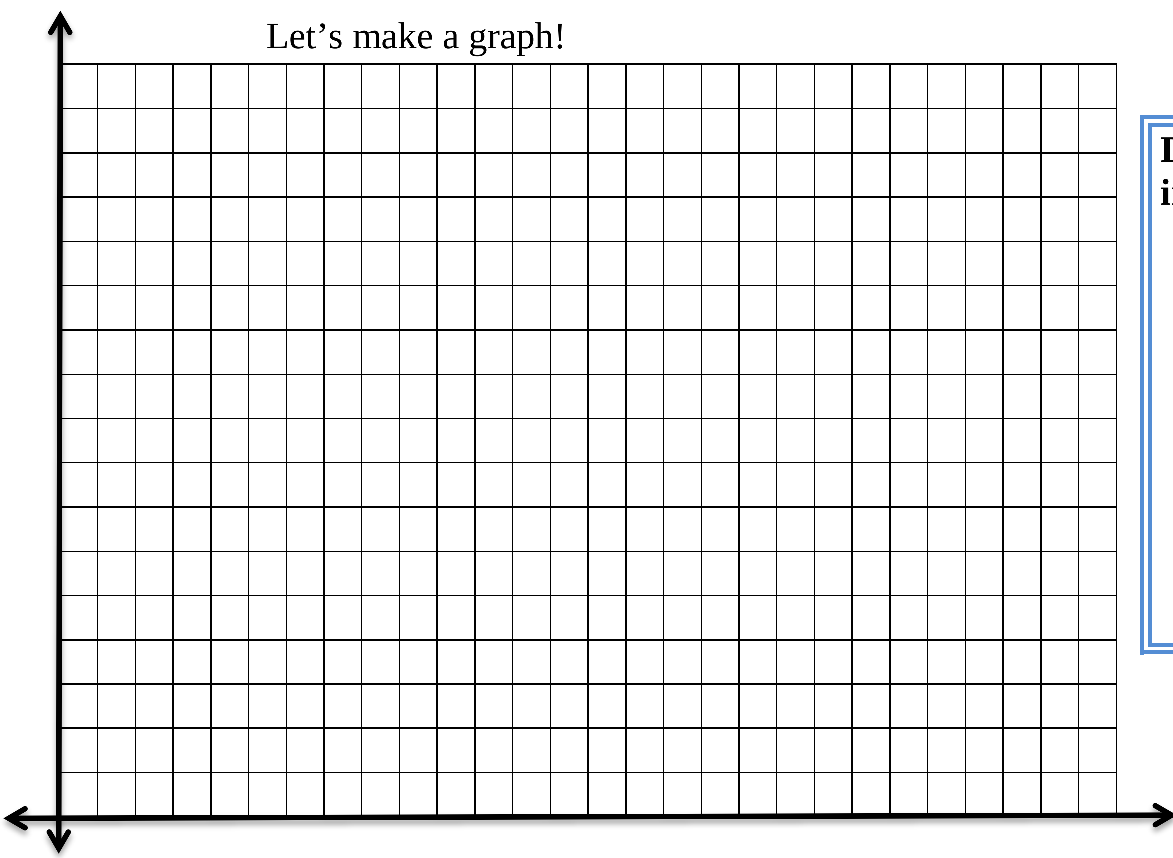
What part is staying the same (constant)? ____

What part is changing (multiplier)? ____

Let's make a chart!

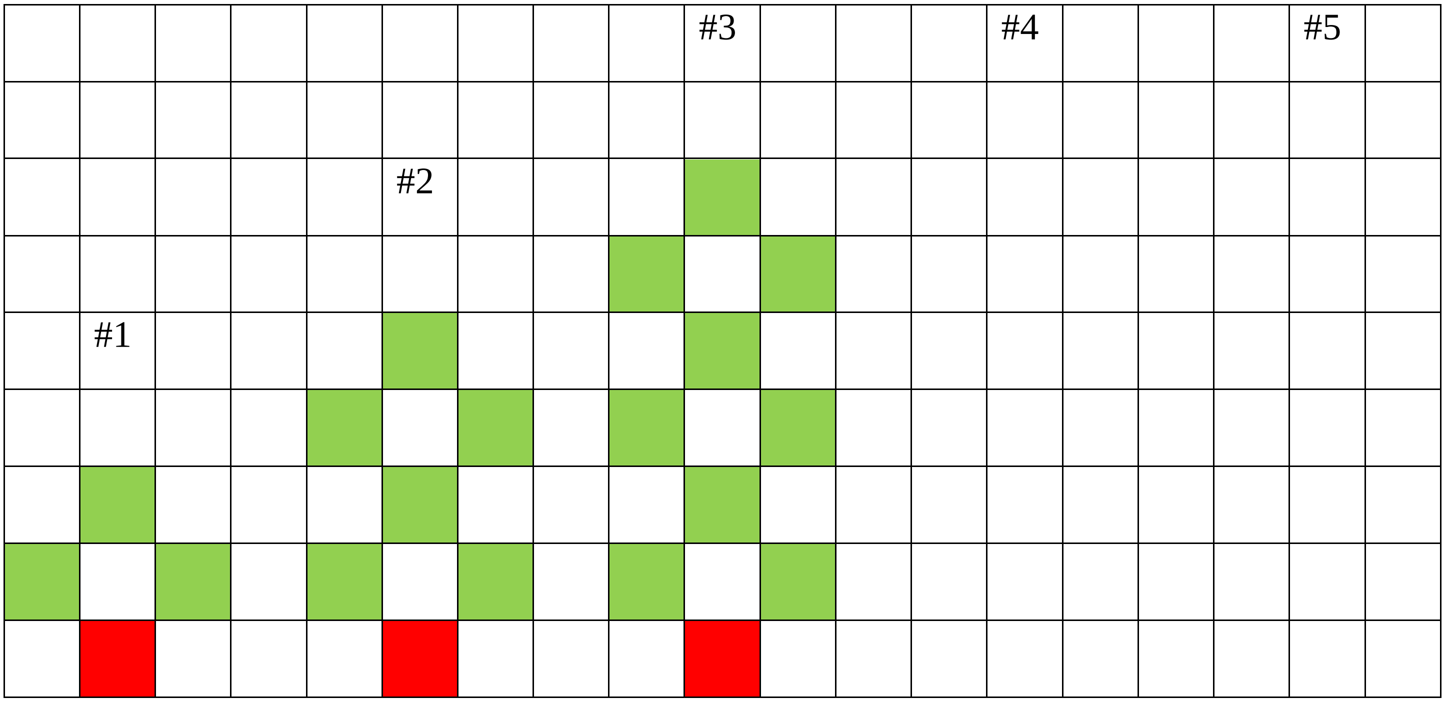
Figure #	Figure 1	Figure 2	Figure 3	Figure 4	Figure 5	Figure 6
Number of toothpicks						
Figure #	Figure 7	Figure 8	Figure 9	Figure 10	Figure 11	Figure 12
Number of toothpicks						

Let's make a graph!



Describe the pattern in words…

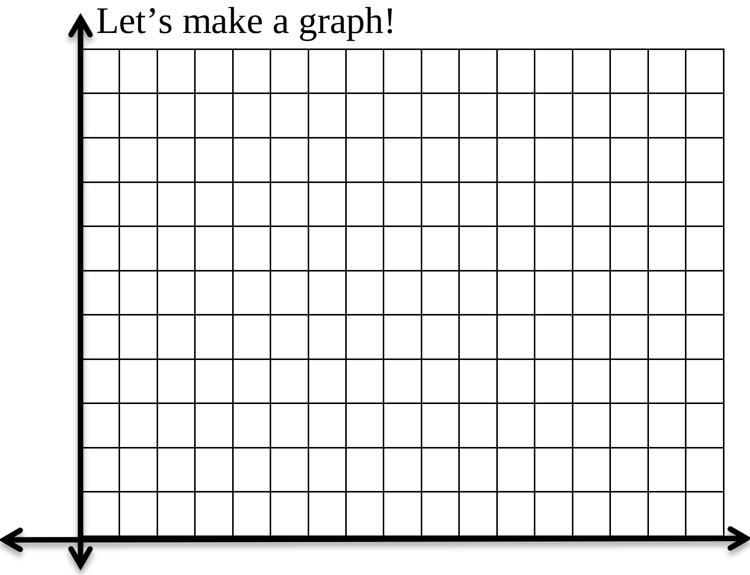
Task #5:



Let's make a chart!

Figure #	Figure 1	Figure 2	Figure 3	Figure 4	Figure 5	Figure 6
Number of squares						

Let's make a graph!



Describe the pattern in words…

1. What part of the pattern is staying the same (constant?)

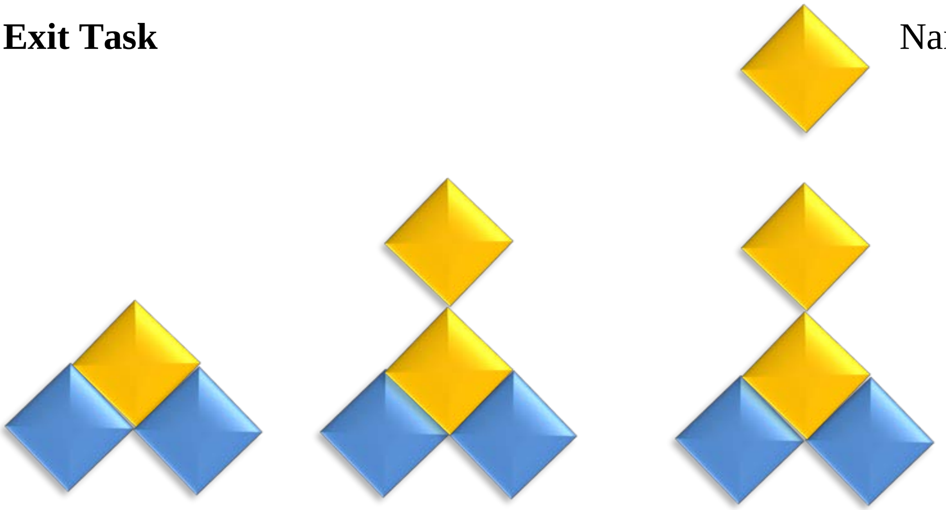
2. What part of the pattern is changing (multiplier)?

3. What would position 30 in the pattern look like?

4. If you had 40 tiles, could you build position 12 in the pattern? Why or why not?

5. If you had 100 tiles, which position in the pattern could you build?

Lesson 2: Exit Task Name:_______________



#1 **#2** **#3**

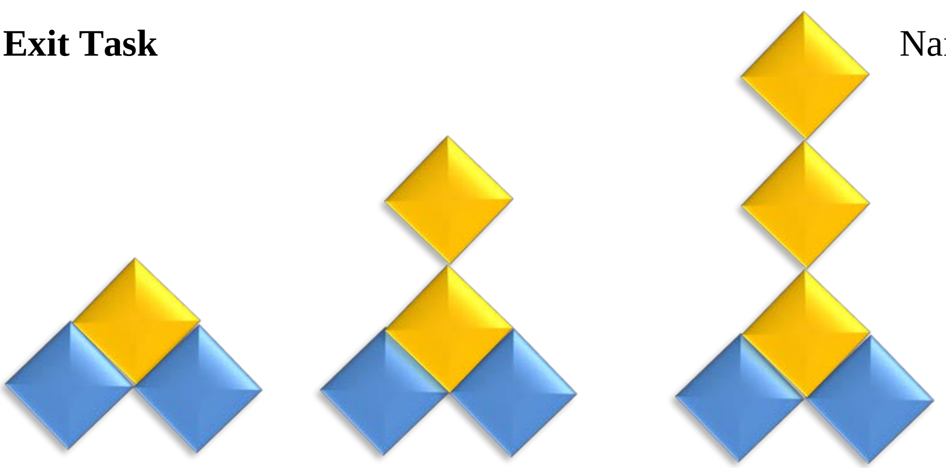
What part of the pattern is staying the same? ____

What part of the pattern is changing? ____

There would be ____________ tiles at the 103rd position in the pattern.

Describe what the 60th position in the pattern would look like.

Lesson 2: Exit Task Name:_______________



#1 **#2** **#3**

What part of the pattern is staying the same? ____

What part of the pattern is changing? ____

There would be ____________ tiles at the 103rd position in the pattern.

Describe what the 60th position in the pattern would look like.

Assignment: Lesson 3 **P9.1**

1. Here is a pattern of squares, each square has a side length of 1 cm. The pattern continues.

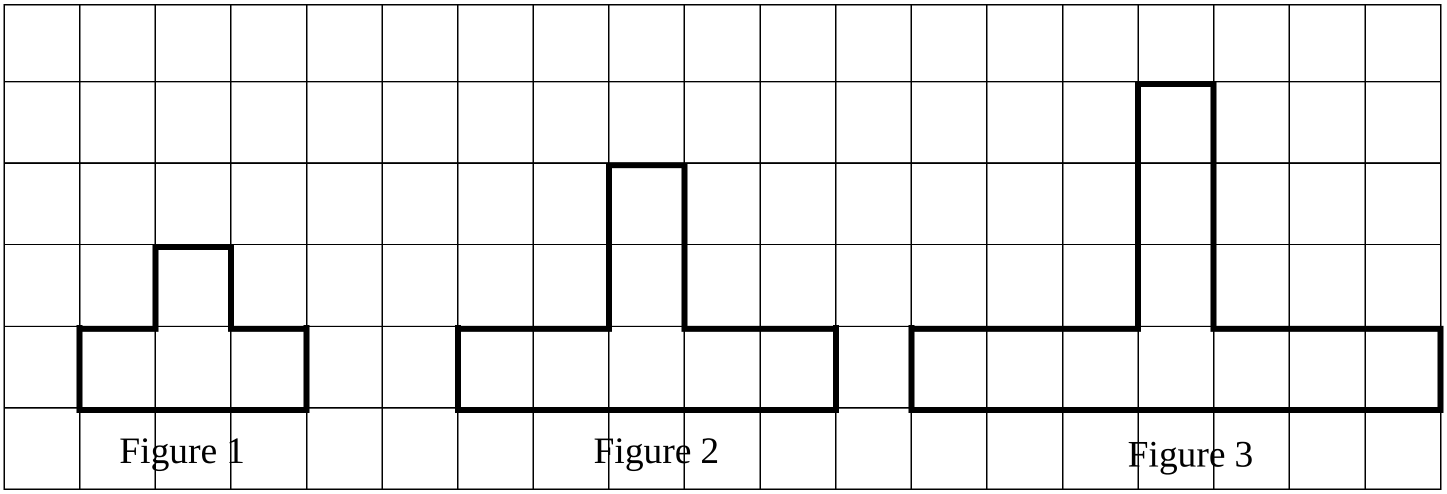
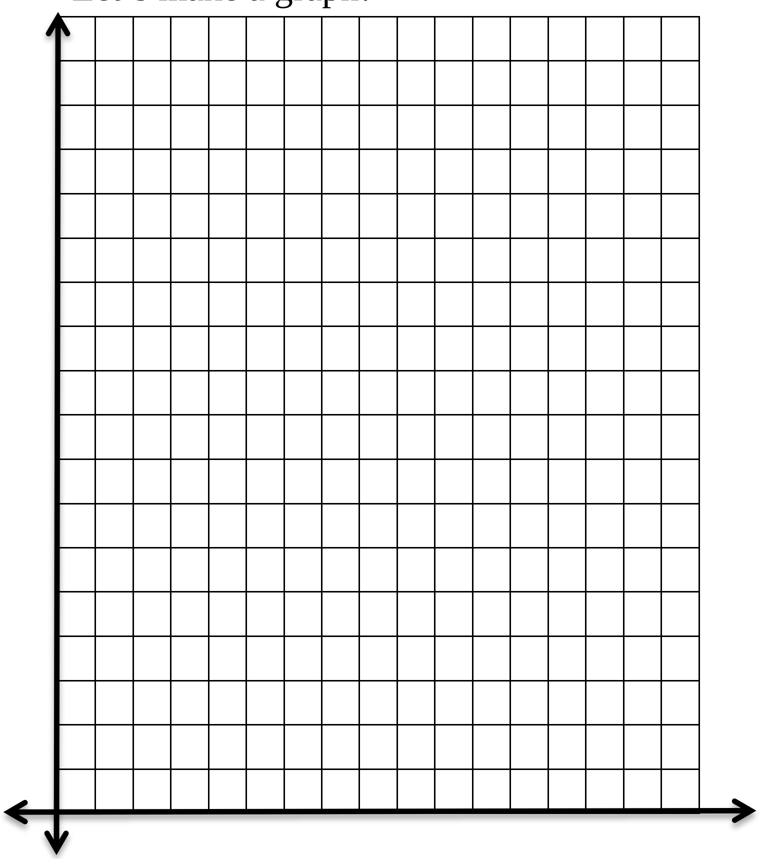


Figure #	Figure 1	Figure 2	Figure 3	Figure 4	Figure 5	Figure 6	Figure 7	Figure 8
Perimeter								

Let's make a graph!



Describe the pattern in words…

What stayed the same in the pattern?

What changed in the pattern?

2. Assuming that the pattern continues. Answer the following questions.

Figure 1 Figure 2 Figure 3

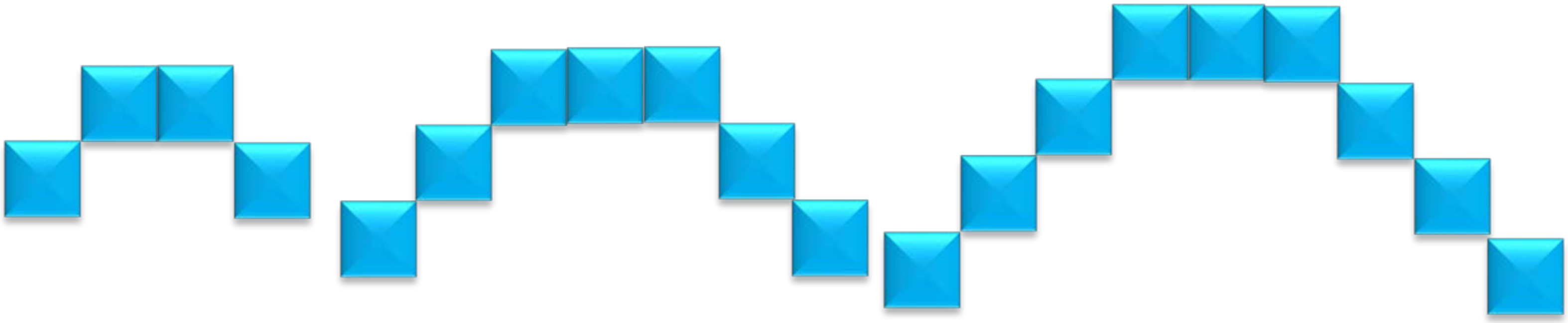
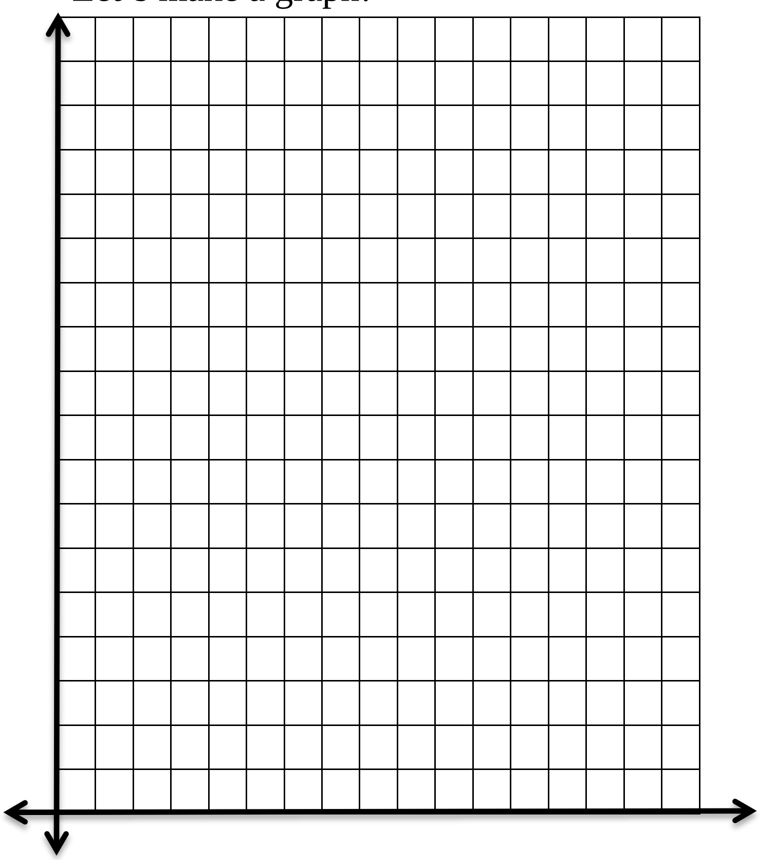


Figure #	Figure 0	Figure 1	Figure 2	Figure 3	Figure 4	Figure 5	Figure 6	Figure 7
Number of Tiles								

Let's make a graph!



Describe the pattern in words…

What stayed the same in the pattern?

What changed in the pattern?

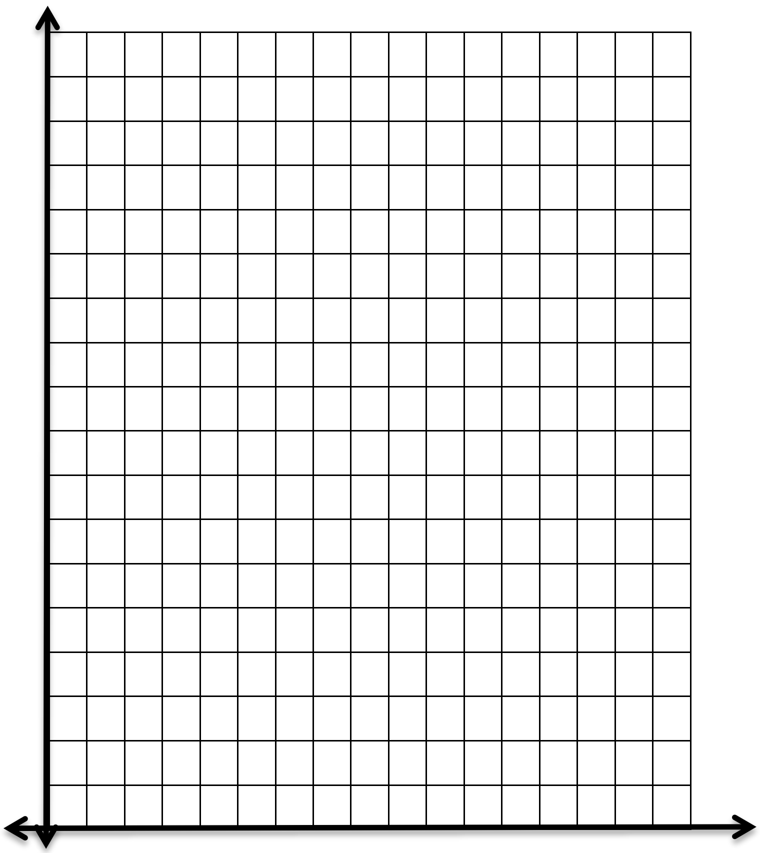
How could you determine how many squares are in figure 2133?

What figure number has 645 squares?

3. Consider this pattern

a. Organize the pattern into a table of values

b. Make a graph



Describe the pattern in words…

What stayed the same in the pattern?

What changed in the pattern?

c. How many hearts are in the 100th term? How did you find your answer?

d. What term has 3199 hearts? How did you find your answer?

Lesson 4: **The Number Transformer** **P9.1**

To The Teacher:

This lesson is geared towards having students using their intuition to find the rule or equation to describe the linear pattern. I have found in the past that grade 9 students struggle when writing the equation when given the chart. My hope is that by introducing the content as more of a puzzle rather than a math problem, students will be more motivated and intrigued to work through tougher problems.

The number transformer idea comes from: http://www.iboard.co.uk/activities/path/knowing-and-using-number-facts_using-inversion/subject/maths

RN #7

Cooperative Learning

RN #5

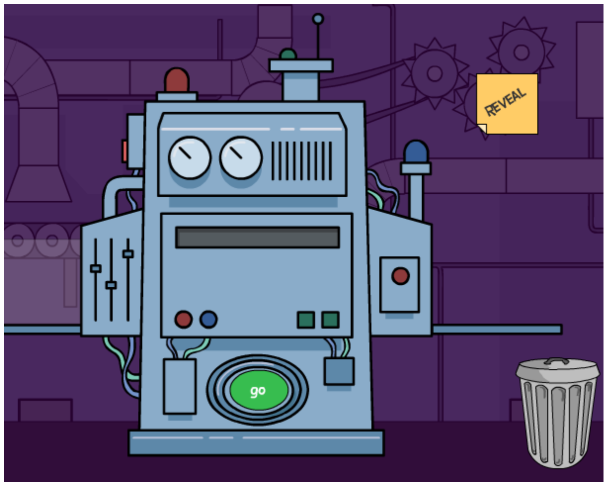
Formative Assessment

Lesson Process:

1. Have students sit in their learning groups.
2. Students are going to need the following for each group.
   - Scissors
   - Large sheet of paper
   - Glue stick
   - Markers
   - Task #6 cut-out page
   - Task #7 cut-out page
3. Begin the lesson by introducing the number transformer.
4. Have students work through task #6 in their groups. Discuss their findings as a class.
5. Have students work through task #7 in their groups. Discuss their findings as a class.
6. Exit slip **RN #5**
7. There is no assignment for this lesson. Lesson 5 continues with the ideas presented in lesson 4.

Introducing the Number Transformer

In mathematics there exists a number transformer that can change one number into another. Often in mathematics the transformer code is hidden and must be found. In this unit, your most important job is to find that code. In order for it to be the right code, it must work for all the numbers in the question.



Task #6 Number Transformer: This table represents clues. The original number is called the input (what is put into the number transformer) and the resulting number is called the output (what comes out of the number transformer).

Challenge: Work with your group to find out what operations are being used on the top numbers to create the bottom numbers (it has to be the same for all of the numbers). You are allowed to use a combination of operations.

Input x	0	1	2	3	4
Output y		3		7	

a. If the number 15 was the input number, what would be the output?

b. Describe the pattern between the input and output values in words.

c. Describe the pattern between the input and output values using numbers and operations. What operations to we apply to x, in order to get y?

d. Write a general rule or equation to find the output (y) given any input (x).

$y =$_______________

Task #6 Number Transformer Cut Outs

Input x	0	1	2	3	4
Output y		3		7	

a. If the number 15 was the input number, what would be the output?

b. Describe the pattern between the input and output values in words.

c. Describe the pattern between the input and output values using numbers and operations. What operations to we apply to x, in order to get y?

d. Write a general rule or equation to find the output (y) given any input (x).

$y =$_______________

Task #7 Number Transformer: This table represents clues. The original number is called the input and the resulting number is called the output.

Challenge: Work with your group to find out what operations are being used on the top numbers to create the bottom numbers (it has to be the same for all of the numbers). You are allowed to try a combination of operations.

Input x	-3	-2	-1	0	1
Output y		-11			4

a. If the number 10 was the input number, what would be the output?

b. Describe the pattern between the input and output values in words.

c. Describe the pattern between the input and output values using numbers and operations.

d. Write a general rule or equation to find the output (y) given any input (x).

e. If I wanted to output the number 15, what number would I have to input?

Task #7 Number Transformer Cut Outs

Input x	-3	-2	-1	0	1
Output y		-11			4

a. If the number 10 was the input number, what would be the output?

b. Describe the pattern between the input and output values in words.

c. Describe the pattern between the input and output values using numbers and operations. What operations to we apply to x, in order to get y?

d. Write a general rule or equation to find the output (y) given any input (x).

$y =$______________

e. If we wanted to output the number 15, what number would we need to input?

Possible Textbook Assignments:

MathLinks (McGraw-Hill Ryerson): Page 216

#1, 4, 5, 6	Take a pattern and come up with a linear equation.
#9	Take a pattern of numbers and come up with a linear equation
#10	Crack the code

Math Makes Sense (Pearson): Page 159

#8, 9, 12 a, c, d, e	Take a pattern and come up with a linear equation.
#11	Crack the code

Note: If students struggle with cracking the code, you may want to save this assignment until after the next lesson.

Lesson 4: Exit Task Name:________________

Find out what operations are being used on the top numbers to create the bottom numbers (it has to be the same for all of the numbers).

Input x	0	1	2	3	4
Output y			4		10

a. If the number 8 was the input number, what would be the output?

b. Describe the pattern between the input and output values in words.

c. Describe the pattern between the input and output values using numbers and operations.

Lesson 4: Exit Task Name:________________

Find out what operations are being used on the top numbers to create the bottom numbers (it has to be the same for all of the numbers).

Input x	0	1	2	3	4
Output y			4		10

a. If the number 8 was the input number, what would be the output?

b. Describe the pattern between the input and output values in words.

c. Describe the pattern between the input and output values using numbers and operations.

To The Teacher:

This lesson will likely take 2-3 days. It begins by working as a group to find a fool-proof way to unlock the mathematical code from the number transformer. Once students have the equation, they can find information about the future based on current data. The next goal is to define some of the terms that students are going to need to know. I have embedded the definitions within a question. Students should be filling out the terms from the front of their student notebook. In order to define what a linear graph, linear equation, and linear table of values is, the student workbook is set up for a concept attainment activity. You can cross off the things that are a "no" and ask students to find the characteristics that make the remaining questions a "yes". I really like the group task in this lesson. It is based on learner generated examples and has students within their learning group create their own linear equation. They then need to create a table of values and graph their relation. Have students hand in their task, check for correctness, and then fill in a blank document with each of the groups learner generated example. Hand out one example to each group and have them work through a different groups question. When they are done, hand it back to the original group and have them correct it.

RN #7

Cooperative Learning

RN #1

Math-in-Context

RN #11

Concept Attainment

RN #12

Learner Generated Examples

Lesson Process:

1. Have students sit in their learning groups. **RN #7**
2. Ask students to incorporate a new question into their task #1 sheet. This may be tough for students because they need to figure out if having a square of 256 squares will work as a shaded square border. After students have had a chance to work through the question, discuss as a class.

3. Have students work in their groups to unlock the mathematical codes from the number transformer in the examples in their workbook. Have students share their answers as a class.

4. Ask students how they crack the code? What are some shortcuts that they used to find out what the code was?

5. Work through task #8 as a class. Define key terms as they come up in the example. There are terms at the front of the workbook that they should be filling out as well. Some of the terms that are covered in the example are: table of values, dependent variable, independent variable, discrete data, input values, and output values. **RN #1**

6. Work with students to develop a definition of what a linear graph looks like. You could go through the examples and just cross off the ones that are not linear and ask students what are the characteristics of a linear graph. I did not include a horizontal or vertical line graph, you may want to ask students if they think they are linear or not by drawing them on the board. **RN #11**

7. Looking at a group of equations, have students determine which ones are linear and which ones are not. You may want to do this as a concept attainment exercise as well, or just by asking students what they think. At the end of the exercise, have students describe the characteristics of a linear equation. **RN #11**

8. Looking at a group of table of values, ask students if they think they are linear or not. This is a tougher exercise because they need to crack the code first and then determine if it is linear. They could also look at the values by which the input values increase, as well as the output values increase to see if they remain constant. However, not all tables will have consistently increasing or decreasing input values. I don't recommend that students rely on the second method to determine if they are linear or not.

9. Have students work through filling in a table of values when the linear equation is known.

10. In the student learning groups, have them, come up with their own linear equation. They will then need to create a table of values and a graph using their own linear equation. A really interesting activity would be as follows: **RN #12**

    a. Each group create their own linear equation. They need to create a table of values, and graph for the linear relation.

    b. Take in one complete sheet from all groups. Look them over to ensure correctness. Have groups fix any mistakes.

c. Create a new worksheet using the generated examples. Give each group one question and have them come up with a table of values and graph. Return the sheets to the original group and have them correct it.

d. An extension to the activity is to give them the table of values and have them come up with the linear equation and graph.

11. Assignment: Lesson 5. The assignment is short for this lesson and you may want to supplement with your classroom assigned resource (textbook).

Possible Textbook Assignments:

MathLinks (McGraw-Hill Ryerson):

None

Math Makes Sense (Pearson): Page 170

#4-5	Are the graphs and table of values linear?
#7	Given a linear equation, fill in the table of values.
#8a, c, d, e	Crack the code, graph
#9	Crack the code
#10	Given a linear equation, fill in the table of values.

The Importance of Finding a Mathematical Equation & Key Terms

Task #1: Let's look back at our shaded square. Will 256 squares work as a shaded square border? What would be the dimensions of that square? Get your poster and add this question in. Work as a team!

Insert your work here for your own personal notes:

Often, we are asked to find information about the future based on current data. In order to do that, we need to unlock the mathematical code from the number transformer. Let's find a way to do that!

Code #1:

Input x	1	2	3	4	5	6
Output y	4	6	8	10	12	14

Code #2:

Figure f	1	2	3	4	5	6
Perimeter P	4	6	8	10	12	14

Code #3

Input x	-2	-1	0	1	2	3
Output y	5	2	-1	-4	-7	-10

Code #4

Input x	-2	-1	0	1	2	3
Output y	-6	-2	2	6	10	14

How do we crack the code?

Key Terms: Before we move through another lesson, it is important to define some of the terms that we will be using. Let’s do it in the context of a question.

Task #8 Find the Perimeter of a Triangle Train

Things to know:

- Each side length is 1 cm
- Interior lines don’t count as part of a perimeter

Figure 1

Figure 2

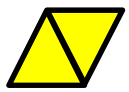
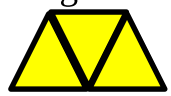


Figure 3

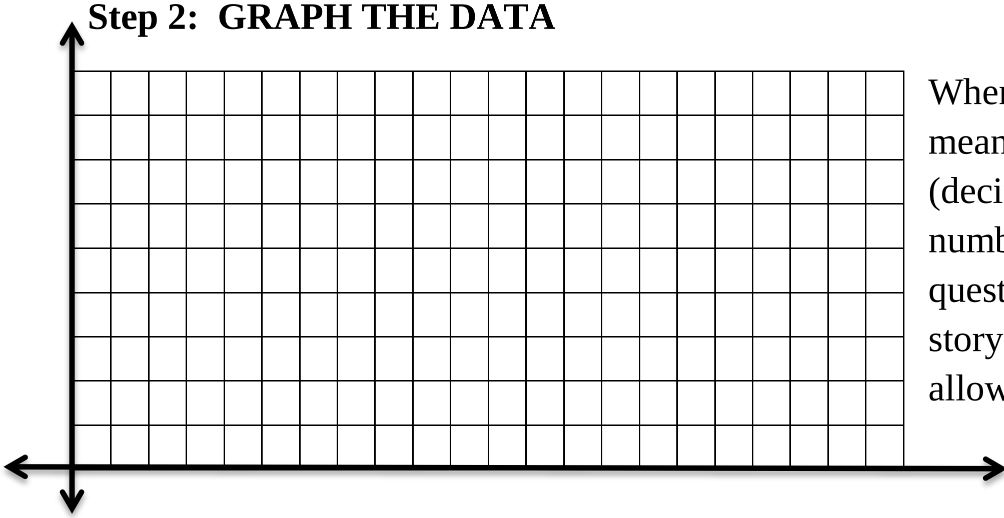


Step 1: TABLE OF VALUES

Input	**Figure #**					
Output	**Perimeter**					

* In a table of values we always put the input number on the top, and the output on the bottom.
* The input represents the dependent variable (goes on the vertical axis of the graph)
* The output represents the independent variable (goes on the horizontal axis of the graph)

Step 2: GRAPH THE DATA



When we join the dots with a line, that means that the numbers in between (decimals and fractions) the whole numbers are allowed to be part of the question. You need to think about the story and if decimals and fractions are allowed.

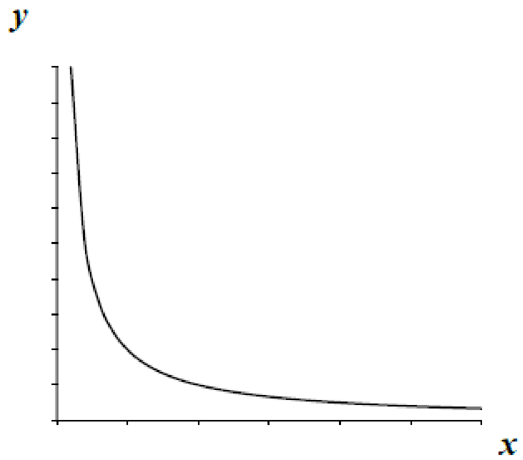
STEP 3: DESCRIBE THE PATTERN IN WORDS

STEP 4: WRITE THE GENERAL EQUATION

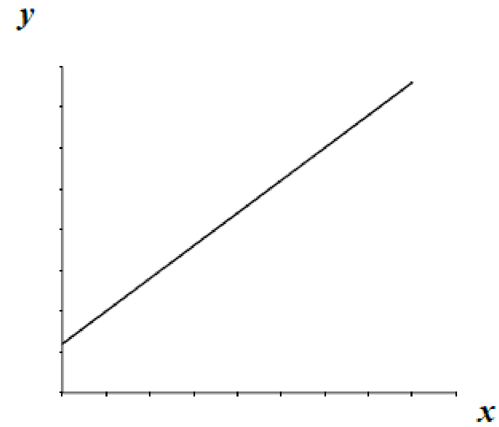
Use the equation to find out what figure will have a perimeter of 522?

Which of the following graphs ARE linear?

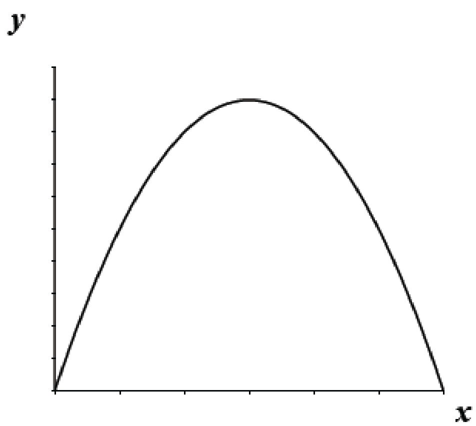
a.



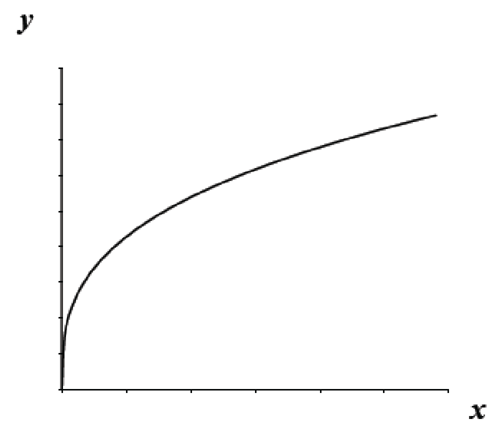
b.



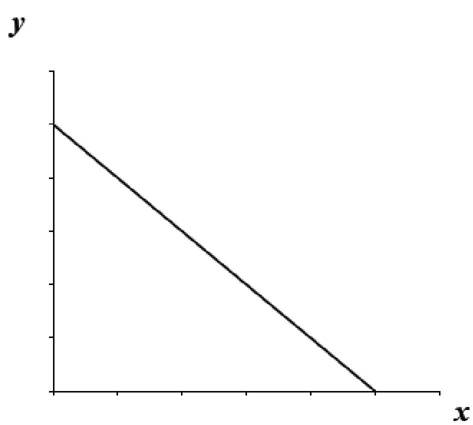
c.



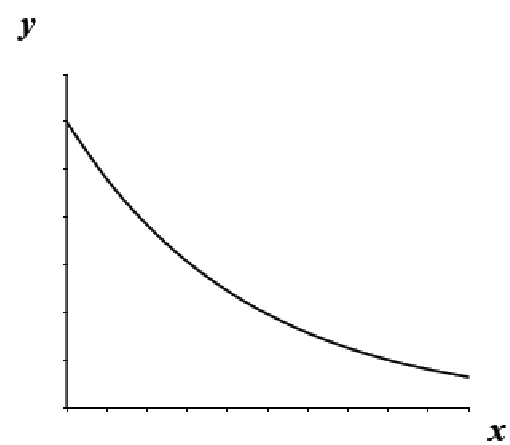
d.



e.



f.



Have you noticed, that all of the graphs we have drawn have gone up by a constant amount? The graphs have always been a straight line. When this happens, it is called a

LINEAR RELATION

Characteristics of a linear **graph**…

With a yes or no, state whether the following equations are linear (yes) or not (no).

a. $2x + y = -3$	b. $x + y = x^2 - 3$
c. $x = 3$	d. $2x^2 + 3x + 2 = 0$
e. $y^3 + 2y = -2$	f. $4x = 2y + 3$
g. $y = -10$	h. $x^2 = 3$
i. $y = 3x + 4$	j. $2x + y = 5 - 2y + 3x$
k. $6y = 2y^2 - 10$	l. $12x = 0$
Characteristics of a linear **equations**…	

With a yes or no, state whether or not the following table of values are linear (yes) or not (no).

a.

x	0	1	2	3	4
y	5	8	11	15	18

b.

x	-3	-2	-1	0	1
y	-8	-7	-6	-5	-4

c.

x	10	8	6	4	2
y	12	8	12	8	12

d.

x	-1	0	1	2	3
y	-6	-3	0	3	6

Characteristics of a linear **table of values**…

Now that we have established what a linear relation, linear table of values, and linear equation looks like, let's get to work!

Example 1: Complete the table of values for each function

a. $y = 3x + 8$

x	y
9	
-6	
3	
-1	
-2	

b. $y = -2x - 1$

x	y
-3	
-1	
0	
4	
10	

c. $y = \frac{1}{2}x - 3$

x	y

d. $y = -\frac{2}{3}x + 1$

x	y

Group Task #9

1. Create your own linear equation. All group members must agree that it is linear. Bonus marks are given for being creative (don't just use easy positive numbers).

2. Using your linear equation, create a table of values.

x						
y						

Show your work here:

3. Graph your linear relation.

Group Task #9

1. Linear Equation

2. Table of values.

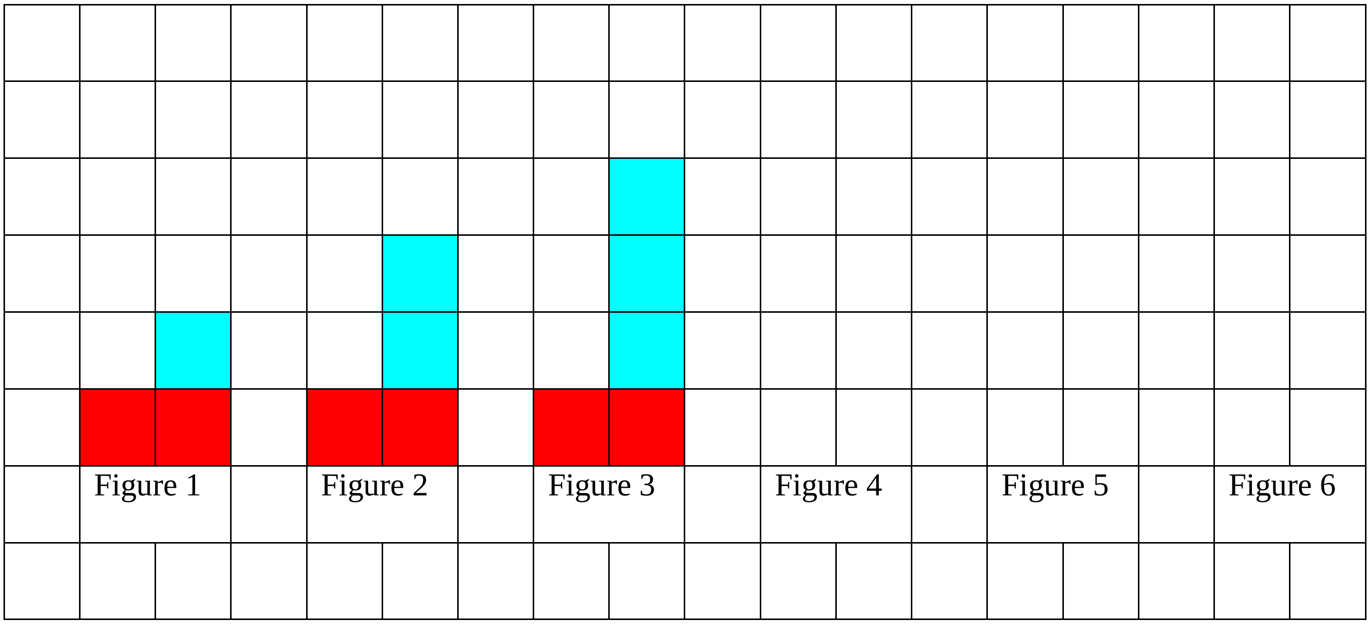
x						
y						

Show your work here:

3. Graph the linear relation.

Assignment: Lesson 5 **P9.1**

1. Using the following figures



a. Draw in the next 3 figures.

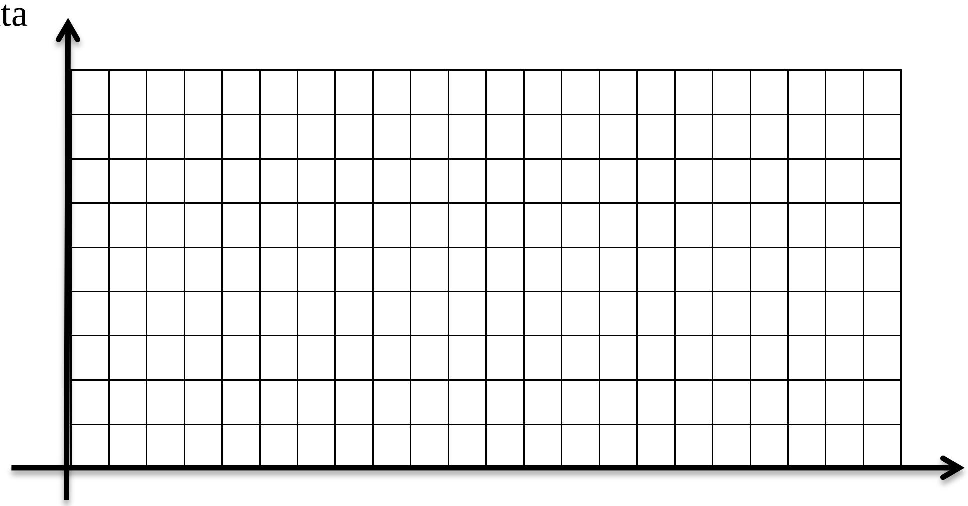
b. Create a table of values using the data from above.

c. Describe how the pattern is working using words.

d. Crack the code.

e. How many squares will be in figure 132?

f. Graph the data



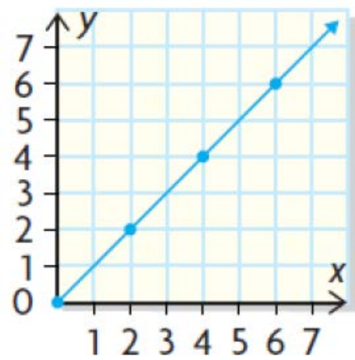
g. Is this graph linear? How can you tell?

2. State whether the given functions are linear or nonlinear:

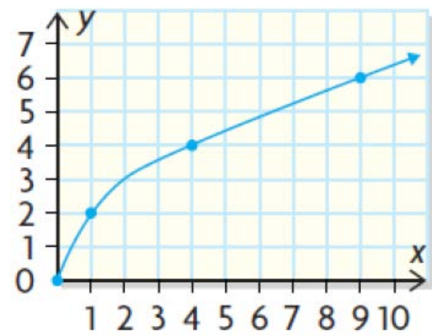
Function	Linear/Nonlinear
a. $y = -6x + 8$	
b. $y = -2x^2 + 1$	
c. $y = 2 + 4x$	
d. $y = 5$	
e. $y = x^3 + 4x$	
f. $2x = 4y + 3$	
g. $y = 2x^2 - 3x + 1$	

3. Identify each relation as linear or nonlinear. Explain how you know.

a.



b.



4. Determine if the table of values represent a linear function or non-linear function

a.

x	y
-2	-5
0	1
4	13
10	31

b.

x	y
-1	-3
0	2
1	6
2	12

5. Crack the code.

a.

r	0	1	2	3	4
p	-3	2	7	12	17

b.

q	z
0	-5
1	-4
2	-3
3	-2
4	-1

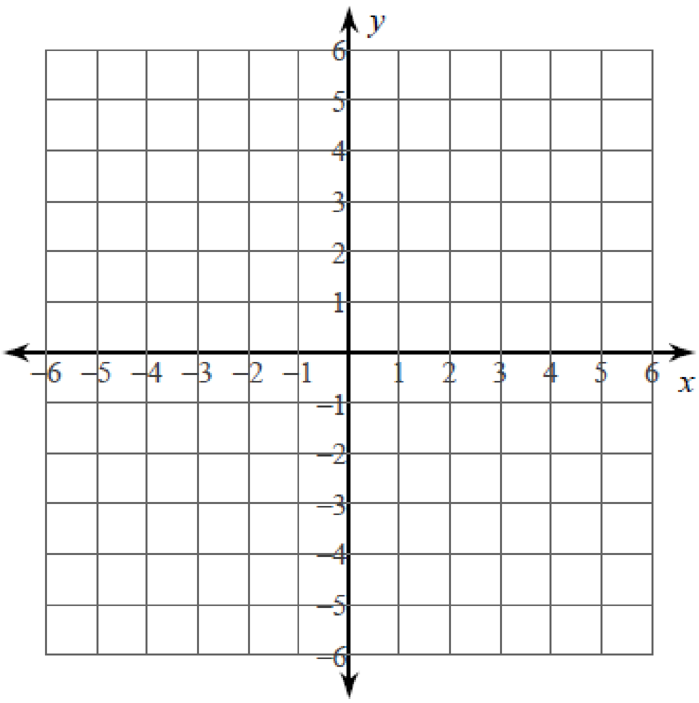
c.

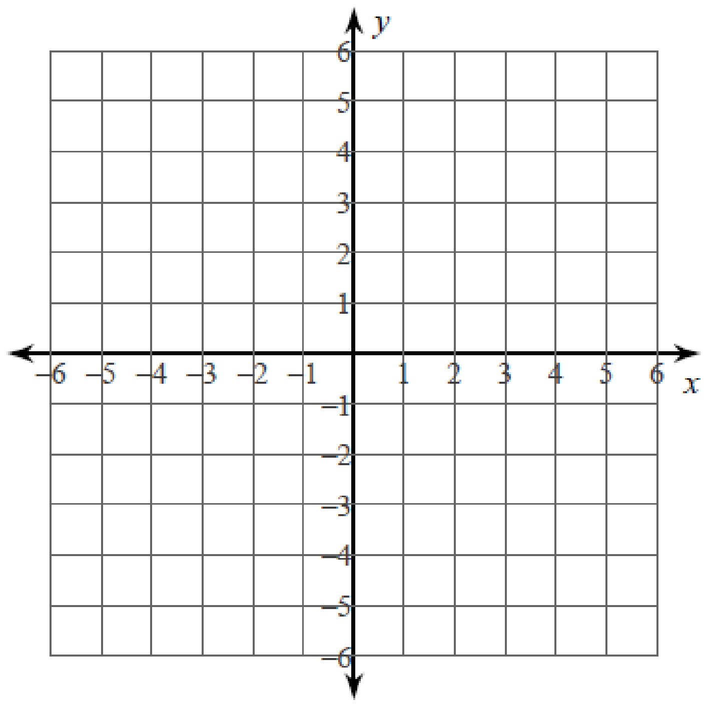
x	0	1	2	3	4
y	-11	-4	3	10	17

6. Create a table of values for each linear relation, then graph the relation.

a. $y = -x + 3$

b. $y = 2x - 3$





To The Teacher:

Continuing on the concepts and ideas worked on in the previous lessons, students will now look at graphing linear relations from three different scenarios; from a table of values, from an equation, and from a story. As with most lessons, it is important that there is a continual dialogue between the teacher and students. Students should be providing most of the instruction of the lesson with the teacher facilitating the discussion and asking key questions to promote understanding.

Lesson Process:

1. Example 1: Using the table of values given in example 1, have students graph the linear relation in their workbooks.

2. Have students "crack the code" that represents the table of values. This is finding the linear equation. Have student share their ideas and come up with a consensus on what the linear equation that represents the table is.

3. Using the graph, have students interpolate the value of y if they know the value of x. They are finding the missing partner to x. Students don't know that this is called interpolation and that term will be introduced in lesson 8.

4. Using the graph have students interpolate the value of x if they know the value of y. They are finding the missing partner to y. The term interpolation will be discussed in lesson 8.

5. Example 2: Using the equation given in example 2, have students create a table of values. You may want to discuss the use of negative values for x. Why doesn't it make sense? This question comes from the McGraw Hill & Ryerson Grade 9 Textbook Page 232.

6. Have students graph the relation. You may want to have a discussion on the importance of selecting numbers and scale for the horizontal and vertical axis.

7. Answer the provided questions in example 2. The students will be extrapolating their answers. The term extrapolation will be discussed in lesson 8. Have students verify their approximations by using the equation. Discuss the difference between approximate solutions and exact solutions.

8. Example 3: Students are not given an equation or table of values. They will need to come up with the equation on their own. This is their first time encountering a question like this so far. You may want to tackle this question together as a class soliciting ideas from

students. Once an equation has been agreed upon, have students create a table of values and graph the relation.

9. Using the equation, have students answer the remaining questions.

10. Assignment: Lesson 6

    You will need to use your classroom resource. Suggestions for an assignment are as follows:

MathLinks (McGraw-Hill Ryerson): Page 239-242

#15	Given table of values: find equation and graph.
#16, 17	Given equation: find table of values and graph
#4, 5, 20	Given story: find equation, table of values, and graph

Math Makes Sense (Pearson): Page 160

#14, 15, 16,	Given story: find equation, table of values, and graph

Math Makes Sense (Pearson): Page 170-173

#10	Given equation: find table of values and graph
#8	Given table of values: find equation and graph
#11, 13	Given story: find equation, table of values, and graph

Graphing Linear Relations

A. From a Table of Values

You are given * **the table of values**

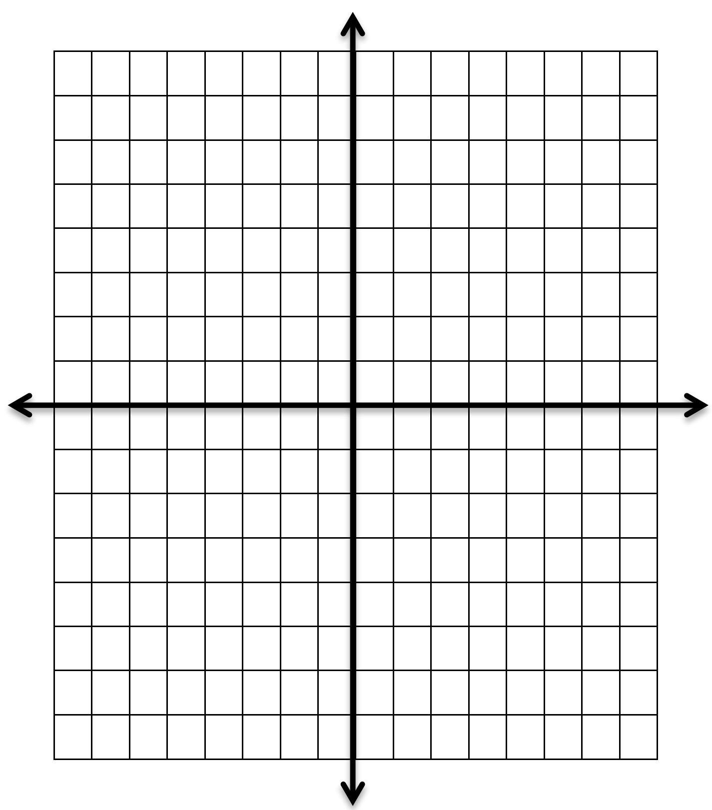
You need to find * **the equation**

* **the graph**

Example 1: Given the following table of values,

a) Graph the linear relation
b) Find the code (equation) that represents the table of values
c) Use the graph to estimate the value of y if $x = -8$
d) Use the equation to verify the value of y if $x = -8$
e) Use the graph to estimate the value of x if $y = -7$
f) Use the equation to verify the value of x if $y = -7$

x	y
-3	9
-2	7
-1	5
0	3
1	1
2	-1
3	-3



B. From an Equation

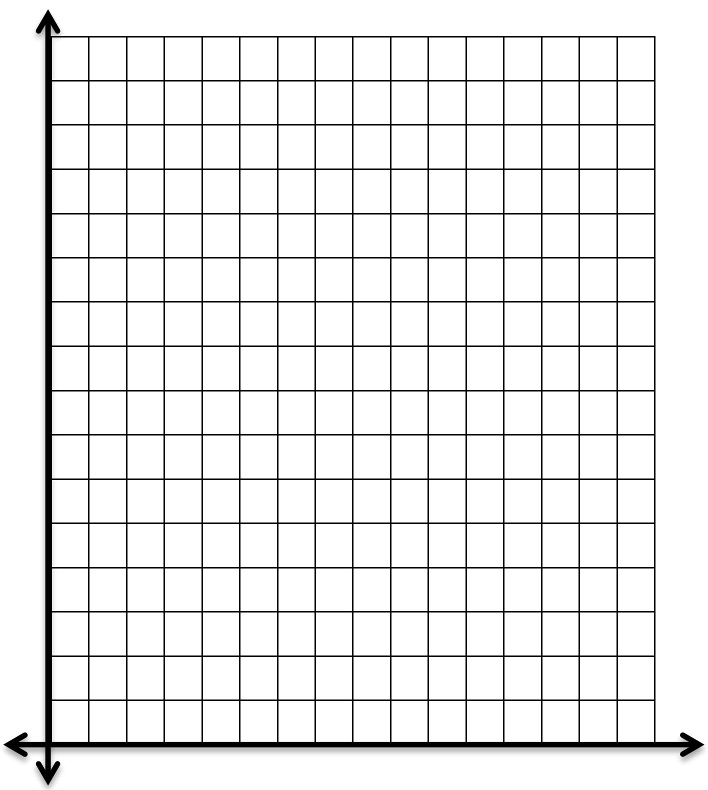
You are given * **the equation**

You need to find * **the table of values**

* **the graph**

Example 2: The world's largest cruise ship, *Freedom of the Seas*, uses fuel at a rate of $12800\ kg/h$. The fuel consumption, f, in kilograms, can be modelled using the equation $f = 12800t$, where t is the number of hours travelled.

a) Create a graph to represent the linear relation for the first $7h$.
b) Approximately how much fuel is used in $11\ h$? Verify your solution.
c) How long can the ship travel if it has approximately $122000\ kg$ of fuel? Verify your solution.



C. From a Story

You are given * **a story about a problem**

You need to find * **the equation**

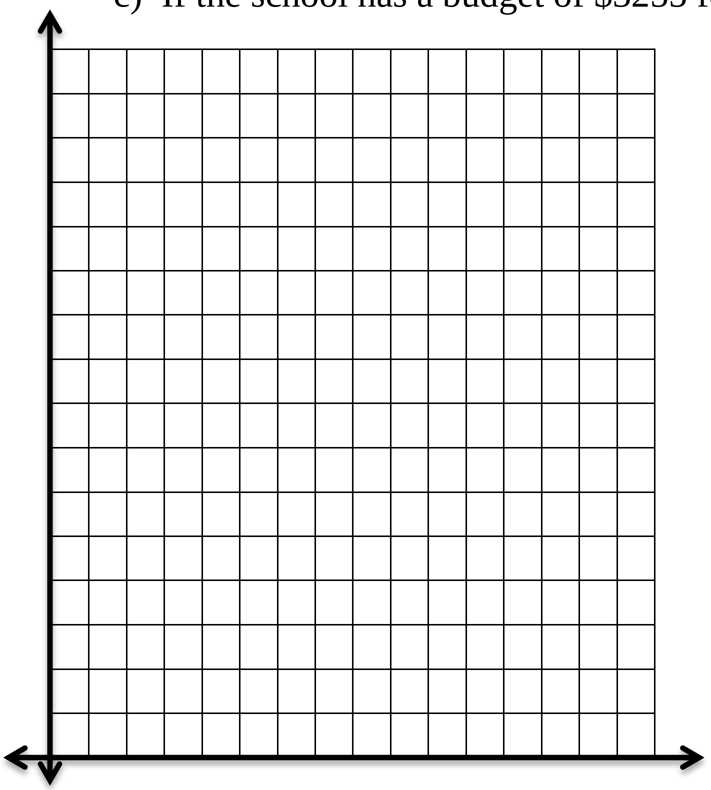
* **the table of values**

* **the graph**

In this order!

Example 3: A school pays a company $220 to design a gym T-shirt. It costs an additional $15 to make each T-shirt.

a) Develop an equation to determine the cost of the T-shirts. Make sure you state what your variables represent.
b) What would it cost to make 253 T-shirts for the grade 9's only?
c) If the school has a budget of $3255 for T-shirts, how many T-shirts can be ordered?



Lesson 7: Graphing $y = a$, $x = a$, and $ax + by = c$ **P9.1**

To The Teacher:

Up until this point, students have only had experience with diagonal lines primarily written in $y = ax + b$ format. This lesson takes into account that we can have horizontal and vertical linear relations.

RN #7

Cooperative Learning

RN #5

Formative Assessment

Lesson Process:

1. Begin the lesson by looking at three linear relations that result in a diagonal line. Have students create a table of values and graph the lines on the provided graph.

2. Have students sit in their learning groups and work through task #10. The table of values in the task will result in a horizontal line and a vertical line. Try to get students to describe a situation that would represent the graph. An idea for the horizontal graph could be someone who had previously been walking, but is now stopped. The vertical line graph is harder to find a scenario for which is interesting to note. An example would be an extension ladder where you are comparing the distance of the base of the ladder to the height of the ladder as it extends up. It can lead to a good discussion on how the horizontal and diagonal lines are more prevalent in society than the vertical line graph. **RN #7**

3. Have students note any differences between the vertical/horizontal table of values to the diagonal table of values they have been working with. Students should notice that in a horizontal and vertical table of values, one of the columns stays the same.

4. Discuss what a horizontal and vertical line equation look like. You may want to provide an example in each of the spaces provided as well as a small table of values.

5. Example 2: Have students graph each of the equations provided. They will need to do some manipulation on b. and c. in order to graph the lines.

6. Have students take another look at diagonal lines again. You may want to talk about the word oblique. This time, the equation will be all jumbled up and students will need to

put it into $y = ax + b$ form. It will make their calculations for their table of values a lot easier if they can do it that way.

7. Example 3: Have students graph $5x - 2y = -10$. They will first need to manipulate the equation before they find a table of values. Have them graph the table of values on the graph provided.
8. Example 4: Have students state if the given equations are diagonal, vertical, or horizontal by sight.
9. Exit task **RN #5**

MathLinks (McGraw-Hill Ryerson): Page 23-242

#6	Horizontal & vertical lines
NONE	Diagonal (oblique) lines: write the linear equation in the form $y = ax + b$, make a table of values, graph
#6, 7, 12	Horizontal, vertical, and diagonal lines in the same question

Math Makes Sense (Pearson): Page 178-180

#4, 5, 6, 7, 8, 12	Horizontal & vertical lines
#10, 15	Diagonal (oblique) lines: write the linear equation in the form of $y = ax + b$, make a table of values, and graph
#11, 13, 18	Horizontal, vertical, and diagonal lines in the same question.

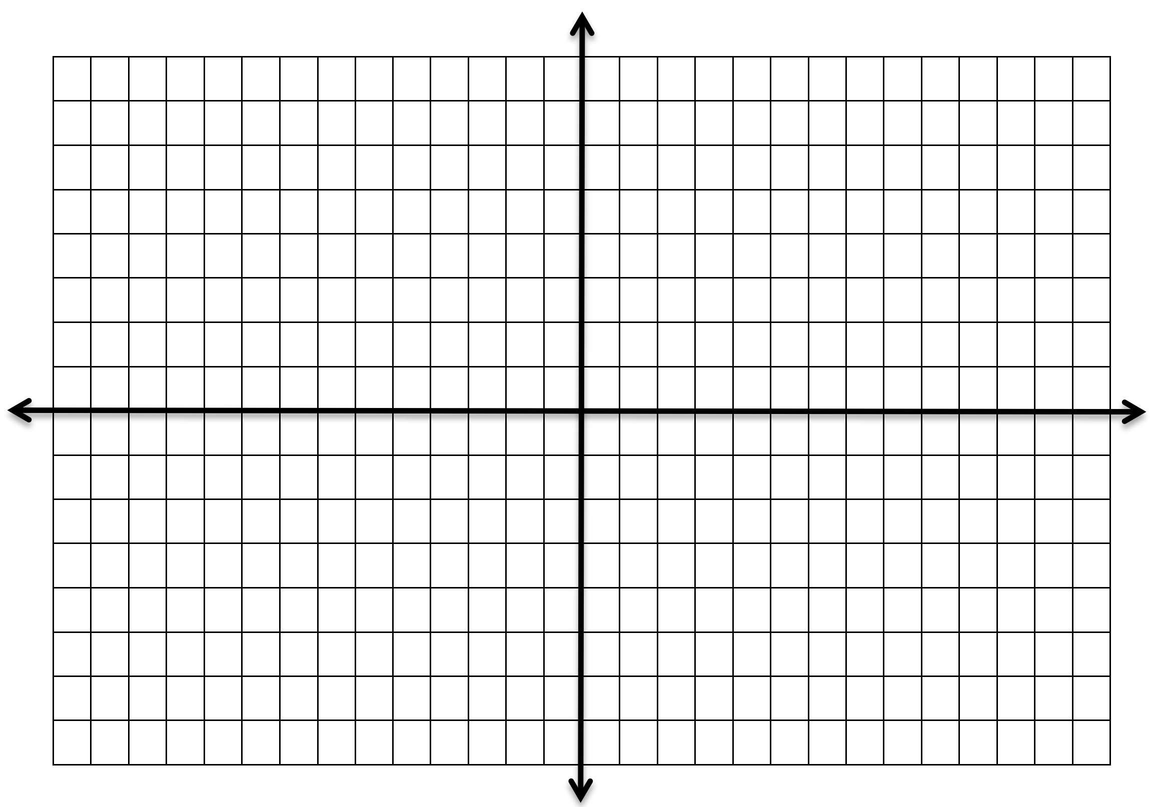
Graphing $y = a$, $x = a$, and $ax + by = c$

A. Graping $y = a$ and $x = a$

So far we have only tackled linear relations of the form $y = ax + b$, where a and b are rational numbers (but not 0).

Example 1: Graph the following

$y = -2x - 3$ $P = 3f - 2$ $s = 2f + 1$

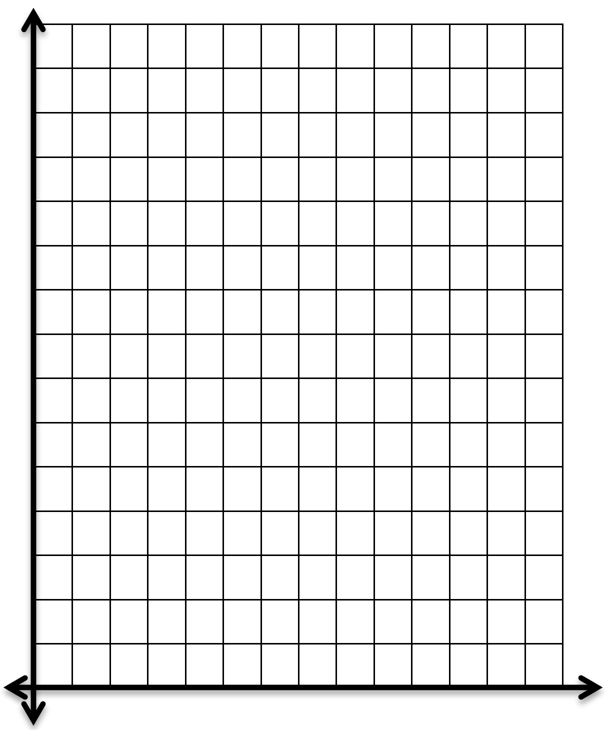


All of these equations resulted in a ____________________________ graph

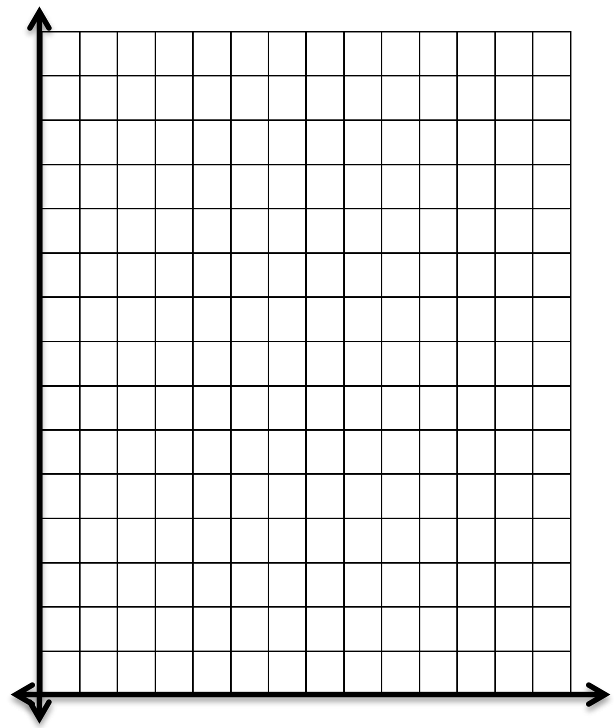
Task #10: Graph the following table of values

Time, x (s)	Distance, y (m)
0	8
20	8
40	8
60	8
80	8
100	8

Distance, x (m)	Height, y (m)
4	1
4	2
4	3
4	4
4	5
4	6



Describe a situation that the graph might represent.



Describe a situation that the graph might represent.

What is different about the above table of values and graphs from the ones we have been working with so far?

Horizontal and Vertical Lines

VERTICAL LINE $x = a$	HORIZONTAL LINE $y = a$

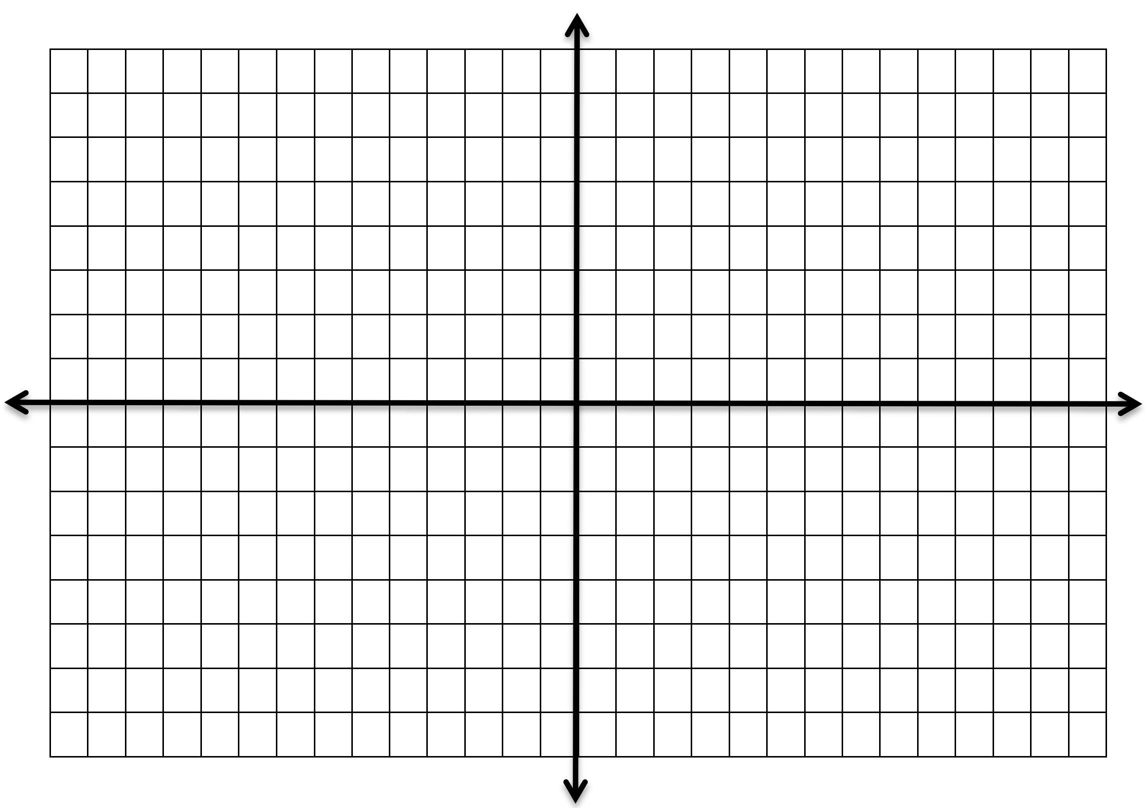
Example 2: Graphing and Describing Horizontal and Vertical Lines

For each equation below:

i) Graph the equation

ii) Describe the graph.

a. $x = -2$ b. $y + 4 = 0$ c. $2x = 7$



B. Graphing $ax + by = c$ (Diagonal Lines)

Anytime a linear equation has an x and a y, it will result in a diagonal line graph. All of the diagonal equations so far have been of the type $y = ax + b$ where the y has been isolated on one side, and the x on the other. Now we are going to work with an equation where all the pieces are jumbled up.

What to do? ____

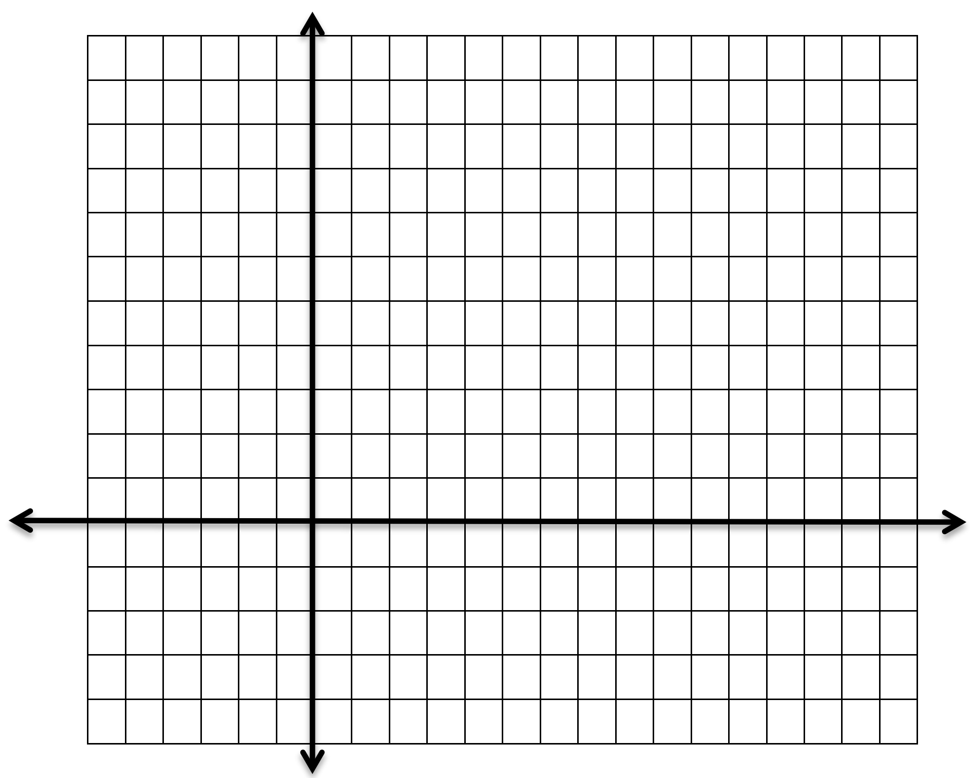
Examples:

Example 3: Graphing and Equation in the Form

For the equation $5x - 2y = -10$

a) Make a table of values for $x = -2, 0, and\ 2$.

b) Graph the equation



Example: State whether the following equations result in a diagonal (oblique) line, vertical line, or a horizontal line.

Equation	$y = 3x + 2$	$x = -8$	$3y + 2x + 2 = 0$	$4 = y$
Horizontal/Vertical/ Diagonal				
Equation	$3x + 7y = -2$	$y = -2x$	$x = 10$	$2y = 2x + 5$
Horizontal/Vertical/ Diagonal				

Lesson 7: Exit Question Name:_____________________

How are equations for diagonal (oblique) lines different from the equations from horizontal and vertical lines?

Lesson 7:Exit Question Name:_____________________

How are equations for diagonal (oblique) lines different from the equations from horizontal and vertical lines?

Lesson 7: Exit Question Name:_____________________

How are equations for diagonal (oblique) lines different from the equations from horizontal and vertical lines?

Lesson 7: Exit Question Name:_____________________

How are equations for diagonal (oblique) lines different from the equations from horizontal and vertical lines?

To The Teacher:

In this lesson students need to find the connection between the equation of a line and the graph of a line. **RN #4**

RN #4

Inquiry

Lesson Process:

1. Example 1: The goal of this example is for students to recognize the link between the graph and equation with respect to the y-intercept and the direction of the line. Have students try to find the link between the linear graph and the linear equation. They may see that the tie to the y-intercept, but they may have troubles finding the slope. I have set up the questions so that students may be able to spot that when the coefficient of the x is negative, the line goes down and when the coefficient of the x is positive, the line goes up. It is recommended in grade 9 to not use the term slope. You can use the word "steepness" in its place if you like.
2. Example 2: Follows the same idea as example 1, comparing the direction of the line and y-intercept with the numbers in the equation.
3. Example 3: Looks at what happens when all of the equations have the same y-intercept and they all have positive slopes. This may take some time to figure out how the coefficients of 1, 2, and 5 are tied to the graphs.
4. Example 4: Using their new found knowledge on counting squares to find the steepness of the line, have students find which graph matches the equation $y = -2x + 3$.
5. Discuss with students that they will see linear equations written in different ways, but it is easiest if we rewrite them all as $y = ax + b$ where a and b are rational numbers.
6. Example 5: Have students practice rewriting linear equations in the form $y = ax + b$

7. Assignment: Lesson 7

MathLinks (McGraw-Hill Ryerson):

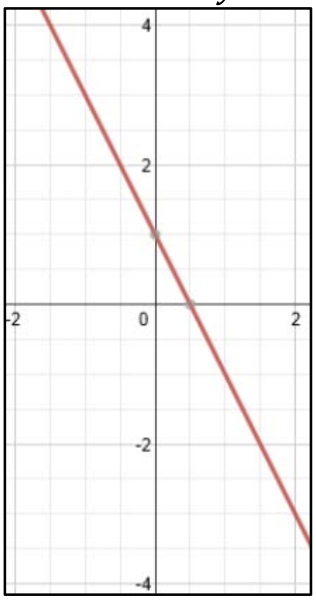
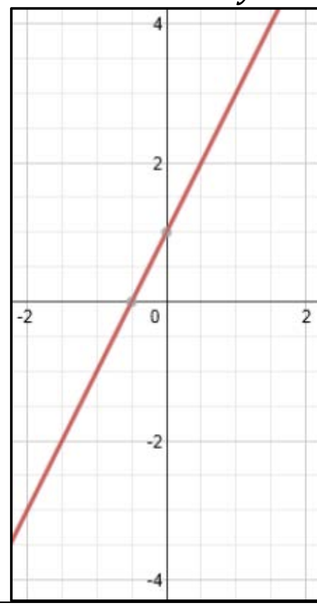
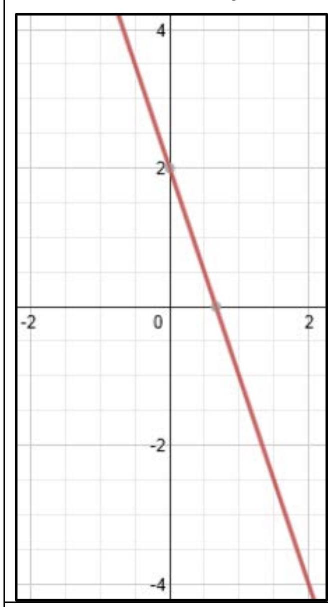
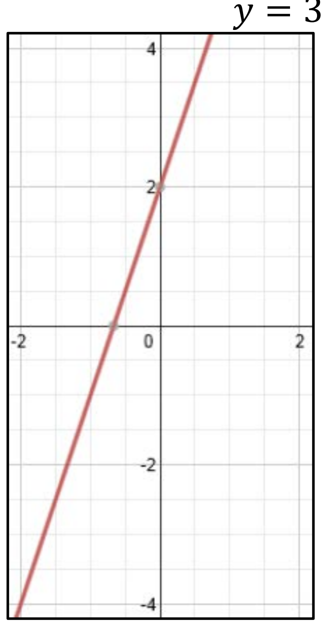
None. MathLinks require students to find the equation of the linear relation from a graph. I did not cover that in my interpretation of the outcome P9.1.

Math Makes Sense (Pearson):

#3, 4, 8	Focus on steepness (slope)
#5	Focus on y-intercept
#6,	Must write equation in $y = ax + b$ form before matching graph
#9, 11, 12	General matching questions

Matching Linear Equations with Graphs

Example 1: Can you find a visible relationship between the graphs and their equations?

$y = -2x + 1$ 	$y = 2x + 1$ 
$y = -3x + 2$ 	$y = 3x + 2$ 

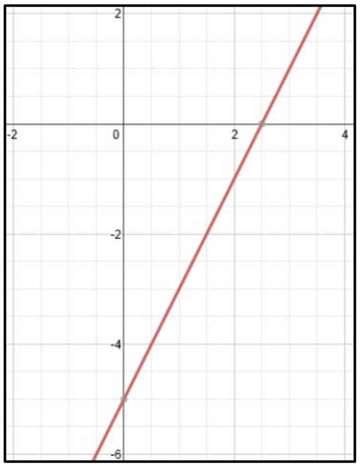
What can the equation tell us?

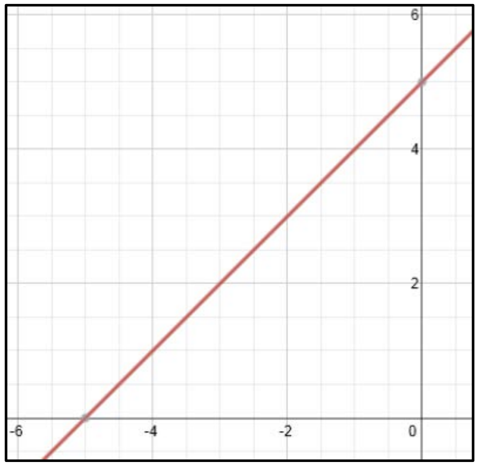
$$y = ax + b$$

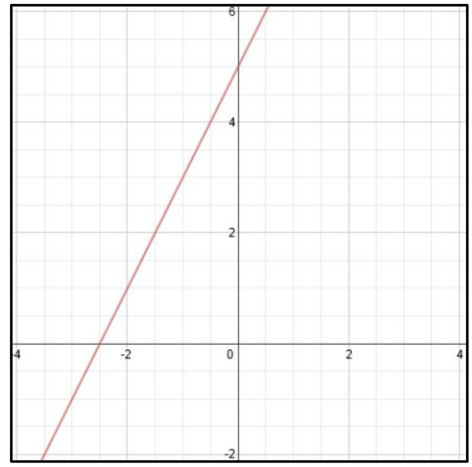
Where: a can be any rational number (except 0)
b can be any ration number

Example 2: Using the ideas from above, match the equations with its graph.

$A.\ y = 2x + 5$ $B.\ y = 2x - 5$ $C.\ y - x = 5$







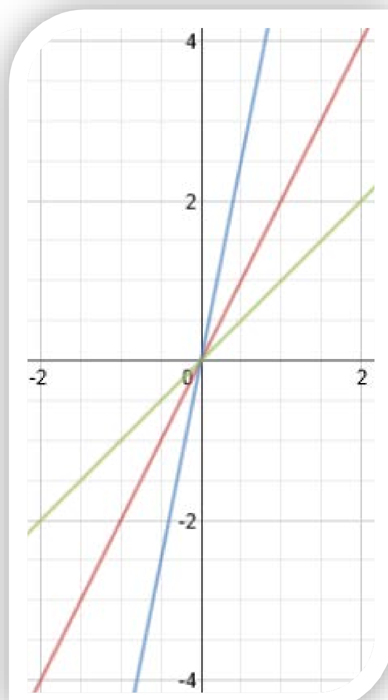
What happens when…?

Example: Match the equations with its graph.

A: $y = x$

B: $y = 2x$

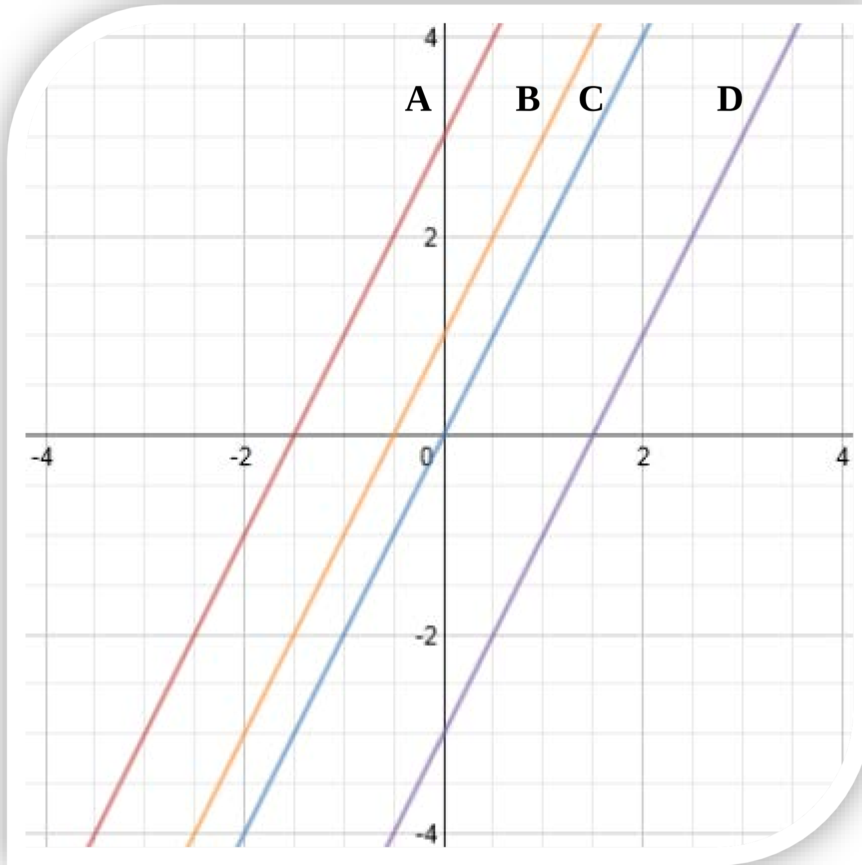
C: $y = 5x$



What do we do when all the equations and graphs have the same y −intercept?

Example 3: Which graph on this grid has the equation $y = -2x + 3$?

You need to be able to explain why you chose that letter.



You will see many linear equations written in different ways. It is always the easiest to interpret if you write it as _________________________.

Example 4: Rewrite the following equations as $y =$ ____ $x +$ ____

a. $2x - y + 5 = 0$ b. $3x + 5 = -2$ c. $y - 2 = 3x$

To The Teacher:

This lesson brings the new terms of interpolation and extrapolation into play. Students will need to interpret the graph to find approximate values. A discussion of what makes an answer exact and what makes an answer an approximation should be part of this lesson.

Lesson Process

1. Example 1: Have students graph the table of values on the given graph and answer the given questions. This would be a good time to discuss that there are times when the table of values is too hard to decode. As a result, we have to use the graph to find values that aren't in the table. Part c. asks why we can't get an exact value for 992.76, the answer is because we don't have the equation and it is too difficult to find (with our current knowledge).

2. Introduce interpolation.

3. Example 2: Have students fill in the table of values, graph the relation, and then answer the questions. You may want to ask, what makes this easier than the last question. The students should note that the numbers are much easier to work with.

4. Example 3: Have students graph $y = 2x - 5$. They will first need to make a table of values. Have them use the graph to find the answers to the given questions. You may want to point out that we can find out if they are exactly right by using the equation.

5. Example 4: As a class, discuss the questions associated with Sara selling chocolate bars.

6. Assignment 8: Use your class resource

MathLinks (McGraw-Hill Ryerson): Page 226-229

#4, 5, 6, 9, 10, 13, 14, 15, 16, 17	Given a graph: answer questions involving interpolation & extrapolation
#7, 8,	Given a table of values: graph and answer questions involving interpolation & extrapolation

Math Makes Sense (Pearson):

#4-10, 12-14	Given a graph: answer questions involving interpolation & extrapolation

Interpreting Graphs

Example 1: The weather balloon recorded the air temperature at different altitudes.

Altitude (m)	350	750	1000	1500	1800
Temperature (℃)	11.4	5.7	2.1	-5.0	-10.0

a. What is the temperature at $700m$?
b. What is the height at -6.5℃?
c. Are we able to get an exact value for temperature at 992.76m? How come? What would we need?

INTERPOLATION:

Example 2: I often like to make predictions on how far I can run at certain paces.

At a speed of 4 minutes per kilometre (1000 metres):

Time (minutes)	2	4	6	8	10
Distance (metres)		1000		2000	

a. Predict how long it will take me to run $10{,}000m$ (10km)

b. Predict how far I will run in 25 minutes

c. What assumptions are we making in this situation?

d. Can we find the equation? If so, what is it?

Extrapolation:

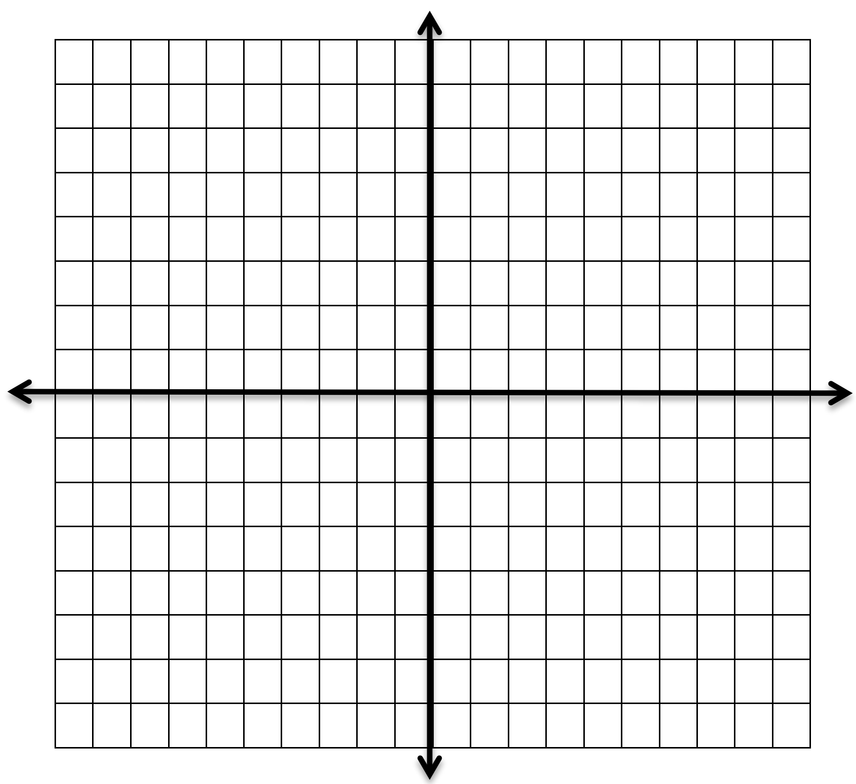
Example 3: Graph $y = 2x - 5$

a. If $x = 9$, what is y?

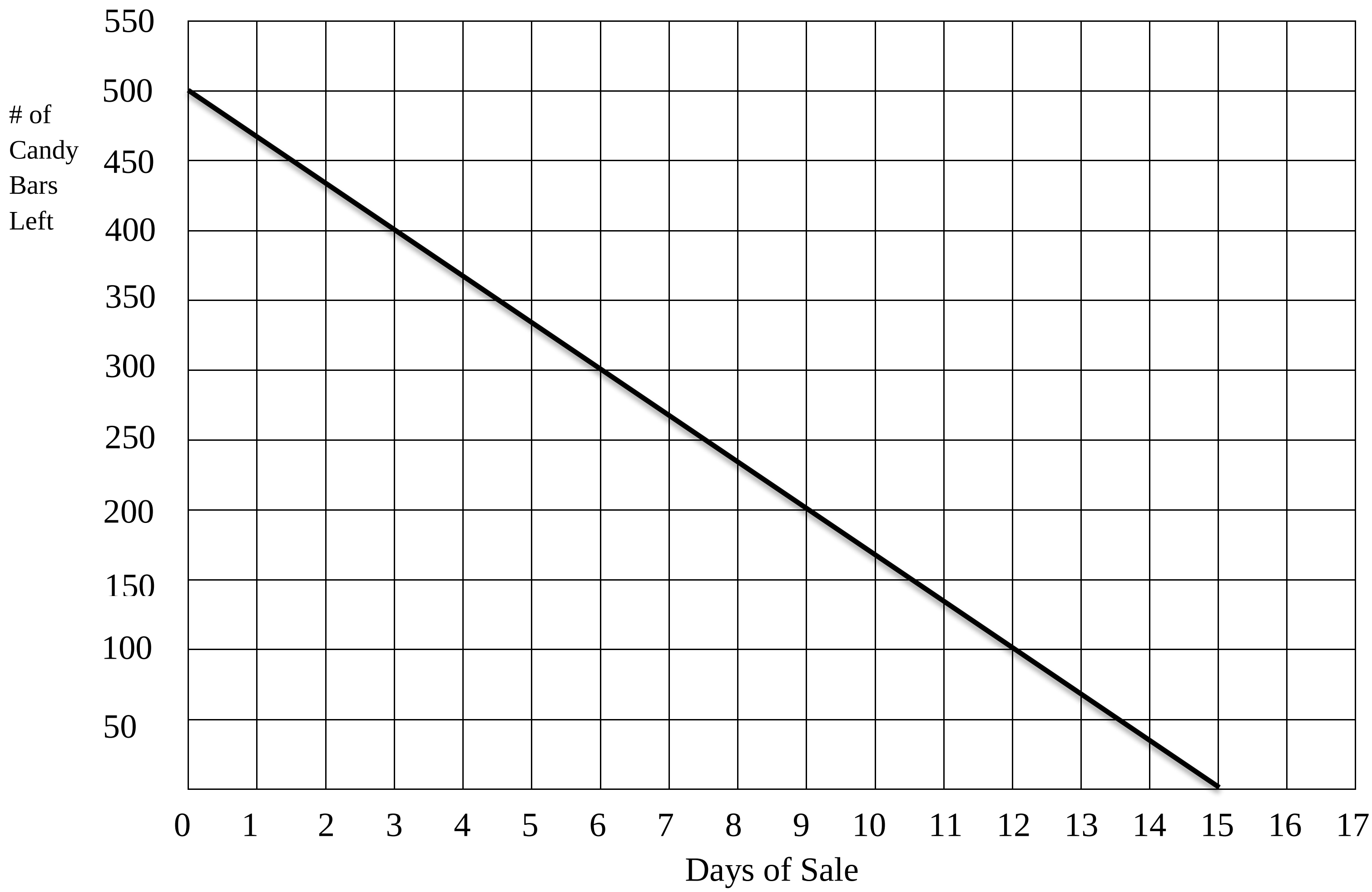
b. If $y = -3$, what is x?

c. If $x = 12$, what is y?

d. If $y = -10$, what is x?



Example 4: Sara is selling candy bars to raise money for a club she belongs to. Here is a graph that displays her efforts in selling the candy bars door to door.



1. Explain why the graph is going down.

2. How many candy bars were there to sell at the beginning?

3. How many days did it take for Sara to sell all of her chocolate bars?

4. Were the candy bars sold at a constant rate?

5. At what day was Sara half-way through her chocolate bars?

6. How many candy bars are left after 3 days?

7. How many days have gone by when there are 100 candy bars left?

Understanding Linear Relations
Squares Game

Question: You have a square game board with dimensions n x n. The game board has a penny in every square except for two. One corner of the board is left empty and the corner furthest from this empty square has a die in it. What is the smallest number of moves required to get the die into the initially empty cell given that the only valid move is to move a marker into an adjacent empty cell (not diagonal)?

1. Try the game with a 3 x 3 board.

2. Try the game with a 4 x 4 game board (create your own on loose leaf)

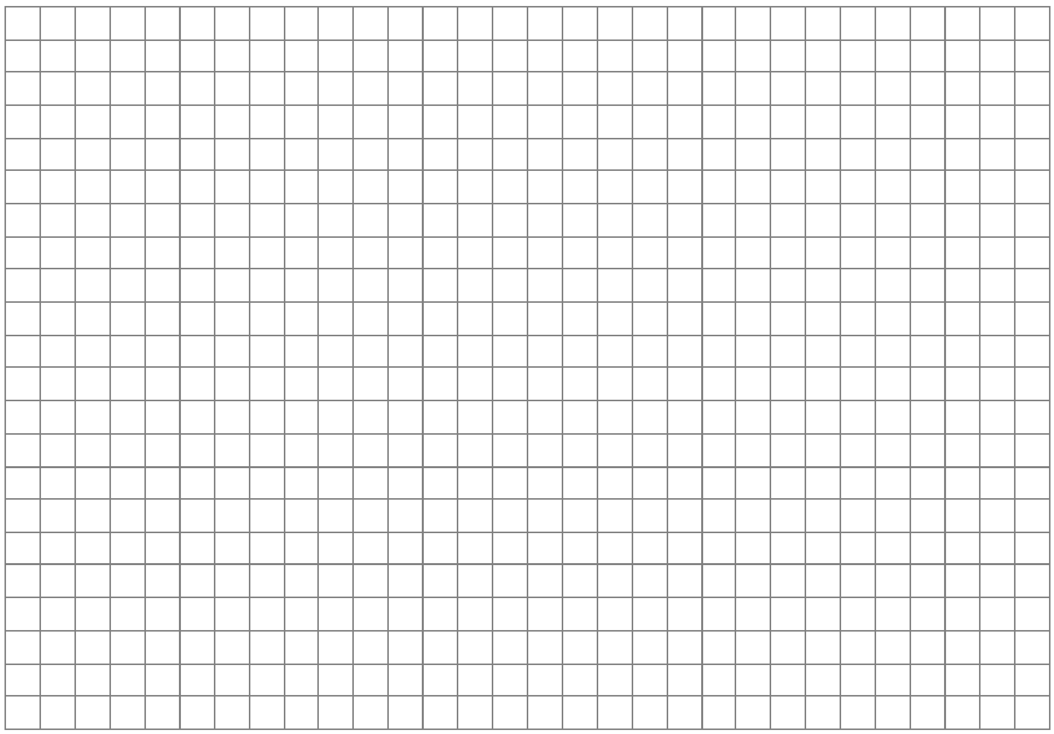
3. Repeat the game with a 5 x 5 game board, then a 6 x 6 game board.

4. Fill in the following table using the values you have found.

Side Length (s)	# of Moves (m)

5. What pattern do you notice from the chart? Can you predict the number of moves on a 7 x 7 board?

6. Graph the results from your chart. Put the n value on the vertical axis and the number of moves on the horizontal axis.



7. What does your graph show for the smallest number of moves for a 2 x 2 game board? (This is called interpolation). Try it – does it work?

8. What does your graph show for the smallest number of moves for a 8 x 8 game board? (This is called extrapolation).

9. Can you come up with an equation to represent this patter? The smallest number of moves, (m), in terms of the game-board side lengths, (s)?

Outcome: P9.2

Model and solve situational questions using linear equations of the form

where a, b, c, d, e, and f are rational numbers

NEW - New for Grade 9 Students

- $ax = b$
- $\frac{x}{a} = b, \ \ a \neq 0$
- $ax + b = c$
- $\frac{x}{a} + b = c, \ \ a \neq 0$
- $ax = b + cx$ *NEW*
- $a(x + b) = c$
- $ax + b = cx + d$ *NEW*
- $a(bx + c) = d(ex + f)$ *NEW*
- $\frac{a}{x} = b, \ x \neq 0$ *NEW*

Lesson 1: What is an Equation?

Lesson 2: Solving Simple Equations

Lesson 3: Solving Equations with Variables on One Side

Lesson 4: Solving Equations with Variables on Both Sides

Lesson 5: Solving Equations Containing Parentheses

Lesson 6: Solving Equations Containing Fractions

Lesson 7: Solving Equations Containing Decimals

Lesson 8: Solving Situational Questions Using Equations

Previous Experience:

Grade 7

Outcome P7.2

Demonstrate an understanding of equations by:

- Distinguishing between equations and expressions
- Verifying solutions to equations

Outcome P7.3

Demonstrate an understanding of one- and two-step linear equations of the form $\frac{ax}{b} + c = d$ (where a, b, c, and d are whole numbers $c \leq d$ and $b \neq 0$) by modelling the solution of the equations concretely, pictorially, physically and symbolically and explaining the solution in terms of the preservation of equality.

Grade 8

Outcome P8.2

Model and solve problems using linear equations of the form:

- $ax = b$
- $\frac{x}{a} = b,\ a \neq 0$
- $ax + b = c$
- $\frac{x}{a} + b = c,\ a \neq 0$
- $a(x + b) = c$

Concretely, pictorially, and symbolically, where a, b, and c are integers.

To The Teacher:

Students have encountered equations in grade 7 and 8. However, they all come into grade 9 with differing skills. It is important to know what skills the students have in your classroom. When I created the lesson I created them under the assumption that they have seen equations before, but may have forgotten how to solve.

One of the new skills for students in grade 9 will be working with variables on both sides of the equation, which is dealt with in lesson 4. Lessons 1-3 are a review of grades 7 and 8.

The lessons in this unit are more traditional than others. Most lessons consist of examples done as a class, students trying some on their own (or with a partner) and then an assignment. You will notice that there are not as many research notes tied to the lessons. However, a common theme of discourse should try to be attained in the classroom. Asking students what they think, asking students randomly for answers, discussing with students why they chose what they did, are all great ways to keep the classroom dynamic.

I have included a start-up question and an exit question for each lesson. **RN #5**

Lesson 1: What is an Equation? **P9.2**

To The Teacher:

The purpose of this lesson is to discuss what an equation is. This is a great opportunity to assess student's prior knowledge. Having students construct their own equations will really help them to understand how to deconstruct it to find out what the variable really represents.

RN #11

Concept Attainment

RN #2

Prior Knowledge

RN #12

Learner Generated Examples

RN #5

Formative Assessment

Lesson Process:

1. Begin with the start-up question. Use this as a jumping off point to discuss what makes an equation.

2. You can ask students what they remember about equations from grade 8. Some key phrases might be; equal sign, isolate the variable, inverse operation. You may even want to ask students to give you an example of an equation (have them make it up). **RN #2**

3. Example 1: Go through the worksheet of "what is an equation?" Students should recognize that equations must have an equal sign. Ask students what it is called when there is no equal sign (expression). You can have many different variables in an equation (formula) **RN #11**

4. Example 2: Have students provide the answers for the true and false questions. For false answers, have them provide counter example.

5. Example 3: Ask students for the answers to the questions in this example. Some ideas of life activities where you have to undo the steps to get back to the beginning are; changing a tire, retracing steps to find a lost item.

6. For section B. a, Have students choose a number and then solicit ideas from students for operations to place on the number. Once you get to the bottom, have students undo those operations to get back to the original number.
7. For section B. b, have students state the 4 main operations used in math and their inverses.
8. For section B. c, have students undo the steps for the wrapped gift.
9. For section B. d, have students take the idea of undoing to deconstruct the equation.
10. Conduct a mini whiteboard activity where students are creating their own simple equations. In the initial activity they try to disguise $x = 5$. Have students share their ideas with the class. You may want to try a few more if students are struggling. Don't forget to try a negative value for the initial number (ie. $x = -4$) or when you are using the operations. **RN #5**
11. For the last activity of lesson 1, have students create their own equations by using the operations of multiplication and addition/subtraction. There is a worksheet that goes along with this activity. Have students fold the paper vertically in half. Because equations can get complicated quite quickly, have them multiply first and then add/subtract. In this way, they will see the reversal process much easier than if they add/subtract first and multiply second. You may want to explain to students that we are beginners at creating equations, so we need to take it slow. Once students have created their 3 equations, have them swap with a partner and try to undo the operations. Don't let them peek on the other side of the paper! **RN #12**
12. Exit question **RN #5**

P9.2 Linear Equations

Lesson 1: What is an Equation? **P9.2**

What is an Equation?

A. What is an equation?

Example 1: Circle the "equation(s)" in each of the circles. Explain why some of them are NOT equations.

$x + 5 = 12$	$x - 5 = 8$
$2 + x = 12$	$3 + x$

$5x = 10$	$2x + 2$
$9x = 18$	$-12x = 24$

$2x - 9 = 11x$	$x = 5x + 12$
$5x - 2 + 3x$	$3x + 4 = 2 + x$

$y = 3x + 1$	$P = 2l + 2w$
$F = \frac{3}{2}C - 32$	$3x + y$

Example 2: Answer True (T) or False (F). Be prepared to justify your answer.

a. Every equation has exactly two sides. ________

b. Every equation has exactly one equal sign. ________

c. Every equation has exactly one variable ________

Example 3: Answer the following with a sentence.

d. What is equality in mathematics?

e. What does the following mean in mathematics? $-3x + 5 = -4$

f. What is an inverse operation?

g. What is something in your life where you have to undo the steps to get back to the beginning?

B. Wrapping and Unwrapping Numbers

a. Let's try wrapping a number and then unwrapping it!

Operations	Number	Operations	Number
Choose a Number			

Let's Reverse the Process

The ENTIRE purpose of solving an equation is to discover the number that has been disguised with operations!

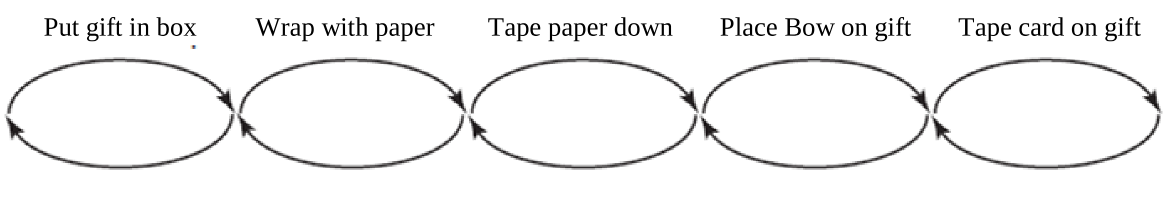
In order to UNDO an operation, you must use its ______________________________.

b. What are the four main operations that we use when solving equations? Can you state their inverses?

4 Main Oper-ations

Operations	Inverse Operation

c. The top row of the arrow diagram shows the steps to wrap a gift. What steps are needed to unwrap the gift?



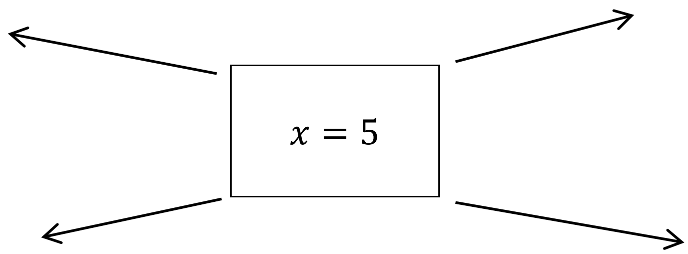
d. Let's look at how someone constructed an equation and the process by which to deconstruct the number.

$$\text{Start Equation} \quad \boxed{x = -2} \underset{\longleftarrow}{\xrightarrow{\text{Multiply by 4}}} \boxed{4x = -8} \underset{\longleftarrow}{\xrightarrow{\text{Add 3}}} \boxed{4x + 3 = -5} \quad \text{End Equation}$$

Lesson 1: Whiteboard Activity **P9.2**

Each student should have a mini-whiteboard, pen, and eraser.

Write in the center of the board:



Have students come up with multiple ways to disguise $x = 5$. Have students conduct 2 or 3 operations on $x = 5$. Have them share their answers on the board. Ask students how we would check all of the equations to make sure they were right.

Lesson 1: Group Activity - Create your own equation

Create Equation #1
Describe Operation

$x =$ ____

____

____

____

____

This Is My Equation ________________

Solve Equation #1
Describe Operation

Equation #1

____

____

____

____

Here is the solution ________________

Create Equation #2
Describe Operation

$x =$ ____

____

____

____

____

This Is My Equation ________________

Describe Operation

Equation #2

____

____

____

____

Here is the solution ________________

Create Equation #3
Describe Operation

$x =$ ____

____

____

____

____

This Is My Equation ________________

Describe Operation

Equation #3

____

____

____

____

Here is the solution ________________

To Teacher:

Students should have experience with this type of equation from grade 8. It might be best to find out what the students remember about solving equations before tackling any questions. I have grouped the questions into three different types; addition/subtraction, multiplication, and division. Use the last lesson's inverse operations discussion to answer the questions as a class.

RN #2

Prior Knowledge

Lesson Process:

1. Before the lesson begins you may want to ask students what are some of the key phrases that some of their other teachers have used when solving equations. Answers could be "isolate the variable", "get x by itself", "what you do to one side, you must do to the other", etc. Discuss with students what each of these phrases mean. You may want to write them on the board as students supply what they remember. **RN #2**

2. I have separated the examples into the operations needed to undo them as well as a section that have negative variables. I have included spaces for checking their solutions.

3. Work through the examples from sections A, B, C, and D. Students should have experience with this type of equation from grade 8. It is important to solicit any previous experience from the students. You may want to ask students:
   a. What does an equation mean?
   b. How do we find out what the solution/answer is?
   c. How do we undo addition, subtraction, division, multiplication?
   d. What is an answer/solution to an equation? **RN #2**

4. Ask students "how you are going to un-wrap equations to find the solution?" Hopefully they will remember the discussion on inverse operations from the day before. By using the idea of inverse operations, work through the examples in the workbook. Check the answers in the space provided.

5. Have students try some on their own or with a partner.

6. Assignment: Lesson 2

Solving Simple Equations

In order to solve equations, you need to use the inverse operations in order to discover what the variable actually equals.

Your goal is to separate the variables and numbers. Isolate the variable!

A. Solving Equations Using Addition and/or Subtraction

1. $x + 3 = -10$	Check:	2. $5 = w - 12$	Check:
3. $-7 = x + 17$	Check:	4. $18 + x = 2$	Check:

B. Solving Equations Using Division

1. $2x = 14$	Check:	2. $-9e = -72$	Check:
3. $-60 = 15x$	Check:	3. $-6x = 18$	Check:

Note: Variables can be on any side of the equation
Variables can be any letter (not just x)

C. Solving Equations Using Multiplication

1. $\frac{1}{2}x = 12$	Check:	2. $\frac{e}{5} = -1$	Check:
3. $\frac{x}{-6} = -4$	Check:	4. $\frac{1}{4}x = -2$	Check:

D. What Happens When The Variable Is Negative?

1. $-x = 10$	Check:	2. $20 - x = 2$	Check:
3. $-3 = -x + 15$	Check:	4. $-7 = -12 - q$	Check:

Lesson 2: Assignment **P9.2**

Solve the following equations. You need to show all of your steps!

1. $x + 8 = 11$

2. $a - 9 = 1$

3. $3 + x = 1$

4. $-8 + e = -8$

5. $12 = 6g$

7. $-4d = -24$

8. $-x + 8 = 14$

9. $\frac{d}{-5} = -4$

10. $3a = 2$

11. $3 = \frac{a}{-10}$

12. $32 = 8x$

14. $-7 = z + 1$

15. $20 = d - 5$

16. $-9 = -4 + k$

17. $x + 9 = 1$

Lesson 3: Solving Equations with Variables on One Side **P9.2**

To Teacher:

Students should have experience with this type of equation from grade 8. Therefore, it is important to tap into their previous knowledge about solving this type of equation. Some students may forget how to solve exactly, but they may remember some key phrases or ideas of solving equations. The start-up question is the exact same as the exit question. I recommend giving it to students, don't correct it, look them over before the lesson to assess student's current understanding of this type of equation. I left the question the same for the exit question to see if students performed at the end of the lesson.

The first activity in the lesson is asking the students what it means to be a solution or a non-solution of an equation. Students should be able to use guess and check techniques to find what numbers work and don't work for the equation.

The questions in the notes are borrowed from Burt Thiessan's math 9 textbook chapter 4.2. I have also used some of the odd questions from the assignment in the same chapter, so if you want to supplement with Burt Thiessan's textbook, assign evens.

RN #5

Formative Assessment

Lesson Process:

1. Begin with the start-up question. Don't discuss as a class since this is also the exit question.

2. Display the question $4x + 9 = 5$ and ask students what value of x make it false. Ask students what value of x make it true. This can lead to a discussion about how **most** equations have only one solution. There are special cases where all numbers are solutions, and where no numbers are solutions. Examples of each type are:

   a. $2x + 3 = 2x + 3$
   All numbers are solutions

   b. $x + 4 = x$
   No solution

3. Discuss with students the importance of the equal sign and how it separates the left hand side from the right hand side. The goal of solving an equation is to discover what x (or any variable) actually is.

4. In the assignment I have asked the students to "code" a few questions. It might be useful to "code" a few of the student examples as well so students know how to do it. Coding is writing down their steps using words. Have students try some questions on their own. They may work in partners if they like.

5. Example 1: Work through the example with the students. Have them check the answers to make sure they are correct.

6. Example 2: Have students try solving the equations on their own. Correct as a class.

7. Example 3: Ask students to find which one of the solutions is incorrect, have them find the error and provide feedback. You may want to suggest to students that they should cover up the student’s work and solve it themselves. When they are done, compare their work with the student’s work to find their errors.

8. Exit question. **RN #5**

9. Assignment: Lesson 3

State a value for x that will make this equation false,

$$4x + 9 = 5$$

Show your calculations and explain your answer.

State a value for x that will make this equation true,

$$4x + 9 = 5$$

Show your calculations and explain your answer.

Solving Equations with Variables on One Side

- The equality sign in an equation separates the equation into a **left-hand side** and a **right-hand side**. The variable can appear on either side.
- Your goal is to get the variable on one side and numbers on the other. The variable has to be number free (actually there is an invisible 1). When you do this, you will discover what the variable really represents.
- Here are some tips
  - Simplify each side first by collecting like terms
  - Undo subtraction and addition first
  - Undo division and multiplication second

Example 1: Solve the following equations and check your solutions.

a. $2x - 7 = -29$	b. $6 - 4t = 54$	c. $100 = -30 - 10y$
Check:	Check:	Check:
d. $-12 = -15a + 8$	e. $3x + 2x + x - 17 = 6$	f. $-23 = 2w - 2 + 4w + 11$
Check:	Check:	Check:

Example 2: Solve the following equations

a. $5w - 6 - 8w = 9$	b. $-8 = w + 5w - 10w - 20$	c. $3x - 8 + 5x - 4 = 8$

Example 3: Error Analysis

Find the ONE error in each of the following solutions

1. $4a - 13 = -31$

$4a - 13 + 13 = -31 - 13$	Add 13 to both sides
$4a = -44$	Simplify
$\frac{4a}{4} = \frac{-44}{4}$	Divide by 4
$a = -11$	Simplify

2. $-23 = 17 - 4x$

$-23 - 17 = 17 - 4x - 17$	Subtract 17 from both sides
$-40 = -4x$	Simplify
$\frac{-40}{4} = \frac{-4x}{4}$	Divide by 4
$-10 = x$	Simplify

3. $3x - 9 - 5x + 4 = -7$

$-2x - 13 = -7$	Combine Like Terms
$-2x - 13 + 13 = -7 + 13$	Add 13 to both sides
$-2x = 6$	Simplify
$\frac{-2x}{-2} = \frac{6}{-2}$	Divide by -2
$x = -3$	Simplify

Lesson 3: Assignment **P9.2**

Solve the following equations. For questions #1-6, code your steps. You do not need to code the remainder of the questions, but you do need to show all of your steps.

1. $3t + 10 = 28$
2. $-3x - 42 = 15$
3. $6x - 50 = 46$
4. $8x - 12 = 8$
5. $13 = -2x - 7$
6. $7x - 21 = -28$
7. $-20 = 40 + 5n$
8. $19 = -5 + 8m$
9. $8m - 3 = 9$
10. $-6 = 3m + 2m + m$
11. $3y - 2y + y = 5 - 11$
12. $12b + 14 - 3b = 16$
13. $7a - 35 = 0$
14. $12x - 2 = 24$
15. $3 = 5t - 11 + 14$

Lesson 4: Solving Equations with Variables on Both Sides **P9.2**

To The Teacher:

Solving equations with variables on both sides is not part of the grade 8 curriculum. This should be a new concept for students. Students are now going to have to use the inverse operations of addition and subtraction to move variables across the equal sign.

The examples in the student workbook are taken from Burt Thiessan's Grade 9 textbook. For the assignment I used questions 7-14 (odd numbers) from section 4.3.

RN #5

Formative Assessment

Lesson Process:

1. Begin with the start-up question. Correct as a class.

2. Discuss with students what it means to have variables on both sides of the equation.

3. Example 1: Work through the example 1 with the class. Since coding is a part of the assignment, you may want to code a few of the questions in the workbook. Have students provide input into what side they want to collect the variables on.

4. Example 2: Students should try these on their own or with a partner. Correct as a class.

5. There is a whiteboard activity that can be done at the end of this lesson or the beginning of the next lesson. It will give you an idea of how students are progressing with equations. In the whiteboard assessment, #7 results with an $x = 0$. Students may find difficulty when the numerical side of the equation results in a 0. You may want to discuss what happens when the numerical side results in a 0. Is zero a solution? **RN #5**

6. Exit Question **RN #5**

7. Assignment: Lesson 4
   Note: in the assignment, question 6 has a solution of zero.

Solving Equations with Variables on Both Sides

- Collect like terms before you start moving terms around
- Choose a side where you will collect the variable (x's) and the other side will then have the constants (#'s without variables).
- Use addition or subtraction to move things to the appropriate sides.
- Use division or multiplication to finally isolate the variable.

Example 1:

a. $2x + 7 = 5x - 5$ Check:	b. $16 - 8x = 30 - 4x$ Check:
c. $-12 - 2x - 3 = -7x + 10x$ Check:	d. $5x - 7 - 3x + 16 = 4x - 11 - 8x + 5$ Check:

Example 2: your turn!

a. $3x - 1 = 7x + 11$ Check:	b. $x - 15 = -4x$ Check:
c. $2x - x + 3x = 9 - x - 14$ Check:	d. $2x - 9 + x = 10 - 5x - 3$ Check:

In order to move forward you need to articulate where you are having problems. Combining like terms, when to add, when to subtract, where to start. Please write any problems you having below.

____

____

Lesson 4: Assignment **P9.2**

Solve the following equations. Remember to show all of your steps and to reduce any fractional answers to lowest terms.

1. $3x - 5 = x + 11$

2. $-5 + 3x = x + 5$

3. $-x - 7 = -3x + 7$

4. $13x = -2x + 30$

5. $3x - 5 = -4x + 2$

6. $3x + 12 = -4x + 12$

7. $9x - 7 = 5x + 13$

8. $4m - 15 - 6m = 13 + m + m$

9. $2x - 5 = 5x + 6 - 4x$

10. $-12y = 65 + y$

11. $4x - 6x + 4 = 5x + 10 - 3x + 3$

12. $3 - 2e = -5e - 42$

13. $9y + 13 + 8y + 7 = 2y$

14. $5k + 1 + 4k + 1 = -10 - k + 11 + 6k$

Lesson 4: Whiteboard Activity **P9.2**

1. $26 = 8 + v$
2. $m + 4 = -12$
3. $p - 6 = -5$
4. $14b = -56$
5. $-6 = \frac{b}{18}$
6. $\frac{v}{8} = 2$
7. $9x - 7 = -7$
8. $-20 = -4x - 6x$
9. $8x - 2 = -9 + 7x$
10. $n + 4n - 11 = 19$
11. $2 - 3x = -2x - 8$
12. $4x + 7 = -3 + 5x$

Lesson 5: Solving Equations Containing Parentheses **P9.2**

To The Teacher:

Students have encountered parentheses in grade 8, but they did not tackle them with both sides of the equation. The student examples in workbook are borrowed from Burt Thiessan's text chapter 4.4.

RN #7

Cooperative Learning

RN #8

Discourse

Lesson Process:

1. Begin with start-up question. Correct as a class. Discuss with students some of the problems they had answering the question.

2. Work through the class examples. Remind students how to distribute. On example 4 they must distribute the negative in front of the bracket. You may want to remind students that there is a 1 in front of the bracket. When checking the answers, #2 works out to be a fractional solution. You may want to check the solution using the decimal rather than the fraction.

3. Have students work in their learning groups. They will need a large piece of paper (bulletin paper), scissors, glue, and a marker. The red equations are the questions and the purple equations are the steps. Students need to separate their large paper into 6 quadrants and glue the 6 red equations on the top of each quadrant. In between each glued slip of paper, they need to write down their steps using the marker. Here is an example of what it might look like. **RN #7, RN #8**

INSERT PICTURE OF WHAT IT WILL LOOK LIKE HERE!

4. Have students share their posters. You may want to have them only share one of the six questions.

5. Assignment: Equations #4

Solving Equations Containing Parentheses

- Use the distributive property and then proceed as normal!
- Collect like terms
- Choose a side where you will collect the variable (x's) and the other side will have the constants (#'s without variables).
- Use addition or subtraction to move things to the appropriate sides.
- Use division or multiplication to finally isolate the variable.

Example 1: Solve the following equations and check your solution.

1. $8(3x+10)=28x-14-4x$ Check:	2. $-2(x-5)-(2x-4)=7(-3+3x)$ Check:
3. $6(2x+8)=2(x-1)$ Check:	4. $-9-(9x-6)=3(4x+6)$ Check:

Lesson 5: Ordering Activity

$4(x-3)=48$	$3x+12=48$
$3(x+12)=48$	$4x-48=4$
$3x+12=60$	$3(x+2)=60$

$4x = 60$	$3x + 6 = 60$
$3x = 36$	$3x + 36 = 48$
$3x = 48$	$4x - 12 = 48$
$3x = 54$	$x = 18$
$4x = 52$	$x = 16$
$3x = 12$	$x = 4$
$x = 13$	$x = 15$
$x = 12$	

Lesson 5: Assignment **P9.2**

Solve the following equations. For questions #1-6, code your steps. You do not need to code the remainder of the questions, but you do need to show all of your steps.

1. $-35 = 5(2x - 3)$

2. $4(-2x - 3) = 36$

3. $-3(3x + 3) = 18$

4. $-24 = -3(-2x - 4)$

5. $2(3y - 3) = 0$

6. $-9(1 + r) - 4 = -2 + 2r$

7. $-6n - 7(3n - 5) = 116$

7. $-2(4v + 5) = -2 - 10v$

8. $10(3 + 3n) = 9 + 9n$

9. $4(5x - 3) = 7(2x + 3)$

10. $x + 2(x + 1) - 5(x - 3) + 3 = 0$

11. $3(m - 3) + 7(m + 3) = 16 - 4(7 - m)$

12. $10k - (2k - 8) - (2k - 3) = -4$

13. $5(2 + 3b) = 15 - (b - 7)$

Lesson 6: Solving Equations Containing Fractions **P9.2**

To The Teacher:

The grade 8 curriculum does contain solving equations with fractions. However, they are limited to variables on one side.

Lesson Process:

1. Begin the lesson with the start-up question. It is an equation containing fractions that they would have seen in grade 8. Correct as a class. Ask students what they remember about solving equations containing fractions.

2. Ask students how many terms are in each of the following in the student workbook. Students may get confused on terms containing parentheses and answer $5(x+3)$ as having 3 terms instead of it just being 1 term. Explain to students that terms are separated with an addition or subtraction symbol unless they are grouped together with a bracket, then it is considered 1.

3. Have students translate the mathematical phrases into English. The goal is for them to understand that $\frac{x}{2}$ and $\frac{1}{2}x$ mean the same thing and that they need to multiply by 2 in order to undo the division of 2.

4. Have students solve the equation $3x - 5 = 7$. Tell the students that I tripled the equation (multiplied it all by 3). What do they think will happen to the solution? Some may say nothing, others may say that it will be multiplied by 3 as well. Have students solve the tripled equation. The solution stayed the same. Discuss with students that multiplying the ENTIRE equation by a number doesn't change the solution. The equality holds.

5. Ask students who enjoys working with fractions. Chances are not many. Discuss with students how we can eliminate fractions from equations by using the lowest common multiple. Work through the student examples in the workbook. Put a large focus on finding the LCM. You may want to ask students if any common multiple will do. What will happen when we don't use the lowest?

6. Example 1: Answer as a class. Have students declare the number of terms and LCM in the question before answering. Have students provide most of the information when solving.

7. Exit question

8. Assignment: Lesson 6

Solving Equations Containing Fractions

A. Review of Past Lessons

Example 1: How many terms are in the following equations? Circle them.

1. $3x + 4 - 2x + 5 = -9x - 2 + 1 - x$ ___ terms

2. $5(2 + 4b) = 18 + 5b - 2(8 + 10b)$ ___ terms

3. $2x + 5(x - 1) = x - (3x - 1) - 2(x + 4)$ ___ terms

4. $\frac{1}{2}x - \frac{3}{4}(x + 2) = \frac{9}{10}(-3x + 1)$ ___ terms

5. $12 - \frac{1}{2}x + 3(5x + 1) = -\frac{1}{8}(x - 1) + x + 5$ ___ terms

Example 2: Answer the following questions

1. Translate $\frac{x}{2}$ into English ____

2. Translate $\frac{1}{2}x$ into English ____

3. How will you undo that type of operation in an equation? _______________

Regular Equation	The ENTIRE equation 3 times the original
4. $3x - 4 = 7$	5. $9x - 12 = 21$

What do we notice about the solutions?

____

C. Equations Involving Fractions

Fractions in equations make it a lot more difficult to solve. Fortunately for us, there is a technique to remove the fractions from the equation! We can use the laws of equality to eliminate fractions and work with just whole numbers! We use the…

LOWEST COMMON MULTIPLE

- Find the first number that all of the denominators will be able to divide into.
- Multiply the ENTIRE equation by that number (LCM).

Example 1: Solve the following equations

a. $\frac{2}{5}x = -16$ LCM = ______ Terms = ______	b. $\frac{3}{4}x = -15$ LCM = ______ Terms = ______
c. $\frac{7}{8}x - \frac{5}{4} = \frac{1}{2}x + \frac{2}{3}$ LCM = ______ Terms = ______	d. $\frac{5}{6}x - \frac{2}{3} = \frac{1}{2}x + \frac{1}{3}$ LCM = ______ Terms = ______
e. $\frac{2}{3}x + \frac{3}{4}(35 - x) = 25$ LCM = ______ Terms = ______	

Lesson 6: Assignment **P9.2**

Solve the following equations. For questions #1-6, code your steps. You do not need to code the remainder of the questions, but you do need to show all of your steps. State the LCM for each question.

1. $\frac{x}{3} = 5$ LCM = _____

2. $\frac{x}{7} - 1 = -6$ LCM = _____

3. $\frac{1}{4}b + \frac{1}{2}b = 3$ LCM = _____

4. $-\frac{2}{7}x = 6$ LCM = _____

5. $2y - \frac{3}{5} = \frac{1}{2}$ LCM = _____

6. $\frac{1}{4} + \frac{1}{2}t = 4$ LCM = _____

7. $\frac{1}{4}x + x = -3 + \frac{1}{2}x$ LCM = _____

8. $m + \frac{2}{3} = \frac{1}{4}m - 1$ LCM = _____

9. $\frac{1}{5}m + \frac{2}{3} - 2 = m - \frac{2}{5}$ LCM = _____

10. $\frac{1}{4}(3c + 5) - \frac{1}{2}(2c + 3) = \frac{1}{2}$ LCM = _____

To The Teacher:

You can choose to teach by multiplying by 10's or to leave decimals as is and use a calculator. I like leaving the decimals alone because that is what they would do if they were in a science class.

Lesson Process:

1. Begin with the start-up question. Discuss the answer as a class.
2. Work through the first equation with the students. Show how (and why) to multiply by 10. You will want to mention that you may need to multiply the entire equation by 100 or 1000 depending on how many decimal places are represented. Answer the same question by leaving the decimals alone and by using a calculator.
3. Example 1: Answer the examples in example 1 as a class. Use a mix of multiplying by 10 and leaving the decimals so students can make up their mind what method they like to use best.
4. Assignment: Lesson 7

Solving Equations Containing Decimals

Good news! All of the same rules from our previous lessons apply when solving equations with decimals in them. There are a few techniques you can use to come up with the solution. Let's look at them.

$$0.4x + 4 = 9$$

Solve by multiplying by 10's, 100's, 1000's etc. to remove decimals and make whole numbers	Solve by leaving decimals alone (calculator recommended!)
$0.4x + 4 = 9$	$0.4x + 4 = 9$

Example 1: Solve the following equations

a. $-6.3n = -8.19$	b. $\frac{x}{1.2} = -7$
c. $0.4x + 3.9 = 5.78$	d. $2.25(x - 4) = x + 3.28$

e. $0.3x - 2.4 = 0.36 + .52x$	f. $3.5x + 0.8 = 18.24 - 5.9x$

Example 2: What way do you prefer to solve equations involving decimals?

a. Multiplying by 10, 100, etc.
b. Leaving the Decimals Alone
c. I like to use both ways

Lesson 7: Assignment **P9.2**

Show all of your steps. If necessary, round to 3 decimal places.

1. $-2.8 = n + 1.3$

2. $-1.5x = -2.55$

3. $-4.84 = -1.3k + 2.7$

4. $0.72 = 0.4(x + 1.4)$

5. $-0.5x - 3.69 = x - 1.9 - 2.39$

6. $3.5(1 + 4s) = 24.5$

7. $6(9f + 8.5) = 18.5$

8. $-2x - 4 + 5.5x = 17.5$

9. $8v + 3 + 9v = -26.5$

10. $-8.5k + 4.5 + 4 = 15.5$

11. $-24.5 - 4.5c = -9(3c + 7)$

Lesson 8: Solving Situational Questions **P9.2**

To The Teacher:

This lesson is intended to be done in groups of 3. Students will need a large sheet of paper, glue, scissors, and markers. They need to come up with an equation that represents the situation and answer the question. They will share their answers with the class when complete.

RN #7

Cooperative Learning

Lesson Process:

1. Have students get into groups. Hand out the necessary items that they need to complete this assignment. Scissors, glue, paper, and markers.
2. Students are to cut out each situation, divide their paper into 6 quadrants, and use the marker to answer each of the questions. They must use and solve an equation. At the end of the class, have each group come up and explain one of the situations to the class.
3. For the second set of situational cards, students are required to come up with two equations and find out when they equal the same number. In order to do this, they must set each equation equal to the other and solve. Have students work in groups like the first activity and share their answers with the class when they are complete. You may want to ask students which situation is better for which numbers if they are done their activity early. For example, if the dog daycares are the same at day 10, then who is the better deal after day 10 and who is the better deal before day 10.
4. If you want to assign a textbook assignment you may want to have students look at the following questions

MathLinks (McGraw-Hill Ryerson): Page 312

#13, 14, 15, 22, 23	Given a story, create and solve the equation

Page 320

#18, 19	Given a story, create and solve the equation
#22	Given the equation, solve

Page 327

#14	Given a story, create and solve the equation

Math Makes Sense (Pearson): Page 273

#9, 13	Creating and solving basic equations
#16	Solving a situation where the equation is already created
#21	Given a story, create and solve the equation

Page 281

#12, 13, 14, 18	Given a story, create and solve the equation

S1

Tim bought a soft drink for 4 dollars and 7 candy bars. He spent a total of 32 dollars.

How much did each candy bar cost?

S2

Sandy had 227 dollars to spend on 9 books. After buying them she had 11 dollars left.

Assuming each book was the same price, how much did each book cost?

S3

Aliyah had $24 to spend on seven pencils.

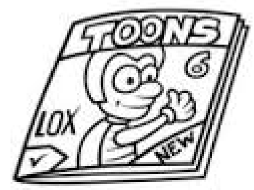
After buying them she had $10 left. How much did each pencil cost?

S4

You bought a magazine for $5 and four erasers.

You spent a total of $25.

How much did each eraser cost?



S5

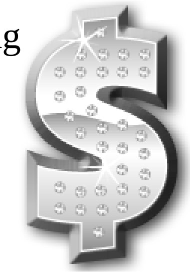
Sumalee won 40 super bouncy balls Playing horseshoes at her school's Game night.

Later, she gave two to each Of her friends. She only has 8 remaining.

How many friends does she have?

S6

How old am I, if 400 reduced by 2 times my age is 244?

S1 Jennifer has \$35.50 and is saving \$4.25/week. Eva has \$24.25 and is saving \$5.50/week.  In how many weeks from now will they have the same amount of money? **Model, Solve, and Verify**	**S2** Two rental halls are considered for the 2014 graduation Reception Hall A costs \$50 per person  Hall B costs \$2500, plus \$30 per person. Determine the number of people for which the halls will cost the same to rent. **Model, Solve, and Verify**
S3 Tamoor and Saba belong to different local fitness clubs. Tamoor pays \$35 per month And a one time registration Fee of \$15.  Saba pays \$25 per month but had to pay a \$75 registration After how many months will Tamoor and Saba have spent the same amount on their memberships? **Model, Solve and Verify**	**S4** Jake plans to board his dogs while he is away on vacation. House A charges \$90 per day plus \$5 per day. House B charges \$100 per day Plus \$4 per day. For how many days must Jake board his dog for boarding house A to be the same price as boarding house B. **Model, Solve, and Verify**

Lesson 1:

Start-Up Question

1. What does an equation look like? Create one!

Exit Question

1. Create an equation that your neighbor could unwrap. (Show the steps that you used to create it)

Lesson 2:

Start-Up Question

1. Solve $5 + x = -13$; show all your steps

Exit Question

1. Solve $\frac{1}{8}x = -1$; show all your steps

Lesson 3:

Start-Up Question

1. Solve $2x + 3 = 1$; show all your steps

Exit Question

1. Solve $2x + 3 = 1$; show all your steps

Lesson 4:

Start-Up Question

1. Solve $9x - 5 = 13$; show all your steps

Exit Question

1. $20 - 7x = 6x - 6$; show all your steps

Lesson 5:

Start-Up Question

1. Solve $5(x - 2) = 55$; show all your steps

Exit Question

1. Solve $-2(x+10)=4(2x-1)$

Lesson 6:

Start-Up Question

1. Solve $6x-4(x+3)=8$; show all your steps

Exit Question

1. $\frac{1}{3}x+1=\frac{5}{6}x-3$

Lesson 7:

Start-Up Question

1. $m+\frac{2}{3}=\frac{1}{4}m-1$

Exit Question

1. $-2.7x+0.4=2.8-1.2x$

Outcome P9.3

Demonstrate understanding of single variable linear inequalities with rational coefficients including:

- Solving inequalities
- Verifying
- Comparing
- Graphing

Lesson 1: Review of Solving Equations

Lesson 2: What is an Inequality?

Lesson 3: Solving Inequalities by Using Addition and Subtraction

Lesson 4: Solving Inequalities by Using Multiplication and Division

Lesson 5: Solving Stories

Previous Experience:

It is interesting to note that there is no mention of inequalities symbols in the K-8 curriculum.

To The Teacher:

Students have no previous experience solving inequalities. It is recommended that this unit follow closely on the heels of P9.2 (solving equations). This way, students can see that the rules for solving equations are almost identical to the rules for solving inequalities.

I separated the operations of addition and subtraction from multiplication and division into two lessons. For this reason, it is important that on lesson 3 the variable is positive before any division takes place because students don't know about dividing or multiplying by a negative number yet.

This unit (like solving equations) is set up more traditionally. There is some space where I have tried to apply more constructivist principles, but the majority of the unit consists of traditional teaching practices. I have included some space where students, with the guidance of the teacher, are generating their own rules. You can apply more reform elements by having students work in groups and by implementing formative assessment techniques.

Lesson 1: Solving Equations Review **P9.3**

To The Teacher:

It is your choice if you want to include this as part of your inequality unit. It depends on how well your students did with solving equations. Some students may need some more help before they move into solving inequalities. This is a good opportunity to rearrange their learning groups depending on their last test results and have them work through the student questions together. **RN #7**

RN #7

Cooperative Learning

Lesson Process:

1. Since this unit follows directly after the solving equations unit, it is a good idea to give the students a chance to work through the different types of equations to ensure that they fix any conceptual errors before they begin working with inequalities.

2. You can choose to do the workbook questions as a class and assignment 1 in their groups.

P9.3 Linear Inequalities

Lesson 1: Review of Solving Equations **P9.3**

Solving Equations Review

Type #1: Basic Equations; solving by using inverse operations.	Type #2: Solving equations with brackets.
1. What is the inverse of additions?	1. Solve $5(x+3)=-35-2(3x-3)$
2. What is the inverse of subtraction?	2. Solve $-2(4x+5)=54$
3. What is the inverse of multiplication?	
4. What is the inverse of division?	Type #3: Solving equations with variables on both sides.
5. Solve $2x=14$	1. $-12-2x-3=-7x+10x$
6. Solve $x-5=10$	2. $-2x-16=28-6x$
7. Solve $8+p=-12$	Type #4: Solving equations with fractions.
	1. $\frac{5x}{9}-\frac{7}{18}=\frac{3x}{2}+\frac{11}{6}$
8. Solve $2n+8=22$	
	2. $\frac{3}{5}x-4=\frac{2}{3}x+1$
9. Solve $5-6x=-31$	

Assignment: Lesson 1 **P9.3**

Solve the equations

1. $6 + 2x = -7 + x$

2. $8w - 12.8 = 6w$

3. $-12a = 15 - 15a$

4. $13 - 3q = 4 - 2q$

5. $4(g + 5) = 5(g - 3)$

6. $-3 = 5x + 3(-2x - 5)$

7. $\frac{x}{3} - 3 = 3$

8. $-4 = -\frac{7x}{6} + \frac{4}{6}$

9. $\frac{x}{4} + \frac{7}{4} = \frac{5}{6}$

10. $2 - \frac{x}{24} = \frac{5x}{24} + 1$

11. $\frac{x}{3}+\frac{x}{4}=x-\frac{1}{6}$

12. $5=2-\frac{x}{2}$

13. $-9-x=-3x+3$

14. $-4(-3x+2)=16$

15. $6x-7(-x+3)=44$

16. $49=7x-7(2x+3)$

17. $-8z+11=-10-5.5z$

18. $\frac{2}{5}(m+4)=\frac{1}{5}(3m+9)$

19. $36=4(-2x+3)$

20. $5x+2(-2x-4)=4$

Lesson 2: What Is An Inequality? **P9.3**

To The Teacher:

It is surprising to note that students have not had a lot of contact with inequality symbols. It is not a specified outcome in the curriculum guide so please be aware that you may have students who are experiencing the symbols for the first time. Some students will read an inequality from left to right regardless of where the variable is ex: $3 < r$ vs. $r < 3$. It is important for students to understand that when reading an inequality (not just the symbol) that we read from the variable first. Students are also learning about the closed and open dot for the first time.

Some students may struggle with number lines. I have included a start-up activity that will provide students with a refresher on number lines and inequality symbols. It will also give you, the teacher, a chance to see what their prior knowledge is.

RN #1

Math-In-Context

Lesson Process:

1. Activity 1: Draw a number line on the board. Have students come up and place their favourite number on the line. Using the student's data, ask students questions such as;
   a. How many students have a favourite number greater than 0?
   b. How many students have a favourite number less than 0?
   c. How many students have a favourite number greater or equal to ____?
   d. How many students have a favourite number less than or equal to ____?

   You may also want to guess a student's number by asking them less than and greater than questions. **RN #1**

2. **Student Workbook:** Introduce the new inequality symbols. You may want to ask students if they have seen them before and if they know what they are called.

3. Example 1: As a class have students create inequalities for the pictures.

4. Have students look at an equation and inequality side by side. Ask students what numbers are solution(s) to each. Students should see that only one number works for the equation but many numbers work for the inequality.

5. Example 2: Have students determine what numbers from the list are solutions to the inequality $x \leq -2$? Students might get a little confused with the negative and what is considered smaller or larger than -2 if they don't have a strong number sense. Have them draw a number line.

6. Before you discuss with students how to read inequalities, have them give a few solutions to the inequalities in the student notebook. After students if they know how to read inequalities. The answer is; read from the variable.
7. Introduce the closed and open dot. Students need to know that the open dot means that the number is not included, but the numbers right up next to it are. The closed dot means that the number is included. You may want to graph each of the inequalities and ask students why you chose the closed dot for the second graph but used an open dot for the first graph.
8. Example 3: Have students graph the solutions on the number line.
9. Example 4: Have students find the inequality when given the number line.
10. Exit slip
11. Assignment: Lesson 2

What is an Inequality?

A. The Inequality Symbols

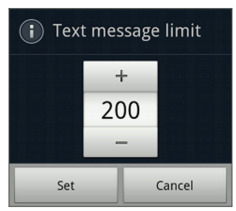
$<$	$\leq$	$>$	$\geq$
________	________	________	________

What is an inequality?

Example 1: Define a variable and write an inequality for each situation.

a. 

____

b. 

____

c. 

____

d. 

____

Difference between an EQUATION and an INEQUALITY

<u>EQUATION</u>	<u>INEQUALITY</u>
Has _______ solution. Example: $h + 3 = 5$	Has _______ solutions Example: $h + 3 < 5$

Example 2: Is each number a solution of the inequality $x \leq -2$?

a. -10 b. 10 c. 0 d. -2.1 e. -1.9 f. -2

How do I read inequalities?

$x < -2$	$-2 < x$	$r \geq 5$	$5 \geq r$

How can I read an inequality?

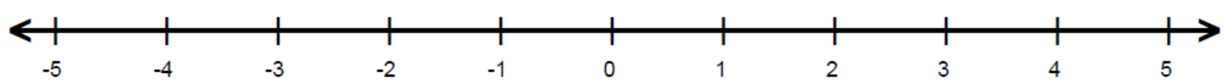
____

____

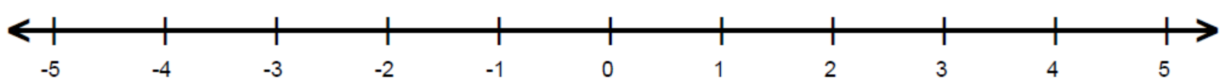
Introduction of the number line – What you need to know

a. Open dot vs. Closed dot

Graph $x < -2$



Graph $x \leq -2$

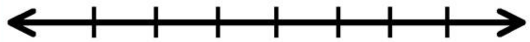


When do you use a closed dot?____

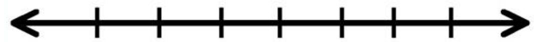
When do you use an open dot?____

Example 3: Graph each inequality on a number line. Make sure you have 4 numbers represented on your line that are solutions of the inequality.

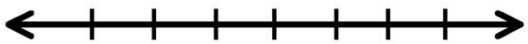
a. $x > -8$



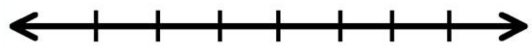
b. $3 < r$



c. $w \leq -4$

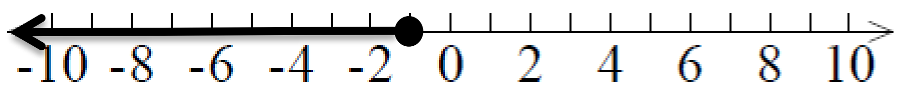


d. $7 \leq x$

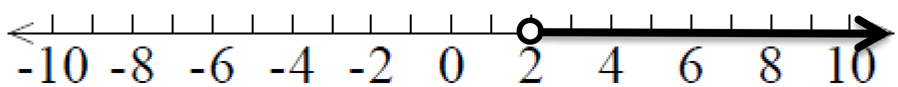


Example 4: State what inequality is represented by each graph

a. ______________________



b. ______________________



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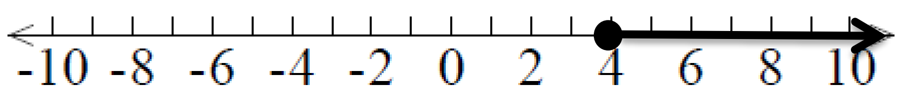

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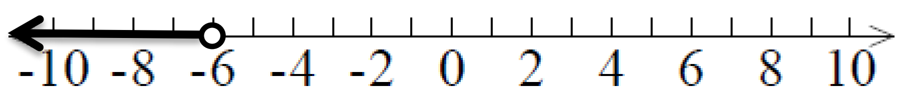
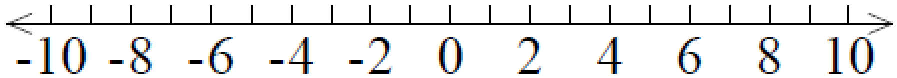


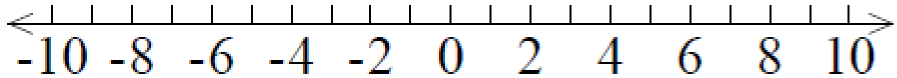
Exit Slip (P9.3) Name:____________________________

Graph the inequality.

1. $x \geq -3$

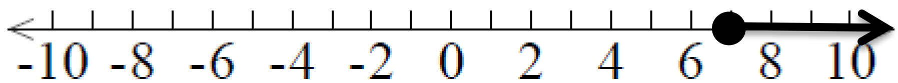


2. $4 > t$



Write the inequality represented by the graph.

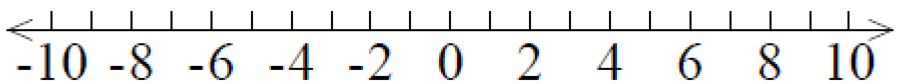
3. ________________________



Exit Slip: (P9.3) Name:____________________________

Graph the inequality.

1. $x \geq -3$



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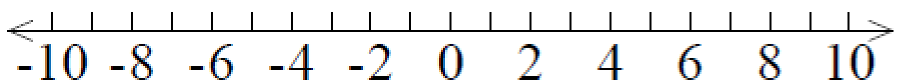

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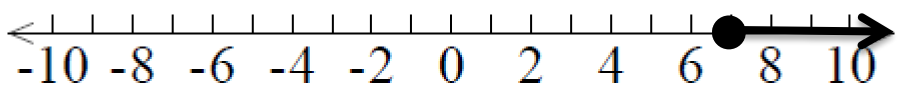

Exit Slip: (P9.3) Name:____________________________

Graph the inequality.

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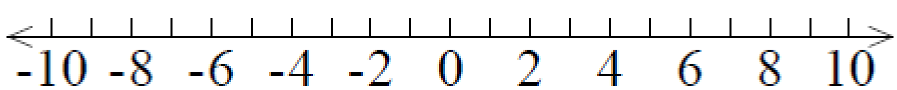
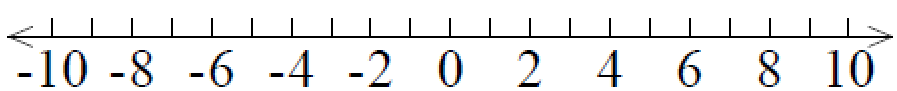
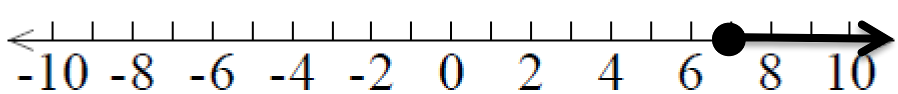


2. $4 > t$



Write the inequality represented by the graph.

3. ________________________



Assignment: Lesson 2 **P9.3**

1. Is each inequality true or false?
   a. $3 < 5$
   b. $-3 < -5$
   c. $-3 \leq -3$
   d. $3.01 < 3$
   e. $-3.01 < -3$
   f. $-5 > -5$
   g. $\frac{1}{3} > \frac{1}{5}$
   h. $-\frac{1}{3} > -\frac{1}{5}$

2. Which numbers are a solution of $x < -5$?
   a. 0
   b. -4.9
   c. -5.01
   d. $-\frac{1}{5}$

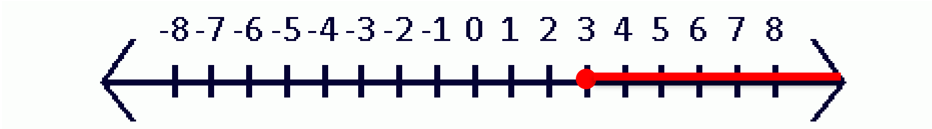
3. Write 4 numbers that are solutions of each inequality.
   a. $b < 4$
   b. $-6 \leq g$
   c. $4 < x$
   d. $y \geq -10$

4. For which inequalities is 5 a possible value of x?
   a. $x > 6$
   b. $x < 5$
   c. $x \leq 5$
   d. $x \geq 5$

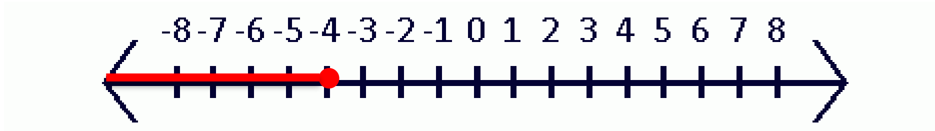
5. Define a variable and write an inequality to model each situation.
   a. The speed limit in school zones is 30 km/h
   b. The maximum number of people allowed in the hall is 600.
   c. In the wrestling class, you must be less than 50kg.
   d. In order to watch the movie you have to be at least 16 years old.

6. Write an inequality whose solution is graphed on the number line.

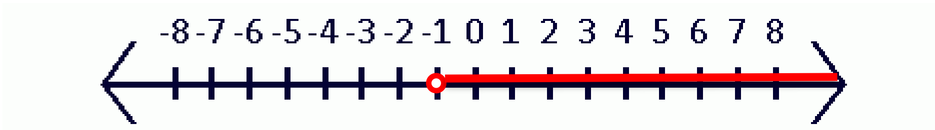
a.



b.



c.



7. Graph the solution of each inequality on a number line.

a. $w > 5$

b. $-3 \geq r$

c. $c \geq 2.1$

d. $d \leq -\frac{1}{2}$

e. $-5 < x$

f. $y \geq 0$

Lesson 3: Solving Inequalities by Using Addition and Subtraction **P9.3**

To The Teacher:

By using a simple inequality such as $-2 < 4$, show students that by adding and subtracting numbers to both sides will still keep the inequality true. Students will then understand that they can add and subtract inequalities the same as equations. What is different is how they interpret their answer. Rather than having 1 solution, they have a large range of numbers that will work. It is your personal preference how many numbers you want your students to put on their number line when graphing the solution.

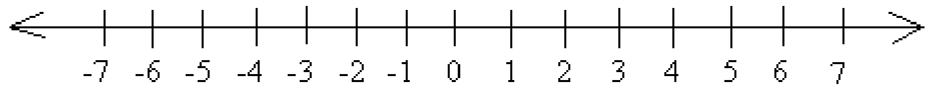
Lesson Process:

1. By using a simple statement such as $-2 < 4$ ask students what happens when we add 2 to each side. Does the inequality still hold true? Is the left hand side still smaller than the right hand side? Do the same with subtraction. You may want to put this in context of equations. When you have $5 = 5$, if we add 2 to each side, do we still have a true statement? Students should understand that we can add and subtract to both sides of the inequality to isolate the variable.
2. Example 1-3: Solve the inequalities with the students. Have them verify their solution by choosing a number in their range and checking. Have them graph their solution on a number line.
3. Assignment: Lesson 3

Solving Linear Inequalities by Using Addition and Subtraction

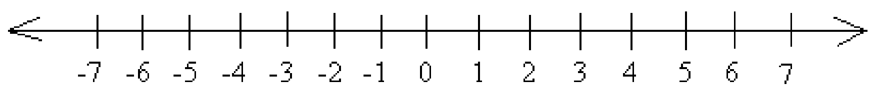
Let's see what happens when we add and subtract to an inequality

$-2 < 4$



Let's add 2 to each side. Does the inequality still hold true?

$-2 < 4$



Let's subtract 2 from each side. Does the inequality still hold true?

What happens when we add and subtract to both sides of an inequality?

Example 1:

a) Solve the inequality: $2.1 \geq x - 3.2$	b) Verify the solution	c) Graph

Example 2:

a) Solve the inequality: $x + 3 < 5$	b) Verify the solution	c) Graph

Example 3:

a) Solve the inequality: $2x + 5 < x - 10$	b) Verify the solution	c) Graph

Assignment: Lesson 3 **P9.3**

Solve each inequality. Check your solution and graph your answer.
Because there is one more important thing to learn tomorrow, it is really important for this assignment that you collect the variable on the side where it will be a $+$ value.

1. $a + 5 < 14$

2. $9k - 12 \geq 80 + 8k$

3. $6y > 14 - 2 + 7y$

4. $q + 10 > 3q - 7 - 3q$

5. $6c - (5c - 7) \leq 12$

6. $a - 12 < 6$

7. $4 < 1 + \frac{n}{7}$

8. $2x + 3 \geq x + 5$

9. $x + \frac{1}{8} < \frac{1}{2}$

10. $3x - 9 \leq 2x + 6$

11. $-0.17x - 0.23 \geq 0.75 - 1.17x$

12. $3(r - 2) < 2r + 4$

13. $1.8w + 4.5 \geq 0.8w - 12.2$

14. $3.8 < 2x - (9 - 1.2x)$

Lesson 4: Solving Inequalities Using Multiplication and Division **P9.3**

To The Teacher:

Often the switching/flipping of the inequality sign when dividing or multiplying by a negative number is often forgotten by students. Hopefully by looking at what happens to simple inequalities; $-3 < 1$ and $-6 < 3$ will give students the opportunity to construct their own understanding of why the sign changes direction when multiplying or dividing by a negative number.

RN #1

Math-In-Context

Lesson Process:

1. Have students look at what happens when we multiply and divide numbers to a simple inequality statement. When the values are positive, the inequality stays true. When we multiply or divide by a negative, the inequality becomes false. Thus, in order to maintain the inequality, we must flip the sign when we multiply or divide by a negative number. **RN #1**

2. Example 1: Complete the inequality examples with the students. You may want students to use a highlighter to emphasize the moments when the inequality sign is reversed. Example 1(i) is from Pearson's Math Makes Sense Grade 9 Textbook page 303

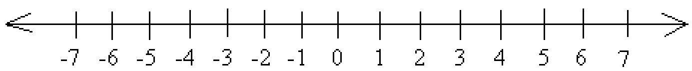
3. Example 2: This example is an inequality story. You may want to save that for the next lesson when students will be working on solving stories. There are no questions like this in the lesson 4 assignment. Example 2 is from Pearson's Math Makes Sense Grade 9 Textbook Page 306.

Solving Linear Inequalities Using Multiplication and Division

Let's look at what happens when we multiply and divide to an inequality.

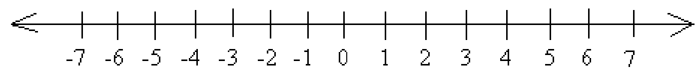
$3 > -1$

Let's **multiply** 2 to each side.
Does the inequality still hold true?



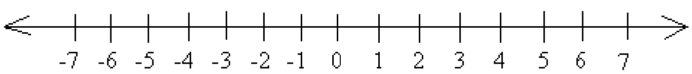
$3 > -1$

Let's **multiply** -2 to each side.
Does the inequality still hold true?



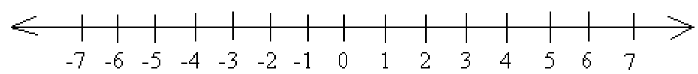
$-6 < 3$

Let's **divide** 2 into each side.
Does the inequality still hold true?



$-6 < 3$

Let's **divide** -2 into each side.
Does the inequality still hold true?



(1) What happens when we **Multiply** or **Divide** by a positive number?

(2) What happens when we **Multiply** or **Divide** by a negative number?

To solve an inequality, we use the same strategy as for solving an equation. However, when we <u>multiply or divide</u> by a negative number, we reverse/flip the inequality sign.

Example 1: Solve the following inequalities. Graph each solution.

a) $-10x \leq 50$ b) $10x \geq -50$ c) $\frac{x}{-3} > -2$ d) $\frac{x}{3} < -2$

e) $6 < 20 - 7x$

f) $2(x + 6) \geq 5x - 9$

g) $-\frac{x}{4} - \frac{7}{2} < \frac{x}{8} + \frac{1}{4}$

h) $3(x - 2) - 5 < 2(x - 1) + 2x$

i) $-2.6a + 14.6 > -5.2 + 10.7a$

j) $21 > -7(x + 2)$

Example 2: A super-slide charges \$1.25 to rent a mat and \$0.75 per ride. Josh has \$10.25. How many rides can Josh go on?

a) Choose a variable, and then write an inequality to solve the problem.

b) Solve the problem.

c) Graph the solution.

Assignment: Lesson 4 **P9.3**

Solve, Check, and graph the solution

1. $-5k < 25$

2. $\frac{x}{-3} > -12$

3. $6x \leq -18$

4. $-2s \geq -4.8$

5. $\frac{x}{4} + 2.5 < 6.1$

6. $1 + \frac{3}{7}x > 13$

7. $12y + 23 \geq -1$

8. $-1 - \frac{m}{4} \leq 6$

9. $9n - 12n + 42 > 0$

10. $6y + 10 > 8 + (y + 14)$

11. $-6x - (2x + 3) \geq 1$

12. $m + 3 - 4m > 2m + 23$

13. $10 + y \leq 8 - (y - 2)$

14. $0.4x - 1.23 > x + 1.17$

15. $0.2(x - 3) < 0.5(2x + 4) - 1$

16. $\frac{x}{2} - \frac{3}{4}(2x - 5) < -\frac{1}{4}$

17. $12.5x - 3.2 \geq 14x - 6.2$

Lesson 5: Solving Stories **P9.3**

To The Teacher:

This lesson is best done in the student's learning groups. Students are to solve each story using poster paper. They should cut out each story and paste it on the page. Below each story will be their solutions. By modelling, students are being asked to create expressions for each of the two situations. They will then put them into an inequality which they will then solve. Once they have their answer, they need to verify that it is correct. Some students may want to work on whiteboards before they use the permanent markers on their poster.

RN #7

Cooperative learning

Lesson Process

1. You may want to include another example of how to solve situational problems before you have students start this activity. Use an example from one of your resources.

2. Hand out to each group
- Scissors
- Copy of the stories
- Markers
- Poster paper
- Whiteboards, dry erase markers, erasures.

3. Students will work in their groups and solve the 6 stories. They need to model each of the situations with an expression, put the expressions together to form an inequality, solve the inequality, and verify that their answer(s) make sense. S5 and S6 are from the McGraw Hill & Ryerson Grade 9 Textbook pg. 364 & 353.

4. Once the groups are complete, have them share 1 or 2 of their stories with the class. Discuss each one and make sure that all the groups agree with the solutions.

5. Students should have the work copied down in their notebooks for future reference.

6. If you want to assign a separate assignment, the questions from the following resources will work.

MathLinks (McGraw-Hill Ryerson): Page 357

#14, 15	Solving inequalities using only addition and subtraction

Page 365

#8b, 9, 10, 11, 12, 13, 14, 15, 16	

Math Makes Sense (Pearson): Page 299

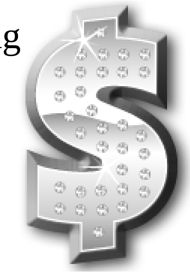
#12, 13	Solving inequalities using only addition and subtraction

Page 306

#10, 13	Solving inequalities using all operations

S1

Jennifer has \$35.50 and is saving \$4.25/week.



Eva has \$24.25 and is saving \$5.50/week.

In how many weeks will Eva start having more in her savings than Jennifer?

Model, Solve, and Verify

S2

Two rental halls are considered for the 2014 graduation Reception

Hall A costs \$50 per person

Hall B costs \$2500, plus \$30 per person.

Determine the number of people for which Hall A will cost less than Hall B

Model, Solve, and Verify

S3

Tamoor and Saba belong to different local fitness clubs.

Tamoor pays \$35 per month And a one time registration Fee of \$15.

Saba pays \$25 per month but had to pay a \$75 registration

After how many months will Tamoor's bill be less than Saba's bill?

Model, Solve, and Verify

S4

Jake plans to board his dogs while he is away on vacation.

House A charges a \$90 one time fee plus \$5 per day.

House B charges \$100 one time fee plus \$4 per day.

For how many days must Jake board his dog for boarding house A to be more than boarding house B.

Model, Solve, and Verify

S5

Abdalla has offers for a position as a salesperson at two local electronic stores.

Store A will pay a flat rate of \$55 per day plus 3% of sales.

Store B will pay a flat rate of \$40 per day plus 5% of sales.

What do Abdalla's sales need to be for store B to be a better offer?

Model, Solve, and Verify

S6

The basketball team here at school wants to buy new jerseys.

Jerseys Unlimited charges \$40 per jersey plus \$80 for a logo design.

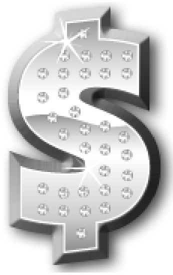
Uniforms R Us charges \$50 per jersey.

How many jerseys does the team need to buy for Jerseys Unlimited to be the cheaper option?

Model, Solve, and Verify

Solving Stories

S1



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Model, Solve, and Verify

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Model, Solve, and Verify

OUTCOME: P9.4

Demonstrate understanding of polynomials (limited to polynomials of degree less than or equal to 2) including:

- Modeling
- Generalizing strategies for addition, subtraction, multiplication, and division
- Analyzing
- Relating to context
- Comparing for equivalency

Lesson 1: Terminology

Lesson 2: Terminology Continued & Like Terms

Lesson 3: Modelling Polynomials with Algebra Tiles & Collecting Like Terms

Lesson 4: Adding Polynomials

Lesson 5: Subtracting Polynomials

Lesson 6: Multiplying Polynomials

Lesson 7: Dividing Polynomials

Previous Experience:

Students have had no experience specifically with polynomials but they have had experience solving equations in grade 8.

To the Teacher:

Although polynomials are a new concept for grade 9 students, they have had experience working with variables when solving equations in grade 8. For some students, they find the numbers, variables, and exponents to be quite abstract. I have found that by using algebra tiles, it gives students the chance to connect an abstract idea to a concrete manipulative. In these lessons, algebra tiles are introduced on the concrete level using the tiles at their desk. Students are then required to draw algebra tiles, and eventually expected to see the mental image of the tiles without having to draw them. Some students may not need algebra tiles to comprehend like terms and the operations involving polynomials. However, for others it may close any gaps they may have with operations involving integers. It also gives those students a chance to understand an abstract concept on a more concrete level. Although algebra tiles can provide a meaningful connection for students, it is important to note that there are some limitations to algebra tiles. There are more limitations than the ones I have noted, but these are the ones I find most prevalent in the classroom.

Limitations:

1. You can't represent every polynomial with algebra tiles. When the polynomials start to involve more variables (or conglomerate variables ie. a^2b) or degrees higher than 2, the model breaks down

2. Often, it feels like using the algebra tiles creates a whole new set of rules and procedures to learn on top of the algebraic method.

3. The whole idea of naming algebra tiles with respect to area goes wayward when the negative tiles are introduced. How do you actually have a negative area?

My suggestion is to explain to students that algebra tiles can only represent some of the simpler variables. Once the polynomials contain more types of variables, we run out of algebra tiles to represent them and we probably don't want to have 8 different sizes and shapes of algebra tiles to sort through. Eventually, the questions are easiest sorted symbolically rather than with tiles. However, the idea of seeing that "like terms" are similar to "like shapes" helps them understand how to collect like terms. It will also give them an idea how addition, subtraction, multiplication, and division works concretely as well. When discussing to students why algebra tiles are named so, it might be best to leave the idea of "area" out of the discussion. Even though it works for the positive tiles, it just perpetrates the myth that some math rules work here, but then they don't work here. Again, it is totally up to you what you decide to do, but I am helping you to avoid an awkward situation such as this…

I was in the middle of a lesson introducing students to algebra tiles and how the name of the tile is associated to the area of the tile. A student puts up her hand and politely asks "but if the names are tied to area, how do we have a negative area? Wouldn't the -1 and -1 on the small square give us a positive area?" As I stood there thinking of a response I decided that the only answer to that question was "I had never though if it that way. I guess we won't bother with talking about area because you are right, it doesn't make sense for the negative tiles."

Even though I was incredibly impressed with her question and insight into the concept, I was at a loss for what to say. In that moment I realized that I was feeding fuel to the fire of "mathematics has so many rules that work for some but not for all" argument. I often avoid tips and tricks that help students learn the content quickly, but the rules aren't founded on "true" mathematics. So for me, the notion of naming the tiles based on area was scrapped because it appears to students that I am just using a rule when it works, and then pushing it off to the side if it doesn't fit in with what I am doing at the time.

To The Teacher:

Having students start with a group discussion on how to translate mathematical symbols, expressions, and equations into English will help down the road when you are in the solving equation unit. It is a great way for you to see what the students know and at the same time back-fill any gaps that some students may have. Some EAL (English as an additional language) students may really appreciate taking the time to show how the mathematical symbol translates into English and vice versa.

RN #2

Prior Knowledge

RN #5

Formative Assessment

Lesson Process:

1. Begin by asking students for their input on how to translate the given mathematical symbols, expressions, and equations in the chart into English. **RN #2**

2. Have students sit in their learning groups and complete the backside of the translation worksheet. Correct the answers as a group.

3. Hand out the whiteboards and give the students the whiteboard questions one at a time. Have students display their answers at the same time. Have groups share their answers and reach a consensus as a class. **RN #5**

4. No assignment for this lesson.

P9.4 Polynomials

Lesson 1: Terminology **P9.4**

Translating English to Math and Math to English

Math	English
$+$	
$-$	
X	
$\div$	
$=$	
x, y, z, a, b, c,…	
$5x$	
$p-9$	
$r+3$	
$5x-10=15$	
x^2	

Translating English to Math and Math to English

A. Translate the following from "math" to English.

1. $+$
2. -5
3. $x-2$
4. $2c+6$
5. $(-3)(p)$
6. $3r=21$
7. $5=2v-8$

B. Translate from English to "math."

1. A certain number.
2. Three plus a number.
3. Five times an unknown number.
4. Six minus a number.
5. Increase a number by ten.
6. A number divided by 5 is equal to 8
7. A number plus 7 is equal to a different number.

C. Find the mathematical statement associated with each of the following situations.

1. A friend of mine is three years older than I am.
2. The skateboard costs five dollars less than I thought it would.
3. Three bags of apples cost twelve dollars.
4. The cost to order 5 books plus seven dollars of shipping is seventy-eight dollars.

Lesson 1: Whiteboard Activity **P9.4**

1. Marion tutors math in the evening. She charges \$30 for high school students and \$25 for elementary students each session. The expression $30h + 25e$ represents her earnings.
    a. What do the variables h and e represent?
    b. How much does Marion make if she works with 3 high school students and 4 elementary students during the week?
    c. Marion decides to increase her fees. \$35 for high school students and \$30 for elementary students each session. What will the new expression be that represents her earnings?
2. To rent a truck at the home store costs the following:

    ### Vehicle Rental Rates & Conditions

    Truck/Van Rental Rates

    First 90 minutes: \$24.95
    Each additional 15 minutes: \$9.00

    Write an expression for the cost.

Lesson 2: Terminology Continued and Like Terms **P9.4**

To The Teacher:

The focus of the lesson is defining the remainder of the terms for the unit. Encourage student participation by having them provide the majority of the input for the lesson. There is a group task included in this lesson for students to work collaboratively. There is a question on the group task that was not addressed in the lesson (such as: provide examples that are NOT polynomials). Students may additionally need to do a quick search on the internet to find out what is and what is not considered a polynomial.

RN #3

Learner Generated Examples

RN #5

Formative Assessment

Lesson Process:

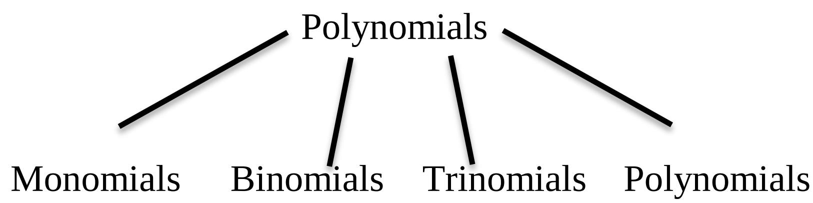
1. You may want to ask students if they know what a term is. It may be difficult for students to articulate what is a term. You may want to ask them for examples of expressions with one and two terms first, and then discuss the definition of a term.

2. Fill in the chart with examples of 1 term and 2 term polynomials. Ask students if they can spot why they are considered 1 term and 2 terms? Why isn't $5ab$ considered 2 terms? Students should recognize that terms are separated by addition and subtraction symbols. $5ab$ is one term while $5a + b$ is two terms.

3. Do the same exercise for 3 terms and 4 terms. You may want to ask students for their own ideas of what 3 term and 4 term polynomials look like.

4. Define a polynomial. Be sure to include that the variables in the polynomial must have positive, whole number, exponents.

5. Example 1: Ask students if they can tell what a monomial, binomial, and trinomial would be by looking at the word. Once you have defined the terms, practice classifying polynomials in example 2. Be careful, students often assume that monomials, binomials, and trinomials are not polynomials. You may want write the following on the board and ask students to discuss.



6. Discuss with students what a degree of a term and a degree of a polynomial is. This is often misunderstood by students so take your time with this one.
7. Example 2: I have left 3 empty spots so you can collaborate with students and generate some class examples. Have students be creative and see if they can create something that would be challenging to find the degree. **RN #3**
8. Example 3: Discuss with students what a coefficient is. Have students use a comma or the word "and" between the numbers. If there is no visible coefficient, students should recognize that the coefficient is 1.
9. Example 4: Ask students if they know what the constant term would be. Give them a hint by asking "what does constant mean when you use the word in everyday life?" Students should know that constants stay the same and never change. As a group, find the constants in the examples. You can use "0" if there is no constant.
10. Example 5: Discuss with students what a variable is. Students should have an understanding of this term because we have used it in the course before. Have students circle the variables in example 6. Students should not circle the exponents, only the letter.
11. Example 6-12: Discuss with students what like terms are. Depending on their grade 8 experience, some students may have experience this. For the majority of students, they will have no experience with like terms. Because this is such an important concept to grasp, I have included a few examples working on this skill. There will be more discussion of like terms in the lesson 3. As a class, work through examples 6-12. Encourage student participation as much as possible to supply most if not all of the answers for the examples. Examples 9 and 10 are learner generated examples **RN #3**
12. There is a short whiteboard activity/exit task if you choose to use it. **RN #5**
13. Assignment: Lesson 2. With students sitting in their learning groups, have them complete the first part of Assignment: Lesson 2, which is a group task. Some students will struggle with the first question. Have them refer back to the definition of a polynomial (positive whole number exponents). The group task is considered an assignment (which is why the individual assignment is shorter).

Terminology & Like Terms

1. Term: ____

Number of Terms	Examples
1	
2	
3	
4	

2. Polynomial: ____

   a. Monomial: ____

   b. Binomial: ____

   c. Trinomial: ____

Example 1: Classify the following polynomials.

a. 5 b. $2x^3 - 2x + 4$ c. $5x^2y^3$

__________ __________ __________

d. $2x$ e. $9a^2 - 5b$ f. a

__________ __________ __________

3. Degree of a Term: ____

Degree of a Polynomial: ____

Example 2: State the degree of the following polynomials.

a. 5	b. $2x^3 - 2x + 4$	c. $5x^2y^3$
________	________	________
d. $2x$	e. $9a^2 - 5b$	f. a
________	________	________
g.	h.	i.
________	________	________

4. Coefficient: ____

____

Example 4: Circle the coefficient(s) in the following polynomials. List the numbers in the space provided.

a. 5	b. $2x^3 - 2x + 4$	c. $5x^2y^3$
________	________	________
d. $2x$	e. $9a^2 - 5b$	f. a
________	________	________

5. Constant Term: ____

____

Example 5: Circle the constant term in the following polynomials.

a. 5	b. $2x^3 - 2x + 4$	c. $5x^2y^3$
________	________	________
d. $2x$	e. $9a^2 - 5b$	f. a
________	________	________

6. Variable: ____

Example 6: Circle the Variable(s) in the following polynomials.

a. 5 ________________

b. $2x^3 - 2x + 4$ ________________

c. $5x^2y^3$ ________________

d. $2x$ ________________

e. $9a^2 - 5b$ ________________

f. a ________________

7. Like Terms: ____

____

Example 7: Circle the polynomials that are like terms with $-3x^2$

$-3x$ $\quad x^2$ $\quad 2x^2y$ $\quad 12x^2$ $\quad 4ab$ $\quad -7x^2$ $\quad -x^2$

Example 8: Circle the polynomials that are like terms with $4ab$

$-ab$ $\quad 2a^2b$ $\quad 4a^2b^2$ $\quad 4a$ $\quad 2ab$ $\quad 4b$ $\quad -6ab$

Example 9: Circle the polynomials that are like terms with $5x$

$-3x$ $\quad x$ $\quad 2x^2$ $\quad 12x^3$ $\quad 4x$ $\quad -7x^1$ $\quad -x^2$

Example 10: List 5 other like terms to $2y^2$

__________ __________ __________ __________ __________

Example 11: List 5 other like terms to $-5xy$

__________ __________ __________ __________ __________

Example 12: Given $3x^2 + 5x - 6xy + y - 12$ answer each of the following

a. What is/are the constant(s)?

b. What is/are the variable(s)?

c. Is this an expression or an equation?

d. Write out each term.

e. How many terms are there?

f. Can we classify this as a trinomial, binomial, or monomial? Why?

g. Identify the coefficient(s)

h. Which term(s) have the highest degree?

i. What is the degree of this polynomial?

j. Write this polynomial in ascending order.

Example 13: Complete the table below.

Polynomial	Number of Terms	Type of Polynomial	Degree of Polynomial	Constant	Variable(s)	Numerical Coefficient(s)
$x^2 + 3x + 1$						
$3y^2 - 4m^3$						
$2a^2b^3c^4$						
		Trinomial	5			
	4				m,n,p	

1. A banquet hall can be rented for parties. An expression for the rental cost is $10n + 450$.
   a. What type of polynomial is $10n + 450$ and what is the degree?
   b. What could the numbers 10 and 450 represent?
   c. How much would it cost to rent the hall for 120 people?

Lesson 2: Group Task (Assignment) **P9.4**

1. Write 3 expressions which are NOT polynomials

a. ____________________ b. ____________________ c. ____________________

2. Give an example of the following polynomials
   a. Trinomial; degree 3 ____________________
   b. Monomial; degree 4 ____________________
   c. Binomial; degree 1 ____________________

3. For the polynomial $5x^2 - 3x - 2$ mark the statements as true or false. Be prepared to state why.
   a. The degree of the polynomial is 3. ___________
   b. The degree of the polynomial is 2 ___________
   c. The coefficient of x^2 is 5 ___________
   d. The coefficient of x is 3 ___________
   e. The constant term is -2 ___________

4. Justify the following statements with examples or counter examples:
   a. We can have a trinomial having a degree 7 ______________________
   b. The degree of a binomial cannot be more than 2 ______________________
   c. A monomial must have a degree of 1 ______________________

5. Complete the entries $-4x^2 + 2xy - y^2 + 4$
   a. Coefficient of x^2 is … ___________
   b. Coefficient of y^2 is … ___________
   c. Degree of the polynomial is … ___________
   d. Constant term … ___________
   e. Number of terms … ___________

1. Complete the following chart.

Polynomial	Number of Terms	Type of Polynomial	Degree of Polynomial	Constant	Variable(s)	Numerical Coefficient(s)
x^2y^3z						
$2x^3 + 3r^3 + 2$						
$2a^2 + 4$						
		Trinomial	3			
	5				w, x, y, z	

2. Write a polynomial that satisfies all of the following statements.
- Is a trinomial
- Has two variables
- Degree of 3
- Has a constant term

3. For each expression, identify the number of terms and whether the expression is a monomial, binomial, trinomial, or polynomial.

a. $9x^2 - 3x + 6$ b. -3 c. $2x - 3$ d. $5r^2 - 3rg + 8dr - g^2$

4. From the list, which terms are like $-7x^2$?

$$7x^2 \quad 7x \quad 6x^2 \quad -7 \quad -5 \quad -7x \quad -3x^2$$

5. From the list, which terms are like $5x$?

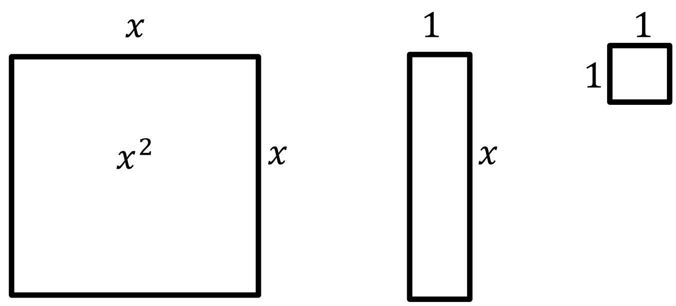
$$5x^2 \quad 4x \quad 3 \quad -8x \quad -5x \quad 9x^2 \quad 5 \quad x \quad -2$$

Lesson 3: Modelling Polynomials with Algebra Tiles & Like Terms P9.4

To The Teacher:

I enjoy teaching with algebra tiles and have included them a lot in this polynomial unit. I find that they help students with their understanding of polynomials. However, I do acknowledge there are some limitations to them. Please refer to the unit introduction for a more detailed description of their limitations and a story of how I got left hanging in front of the classroom. Below is a quick synopsis of why I don't like linking area to the algebra tiles anymore.

The idea of algebra tiles is based on the premise that the x^2 algebra tile is created from a square having dimensions x and x, the x tile is created from a rectangle having dimensions 1 and x, and the 1's square is created from a square having dimensions 1 and 1. See below:



No problem, easy enough right? Until you have a student ask, "if they are based on area, why do we have negative areas?"

RN #6

Representation

RN #5

Formative Assessment

Lesson Process:

1. The first part of the lesson is devoted to remembering some of the important (and often forgotten) terms from the previous lesson.

2. Example 1: Students have experience with this type of question from the lesson 2 group task. Have students supply which expressions are polynomials and which are not polynomials. They need to be able to say why an expression is not a polynomial.

3. Example 2: Have students supply the answers for chart. This is a review of lesson 2.

4. Establish a student generated definition for like terms. Have students circle the like terms with $3x$.

5. Example 3-4: Based on lesson 3, students should be able to answer these examples.
6. Example 5: Students often don't understand how to properly rewrite a polynomial. They usually leave the term's sign behind when rearranging to combine like terms. This will hopefully explain why the negative or positive in front of the term is important.
7. Introduce students to algebra tiles. It is important to write in that the large square can represent any variable squared (x^2, b^2, y^2 etc.) and that the rectangle would then represent the corresponding single variable ($x, b, y,$ etc.).
8. Discuss with students how to draw algebra tiles (shade for negative, leave white for positive).
9. Example 6-7: Have students model polynomials with algebra tiles as well as translate the algebra tiles to a polynomial.
10. With students, define what "zero pair" means. They may have heard this term when working with integers in elementary school.
11. Introduce students to zero pairs with algebra tiles. Have students provide numerical as well as monomial examples of zero pairs in the provided space.
12. Example 8-9: Have students cross off the zero pairs and simplify the polynomial. You may want to hand out algebra tiles to the students so they can use the manipulatives at their desk.
13. Example 10: Use algebra tiles AND the algebraic method to simplify the given polynomials. On 10b, the question is far too big to use algebra tiles so students need to rearrange the terms and collect like terms. On the last polynomial, you are not able to use algebra tiles either. Have students explain why that is (because we don't have enough tiles to represent the different terms).
14. I have included a whiteboard activity and an exit task to assess student's understanding of the lesson. **RN #** 5
15. Assignment: Lesson 3

Modelling Polynomials with Algebra Tiles

A. Review of Terminology

Example 1: A polynomial must be one term or the sum or difference of terms whose variables have POSITIVE WHOLE number exponents. An expression that contains a term with a variable in the denominator such as $\frac{3}{n}$, or the square root of a variable, such as $\sqrt{p}$ is NOT a polynomial. Which are polynomials? (WHY or WHY NOT?)

a) $5 + 6x$ b) $\frac{1}{x^2} + \frac{5}{2x} - 1$ c) $\frac{1}{3}x$ d) $5\sqrt{x}$ e) 11

Example 2: Name the coefficient, variable, constant, and degree of each monomial

	$-2x^2$	f	$3x^2 + 2x - 10$	$9f^3 - 8f^2 + 12$
Degree				
Variable				
Coefficient				
Constant				

In your own words, what is a like term? How can you tell?

Example 3: Circle all the terms below that are like terms with $3x$.

$2x$ x^2 $3y$ x xy $3xy$ p^2 x^3 $8x$ $2308x$

Example 4: Write three terms that are like $5xy^2$.

_______________ _______________ _______________

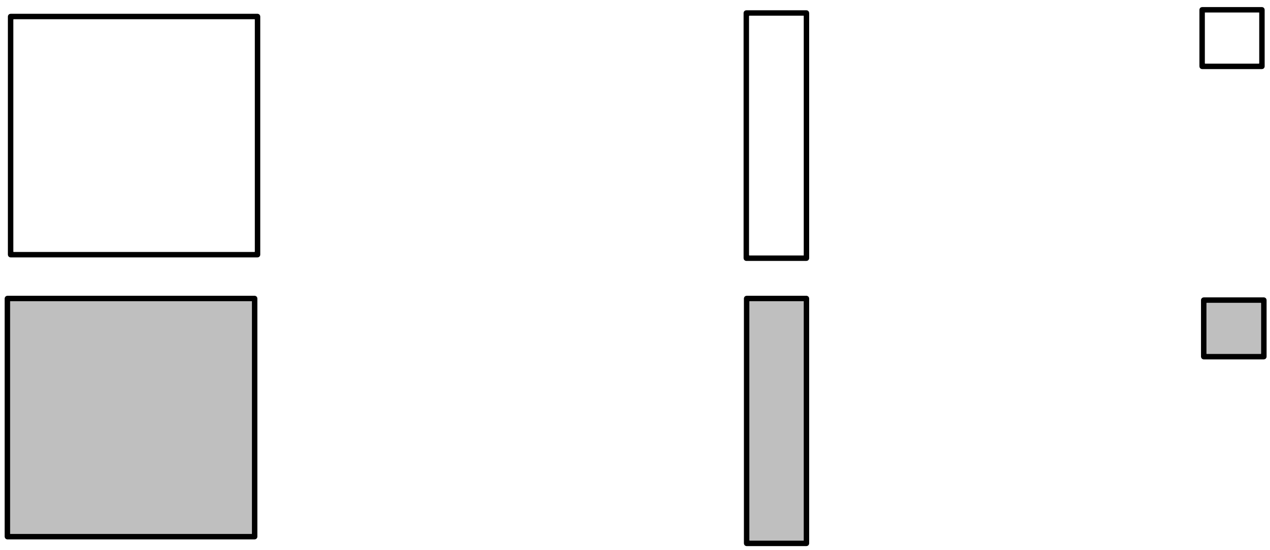
Example 5: We know that $-8m$ and $7m$ are like terms.

a. What does the -8 in $-8m$ tell us? b. What does the 7 in $7m$ tell us?

c. Explain why these are like terms.

B. Algebra Tiles

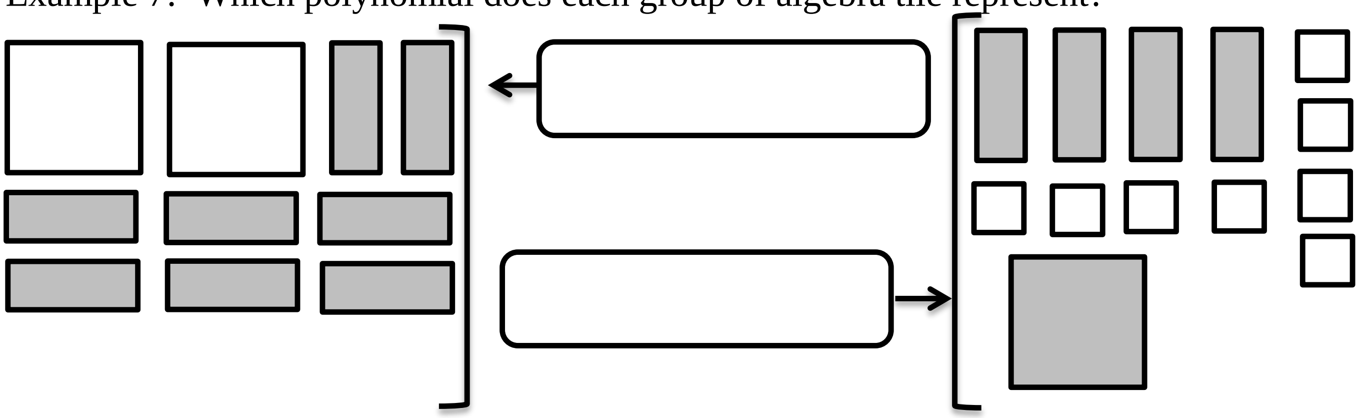
- When we are first working with polynomials, we often use algebra tiles to help us with like terms.
- Unfortunately, they only work for polynomials with one variable up to a degree of 2.
- You may see a variety of colours when working with algebra tiles.
- When drawing your own algebra tiles, shade for negative and leave white for positive.



Example 6: Use algebra tiles to model each polynomial.

a. $-3x^2$ b. $4b^2 - b + 3$ c. $6a - 3$

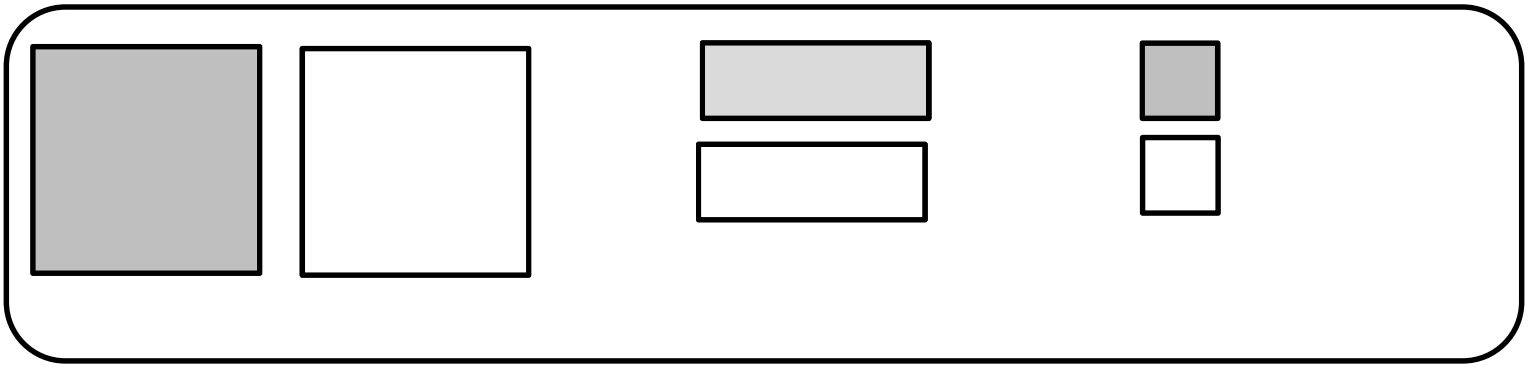
Example 7: Which polynomial does each group of algebra tile represent?



C. Zero Pair

Zero Pair: ____

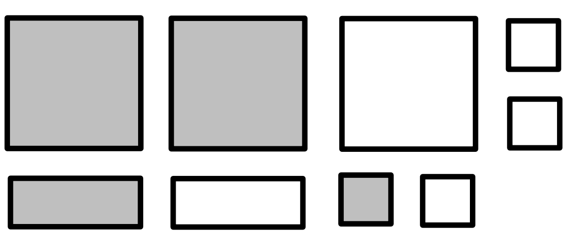
Examples of Algebra Tile Zero Pairs



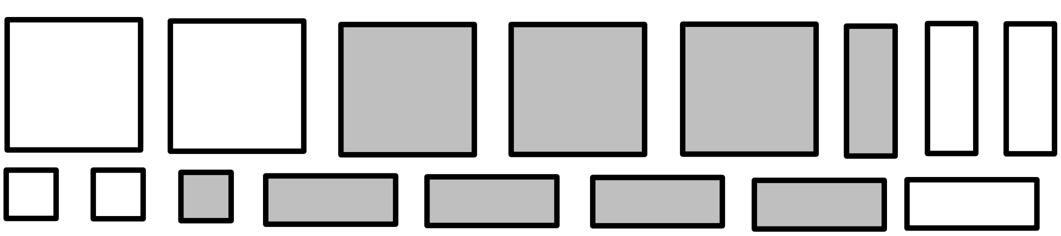
Examples of **Numerical** Zero Pairs

Examples of **Polynomial** Zero Pairs

Example 8: Write the **simplified** polynomial represented by the following algebra tiles.



Example 9: Write the **simplified** polynomial represented by the following algebra tiles.



D. Using Algebra Tiles for Combining Like Terms

Algebra tiles that are the same size and shape are **like terms**.

Example 10: Use algebra tiles AND the algebraic method to simplify the following polynomials.

a. Simplify $4n^2 - 1 - 3n - 3 + 5n - 2n^2$

Tile Model	Algebraic Method

b. Simplify $14x^2 - 11 + 30x + 3 + 15x - 25x^2$

Tile Model	Algebraic Method

c. Simplify $3xy - y^2 - 5x^2 + 3xy - x - 4y^2$

~~Tile Model~~	Algebraic Method
Why can't we use tiles for this polynomial?	

Lesson 3: Assignment **P9.4**

1. Which of the following expressions are polynomials? For the ones that are not, circle the problem area(s) of each polynomial.

   a. $6 + 3d$ b. $\sqrt{3} + x$ c. $\sqrt{x} + 3$

   d. $\frac{1}{x^2} - \frac{2}{x} + 5$ e. $\frac{2}{3}x^2 + 4x$ f. 0

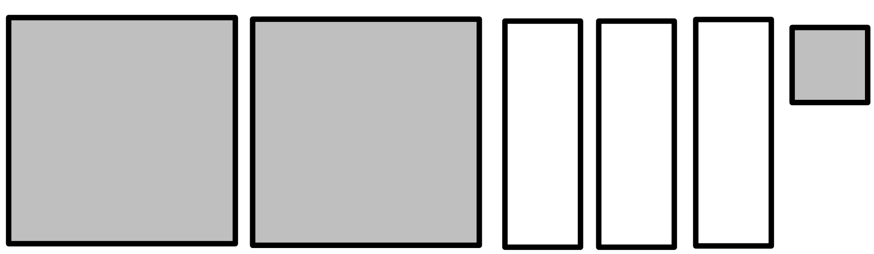
2. Use algebra tiles to model each polynomial (if possible). Sketch the tiles. Shade in for negatives.

   a. $5x^2 - 2x + 1$ b. $2b^2 + 3$ c. $-3f$

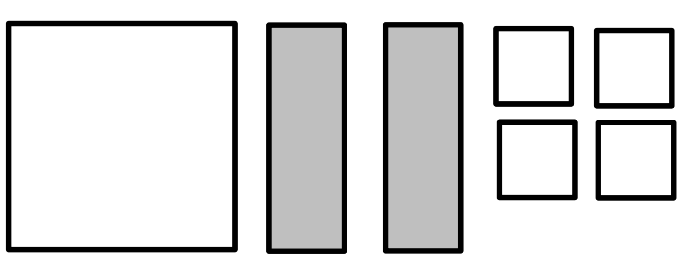
   d. 2 e. $-2d^2 + d - 2$ f. $5xy - 6zx$

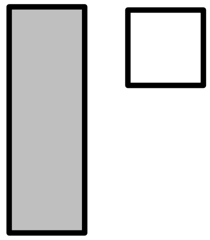
3. Which polynomial does each collection of algebra tiles represent?

   a.



   b.

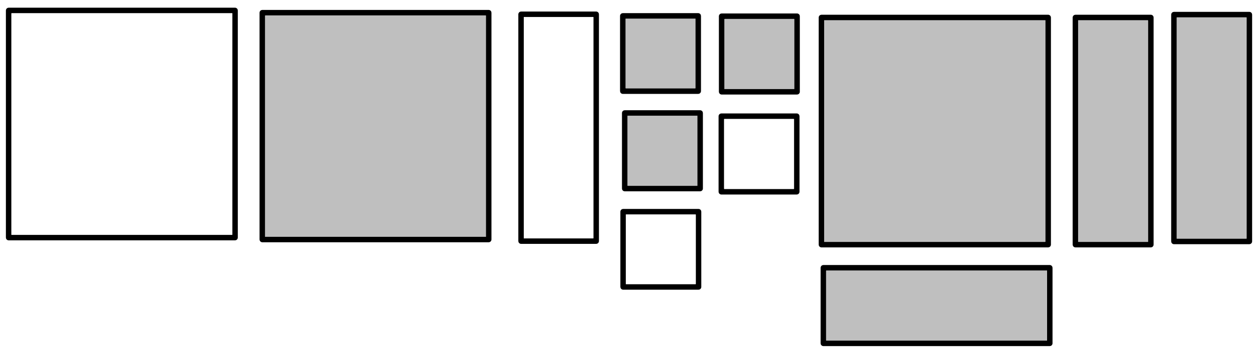


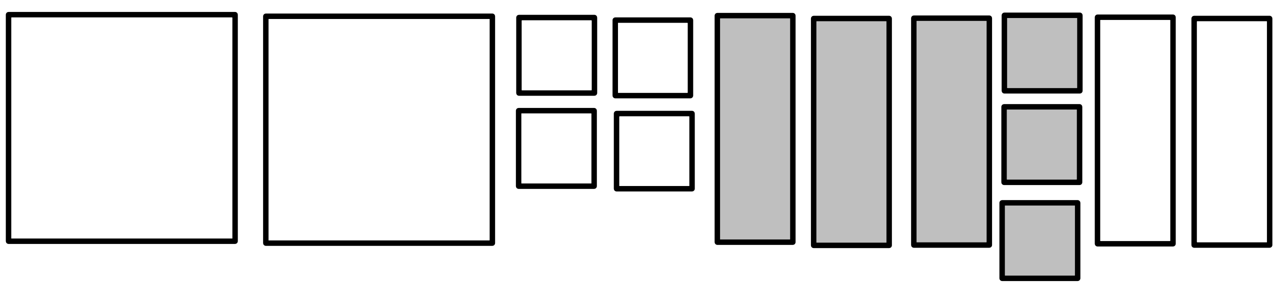
c. 

4. From the list, which terms are like $-2x$? (circle all that apply)

$$-2x^2 \quad -2 \quad 8x \quad x \quad -11x^2 \quad 2x \quad 2x^2$$

5. Write the ***simplified*** polynomial represented by the following algebra tiles.

a. 

b. 

6. Use algebra tiles and **simplify the polynomials**.

a. $3a + 5 + 4a + 2$

b. $2x^2 - 3x + 5x + 6x^2$

c. $2b^2 - 3b + 5 - 3b^2 + 4b + 2$

d. $-2a^2 - 3a^2 + 7a - 2 - 2a - 4$

7. Simplify each polynomial **without** algebra tiles.

a. $5x - 9 + x^2 - 3x + 3$

b. $3x^2 + 9x - 2 + 2x^2 - 3x + 3$

c. $6x^2 - 3x + 4x - 6 - 10x^2 + 5x - 3$

d. $6r - 4r - 3 - 6$

e. $2x^2 + 5y - x^2 - 3y + 1$

f. $4r^2 + rs - 7 + s^2 + 6rs - 12$

g. $3x^2 - 7 + 2x - 3x^2 + 7 - 2x$

h. $r + s - 1 - 6rs + 5s + 2r - 8$

Lesson 3: Exit Task (P9.4) Name:______________________

1. $x^2 - 2x + 3x^2 - 5x + 3$

2. $-3x^2 + 4x + 3 - 2x^2 - 7x + 10$

Lesson 3: Exit Task (P9.4) Name:______________________

1. $x^2 - 2x + 3x^2 - 5x + 3$

2. $-3x^2 + 4x + 3 - 2x^2 - 7x + 10$

Lesson 3: Exit Task (P9.4) Name:______________________

1. $x^2 - 2x + 3x^2 - 5x + 3$

2. $-3x^2 + 4x + 3 - 2x^2 - 7x + 10$

Lesson 3: Exit Task (P9.4) Name:______________________

1. $x^2 - 2x + 3x^2 - 5x + 3$

2. $-3x^2 + 4x + 3 - 2x^2 - 7x + 10$

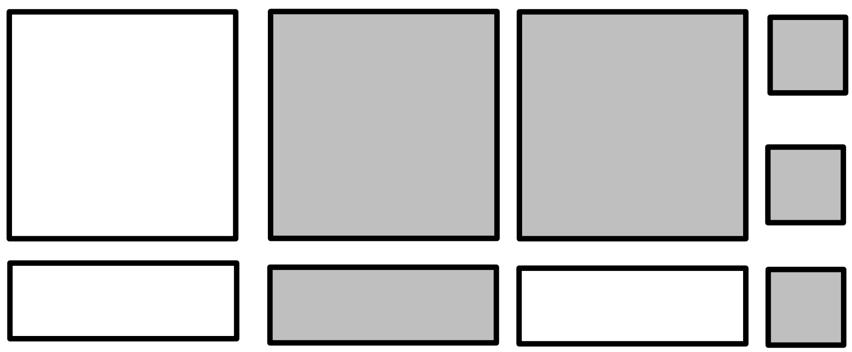
Lesson 3: Whiteboard Activity **P9.4**

Model the following polynomials using algebra tiles.

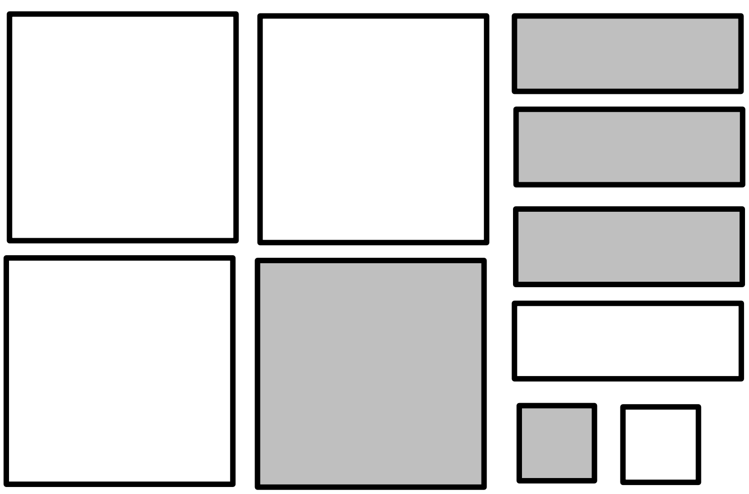
1. $7x + 3$
2. $4x^2 - 3x + 2$
3. $x^2 - 3x - 2$

Write the ***simplified*** polynomial represented by each group of algebra tiles.

4.



5.



Use algebra tiles to model the polynomials and then simplify by collecting like terms.

6. $x^2 - 2x - 3 + 3x^2 + x - 9$
7. $-4x^2 - 2y^2 + 3xy + x^2 + y^2 - 4xy$
8. $6x^2 - 9x^2 - 2x^2$
9. $2x - 7 + 3x + 4 - 9x + 6$
10. $-4 - 3x - 3x^2 - 0 + 5x^2 + 4x - 6x^2$
11. $3x^2 - 5x + 4 + 2x - 6$
12. $2x^2 - 7x + 4 - x^2 + 2x + 2$
13. $3x^2 + 2x - 4 + x^2 - x - 2$
14. $-3x^2 + 5x - 2 - 1 + x + x^2$
15. $-x^2 - x - 1 + 1 + x + x^2$
16. $x^2 + 3x - 4 - 2x + 6$

To The Teacher:

This lesson is an extension of combining like terms. If students are struggling with adding, encourage the use of algebra tiles. The assignment for this lesson doesn't come until after the subtracting polynomials lesson. I have included a whiteboard activity for extra practice if you want students to have more practice before the subtracting lesson.

RN #6

Representation

RN #5

Formative Assessment

Lesson Process:

1. Example 1: have students model the addition of the polynomials. Have them remove their zero-pairs and translate their answer back into algebraic terms. For the algebraic method, have students write $(2x^2 - 3x + 4) + (-4x^2 - x + 2)$. You may want to drop down to example 2 before you add the polynomials together (see below). Discuss the importance of brackets to keep the polynomials grouped together. You may want to ask the students "what the next step is?" or "how do I get to the simplified answer from the left hand column?". Distribution of the $+$ sign through the second bracket is the key. It is important to acknowledge the distribution so they don't think that when we get to subtraction that distribution is an isolated rule. **RN # 6**

2. Example 2: this example will help students understand that the method used to add integers is the same for polynomials. Have students answer example 2 and then ask the students how they got their answers.

3. Example 3: work through as a class. Ask students for their input as much as possible.

4. Example 4: gives students the opportunity to try some on their own or to continue working as a group.

5. Example 5: this example may prove to be difficult for some students depending on their understanding of perimeter. You may want to ask students "what is perimeter?" Ask students for their input as much as possible when answering the question.

6. Example 6: this example requires students to find the error in a make-believe student's work. In this case, I made the error a procedural one (although a case could be made that

it is conceptual). Steps 2 and 3 are correct, the error is in the last step where they combined $-3x$ and $-x$ and got $-3x$. The make-believe student didn't recognize the $-x$ as having a value of $-1x$. Ask students to do this example on their own and have a few students describe the feedback that they wrote to the student. When asking them to provide feedback, make sure they understand that they want to give the student help so they recognize what their error was and they don't make the same mistake again. Circling the mistake isn't good enough feedback.

7. There is not a formal assignment dedicated to this lesson. It is combined with lesson 5 (subtracting polynomials). You can choose to assign the addition questions from the assignment.

8. Whiteboard Activity: Lesson 4 **RN # 5**

Adding Polynomials

In order to add polynomials, you need to combine like terms. We are going to develop a strategy to add polynomials with and without using algebra tiles.

Example 1: Add the following trinomials together.

$$2x^2 - 3x + 4 \text{ and } -4x^2 - x + 2$$

Model	Algebraic

Brackets are used to group polynomials. When there is a "friendly" addition sign between the brackets, you can distribute (multiply) the $+$ sign through the second bracket. But we all know that multiplying a $+$ sign through doesn't change anything.

Example 2: Add the following integers

a. $(5) + (3)$ b. $(-5) + (-3)$ c. $(-5) + (3)$ d. $(5) + (-3)$

Polynomials follow the same rule as integers.

Example 3: Add $(2r - 3) + (2r^2 - r - 1)$

Example 4: Add the following polynomials

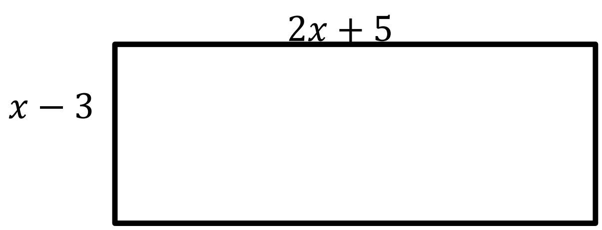
a. $(4x - 6) + (-8x + 11)$

b. $(-x - 7) + (5 - 2x)$

c. $(6k^2 - 2k) + (3 - k + 2k^2)$

d. $(-x^2 + 3 - 7x) + (7 + x^2 + 10x)$

Example 5: Find the perimeter of the following rectangle.



Example 6: A student added $(2x^2 - 3x + 5)$ and $(-3x^2 - x - 1)$ as follows.

$$(2x^2 - 3x + 5) + (-3x^2 - x - 1)$$

$$2x^2 - 3x + 5 - 3x^2 - x - 1$$

$$2x^2 - 3x^2 - 3x - x + 5 - 1$$

$$-x^2 - 3x + 4$$

Is the student’s work correct?

If not, find the student’s mistake and provide feedback so they can recognize their error.

Lesson 4: Whiteboard Activity **P9.4**

Add the following polynomials

1. $(-2x^2 + 3x - 2) + (-x^2 + 4x + 8)$
2. $(3x - 5) + (9x - 1)$
3. $(4x^2 - 10) + (-3x^2 + 9)$
4. $(-x^2 + 7) + (x^2 - 7)$
5. $(12a^2 - a - 3) + (3a^2 + 6)$
6. $(-3y^2 + y) + (-3y - 5)$
7. $(f^2 + 5f + 12) + (f^2 - 3f - 10)$
8. $(4x^3 + 3x^2 - x + 2) + (-2x^3 - 4x)$
9. $(-2v^4 + 7v^3 - 2v^2 + 3v - 1) + (-6v^4 - 8v^3 - 7v^2 - 2v - 9)$

To The Teacher:

Subtracting polynomials poses a very interesting question for the teaching and learning of mathematics. When you teach subtraction, do you add the opposite or distribute the negative? When I ask mathematics teachers what method they prefer, I get different answers for what they do in their grade 9 class. Some teachers change the subtraction statement to an addition statement and "add the opposite". Others teach it by distributing the negative sign through the back bracket. I decided to include both in the lessons for two reasons. First, many students will have different mathematics teachers during their high school career. By showing both methods it will cover their knowledge attained from previous classes. It will also prepare them for teachers who explain it a different way than their preferred method in future years. Secondly, understanding distribution is an absolute necessity for polynomial multiplication. Being exposed to the method is important for that lesson. Once students are exposed to both methods, they can choose which one works best for them.

One could argue that "adding the opposite" and "distributing the negative" are the same thing. I disagree. One is changing the expression to an addition statement while the other is multiplying the -1 in front of the bracket through.

RN #2

Prior Knowledge

RN #5

Formative Assessment

RN #6

Representation

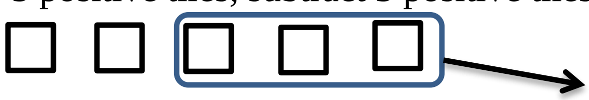
Lesson Process:

1. Begin the lesson by asking students "what is subtraction?"

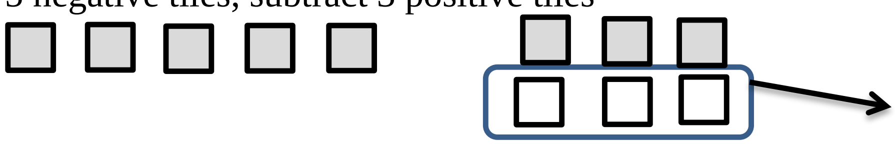
2. Example 1: Method 1, review with students how to subtract on a number line. I prefer to talk about it as "difference". What is the difference between 5 and 3 on the number line? What is the difference between -5 and 3 on the number line? This is a good time to assess how if they see it as "add the opposite" or "distribute (multiply) the signs".

3. Example 1: Method 2, discuss with students how to subtract using algebra tiles. Students may remember this technique from grade 7. Instead of using algebra tiles they probably used counters in elementary school. **RN # 2**

    a. $(5) - (3)$ You may want to discuss if the brackets are necessary.
    5 positive tiles, subtract 3 positive tiles

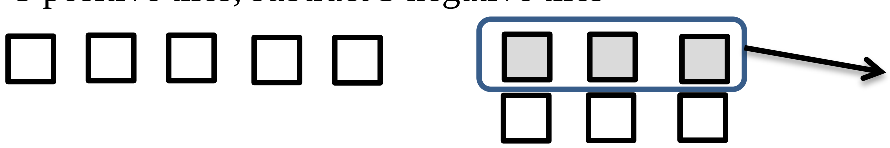
    

    b. $(-5) - (3)$ You may want to discuss if the brackets are necessary.
    5 negative tiles, subtract 3 positive tiles

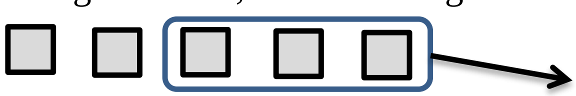
    Uh Oh! We don't have any positive tiles to remove! Let's add in zero pairs and remove the 3 positive tiles.

    c. $(5) - (-3)$ You may want to discuss if the brackets are necessary.
    5 positive tiles, subtract 3 negative tiles

    Uh Oh! We don't have any negative tiles to remove! Let's add in zero pairs and remove the 3 negative tiles.

    d. $(-5) - (-3)$ You may want to discuss if the brackets are necessary.
    5 negative tiles, subtract 3 negative tiles.

4. Warning: For the next two methods you may want to discuss with students that they are going to feel more comfortable with one than the other depending on the way they have been taught in the past. You are going to show both methods but they can choose which

one they are going to use for assignments and examples. You may even want to display a question on the board and ask students to answer, such as $5 - (-10)$ and see which students added the opposite and which students multiplied the signs together.

5. Example 1: Method 3, adding the opposite. Tell students that all subtraction statements can be written as addition statements by the method of “add the opposite”.

   a. $(5) - (3) = 2$
      $(5) + (-3) = 2$

      * This question is best left as it is. Easily interpreted this way.

   b. $(-5) - (3)$
      $(-5) + (-3)$
      -8

   c. $(5) - (-3)$
      $(5) + (+3)$
      $(5) + (3)$
      8

   d. $(-5) - (-3)$
      $(-5) + (+3)$
      $(-5) + (3)$
      -2

6. Example 1: Method 4, distribution. Tell students that another method is to multiply the – or -1 into the bracket. For questions a and b, it doesn’t really work well because the question doesn’t change when you multiply the negative through. However, for c and d the questions changes dramatically. For some students, this is familiar to them.

   c. $(5)\overset{+}{-(-}3)$
      $5 + 3$

   d. $(-5)\overset{+}{-(-}3)$
      $-5 + 3$

7. Example 2: Practice what methods 2, 3, and 4 look like with polynomials. Explain to students that they are expected to be able to model method 2, but they can choose between methods 3 and 4 depending on what they like and remember the best.

8. More practice questions are included in the Whiteboard Activity: Lesson 5 **RN #5**

9. Included is an exit task **RN #5**

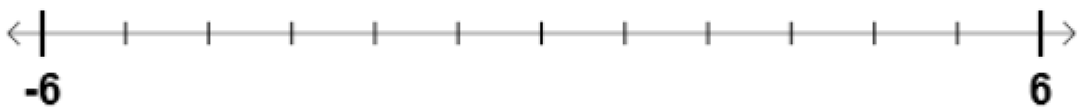
10. Assignment: Lesson 4 & 5

Subtracting Polynomials

Before we start subtracting algebraically, we should review what it means to subtract.

Example 1: Subtract

Method 1: Number Line



a. $(5)-(3)$ b. $(-5)-(3)$ c. $(5)-(-3)$ d. $(-5)-(-3)$

Method 2: Algebra Tiles

a. $(5)-(3)$ b. $(-5)-(3)$ c. $(5)-(-3)$ d. $(-5)-(-3)$

Method 3: Numerically (add the opposite). Adding the opposite is the EXACT same as subtracting! You can represent any subtraction statement as the addition of the opposite. Let's experiment with some subtraction statements and convert them into addition statements. a. $(5)-(3)$ b. $(-5)-(3)$ c. $(5)-(-3)$ d. $(-5)-(-3)$
Method 4: Numerically (distribution). The subtraction sign is representing a -1 that can be multiplied to the value(s) behind it. a. $(5)-(3)$ b. $(-5)-(3)$ c. $(5)-(-3)$ d. $(-5)-(-3)$

We can apply the same strategies when working with polynomials. Let's practice all of the methods.

Example 2: Subtract the following statements

a. $(5x-2)-(2x+4)$

Algebra Tiles	Method 3: Add the Opposite	Method 4: Distribute the Sign
$(5x-2)-(2x+4)$	$(5x-2)-(2x+4)$	$(5x-2)-(2x+4)$

b. $(6x^2 - 2x + 3) - (-x^2 + 4x - 2)$

Algebra Tiles	Method 3: Add the Opposite	Method 4: Distribute the Sign
$(6x^2 - 2x + 3) - (-x^2 + 4x - 2)$	$(6x^2 - 2x + 3) - (-x^2 + 4x - 2)$	$(6x^2 - 2x + 3) - (-x^2 + 4x - 2)$

c. $(-2x^2 + 7x - 2) - (3x^3 + 2x - 1)$ Use your own method

d. $(-x^2 + 4x - 2) - (4x^2 + 2x - 5)$

e. $(14y^2 + 3xy + x^2 - 5) - (3xy + 4x^2 + 12y^2 - 10)$

Lesson 4 & 5: Exit Task (P9.4) Name:____________________

Add or subtract as indicated

1. $(2x^2 - 3x + 8) - (5x^2 - x + 15)$

2. $(x^2 + 3x - 4) + (7x^2 - 7x - 1)$

Lesson 4 & 5: Exit Task (P9.4) Name:____________________

Add or subtract as indicated

1. $(2x^2 - 3x + 8) - (5x^2 - x + 15)$

2. $(x^2 + 3x - 4) + (7x^2 - 7x - 1)$

Lesson 4 & 5: Exit Task (P9.4) Name:____________________

Add or subtract as indicated

1. $(2x^2 - 3x + 8) - (5x^2 - x + 15)$

2. $(x^2 + 3x - 4) + (7x^2 - 7x - 1)$

Lesson 4 & 5: Exit Task (P9.4) Name:____________________

Add or subtract as indicated

1. $(2x^2 - 3x + 8) - (5x^2 - x + 15)$

2. $(x^2 + 3x - 4) + (7x^2 - 7x - 1)$

Lesson 5: Whiteboard Activity **P9.4**

Subtract the following polynomials

a. Using algebra tiles

1. $(2x^2 - 3x + 5) - (x^2 - 2x + 3)$

2. $(-5x^2 + 7x - 2) - (-2x^2 - x + 3)$

b. without algebra tiles

1. $(3x^2 - 2x + 4) - (-3x^2 + x + 1)$

2. $(5x - 1) - (-3x + 1)$

3. $(-4x^2 + 2) - (-2x^2 + 2x - 1)$

4. $(8x^2 - 2x + 4) - (-9x^2 - x + 10)$

5. $(4f^2 + 3f + 8) - (7f^2 + 6f + 10)$

6. $(-9x^2 - 6x - 1) - (-2x^2 - 3x - 5)$

7. $(2x^3 + 3x^2 - 5x + 8) - (6x^3 - 2x^2 + 6x - 1)$

Lesson 4 & 5: Assignment **P9.4**

1. Use algebra tiles to model each difference. Record your answer.

a. $(5x+4)-(6x+7)$

b. $(-5x^2-2x+3)-(-2x^2-3x-1)$

2. Subtract the following without using algebra tiles.

a. $(7x+12)-(4+3x)$

b. $(8u^2-3u)-(u)$

c. $(2x^2+7)-(7x^2-2)$

d. $(7x^2+13x-8)-(17x^2-4x+6)$

e. $(4a+12)-(-2a-2)$

f. $(-3x^2+2x-3)-(5x^2-10x+7)$

g. $(-5x^2-17x+1)-(-3x^2+12x-10)$

h. $(-5x^2-11x+7)-(-8x^2+9x-10)$

i. $(-5a+12b-5c-4d)-(-2a+7b-6c-4d)$

3. Add or subtract as indicated.

a. $(5x - 9) + (-3x - 10)$

b. $(7x - 10) - (2x + 3)$

c. $(-2x^2 - 3x + 4) - (x^2 + 3x + 4)$

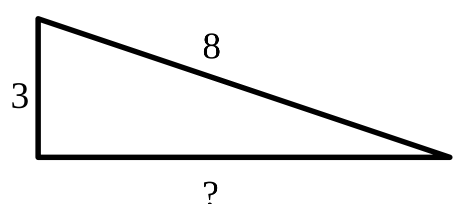
d. $(5x^2 - 3x + 10) + (-5x^2 + 3x - 10)$

e. $(10x - 6) - (10x - 6)$

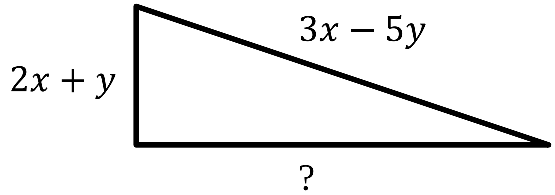
f. $(-3g^2 + 2gh + h^2) + (6gh - 3h^2 + 5g^2)$

g. $(xy - 3y + 2x^2 + 5y^2 - x) - (y^2 + 9x - 4y - 2xy + 12x^2)$

4. If the perimeter of the triangle below is 16, find the length of the missing side.



5. If the perimeter of the triangle below is $7x + 2y$ find the length of the missing side.



Lesson 6: Multiplying Polynomials **P9.4**

To The Teacher:

The lesson starts with a discussion of what multiplication is and how to represent it visually. By relying on their previous knowledge on multiplication of integers, students will extend their knowledge to polynomial multiplication. Students will also need to recall their exponent knowledge in order to multiply monomials. I recommend students working with algebra tiles at their desk or with virtual algebra tiles, and then sketch the tiles in their notes. As with most other lessons, it is best to solicit as much information from the students as possible.

RN #6

Representation

RN #2

Prior Knowledge

RN #5

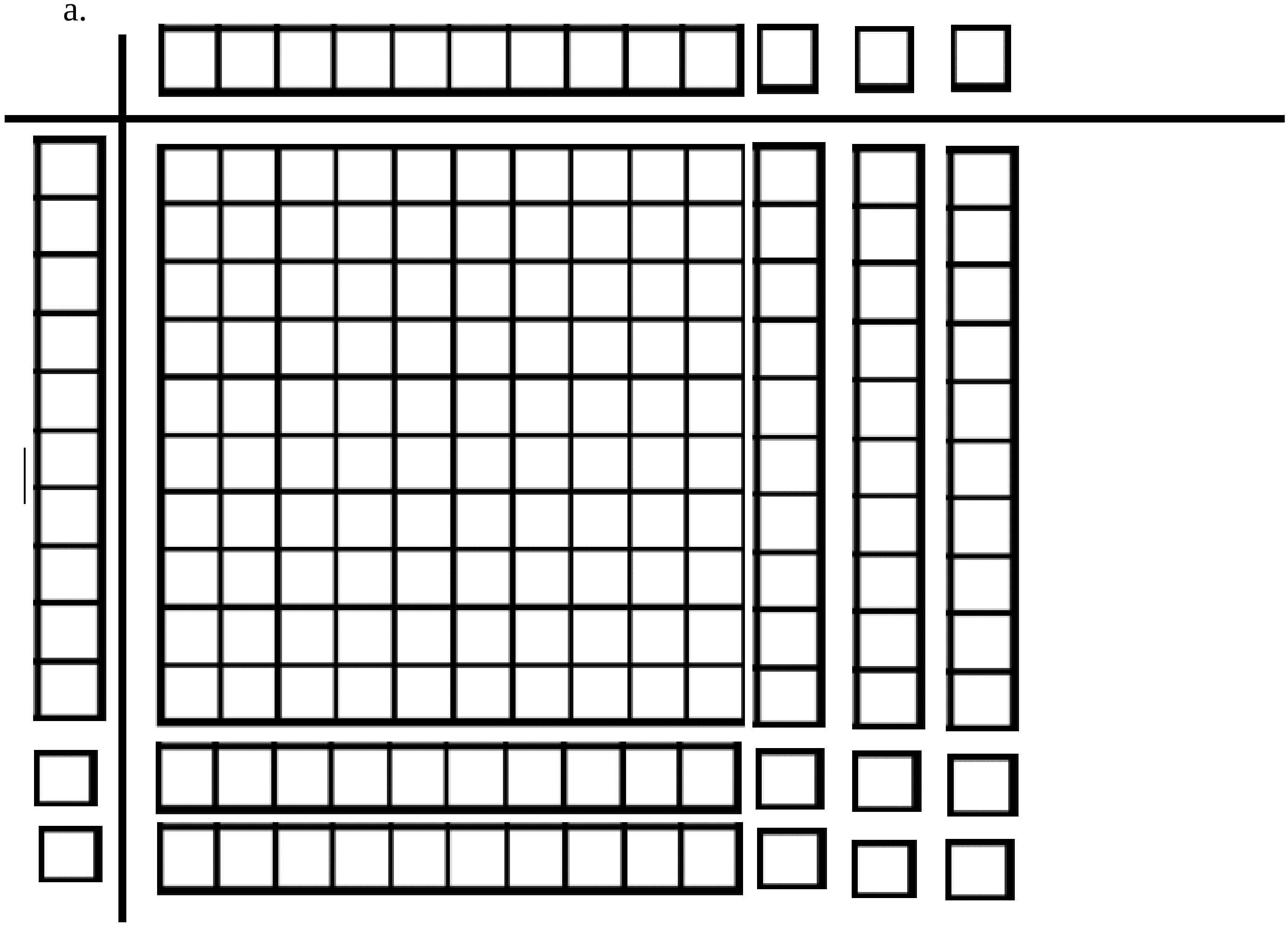
Formative Assessment

Lesson Process:

1. Begin the lesson by asking students what is multiplication. It is important that they see it as scaling (making something bigger or smaller) rather than repeated addition. Have students complete example 1 and ask how they got their answers.

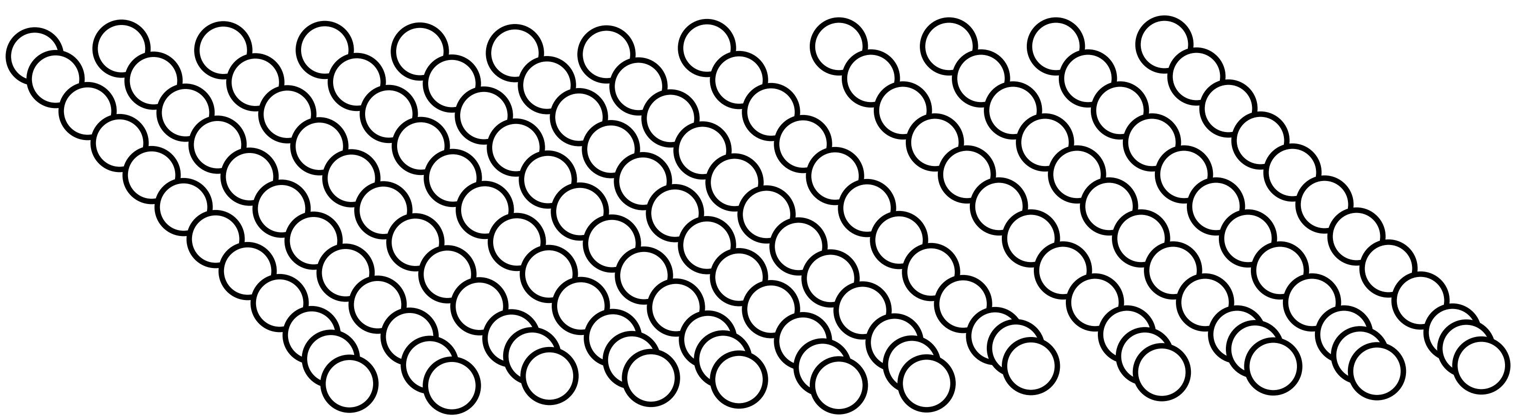
2. Example 2: Ask students to show 13×12 using a model. Depending on their elementary school experience, here are some examples of what students may draw. **RN # V, RN # PK**

   a.



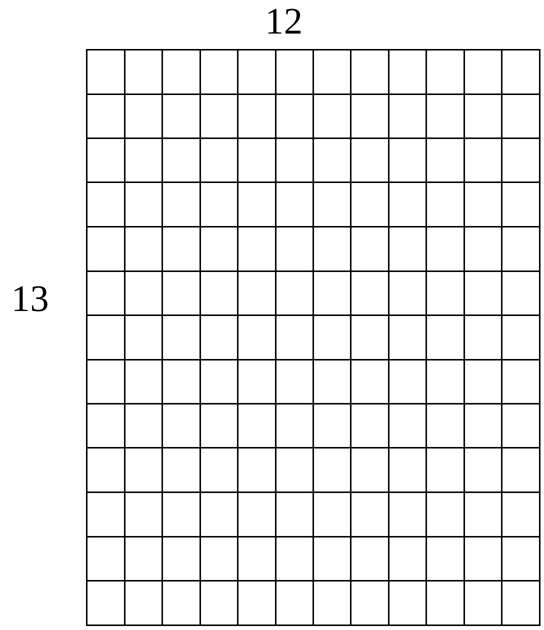
$100 + 30 + 20 + 6 =$_____

   b. 12 groups of 13 OR 13 groups of 12



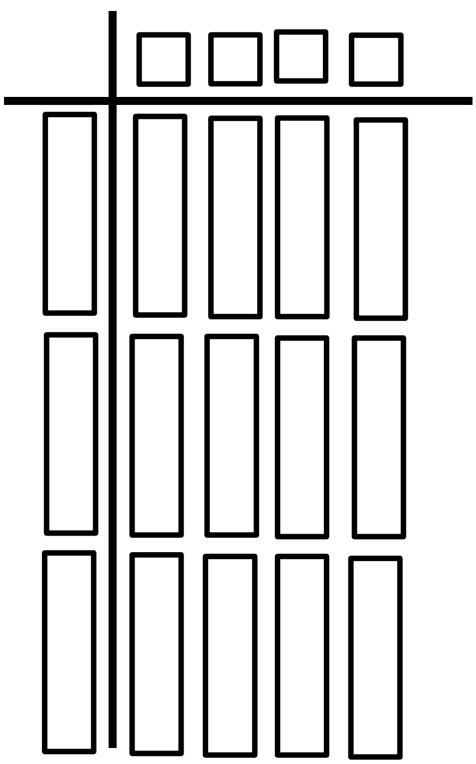
Students may also draw the following (13 groups of 12)

How many squares?

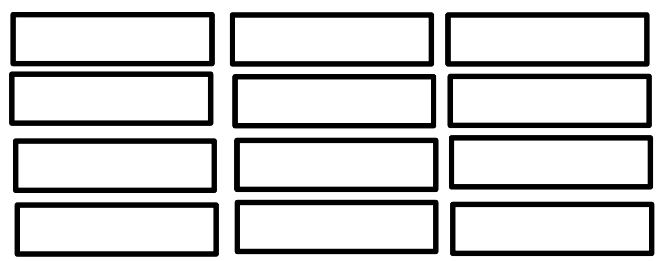


3. Example 3: using the idea of multiplying base 10 blocks. Ask students if they would be able to multiply $(4)(3x)$ using algebra tiles. **RN #6**

Method #1

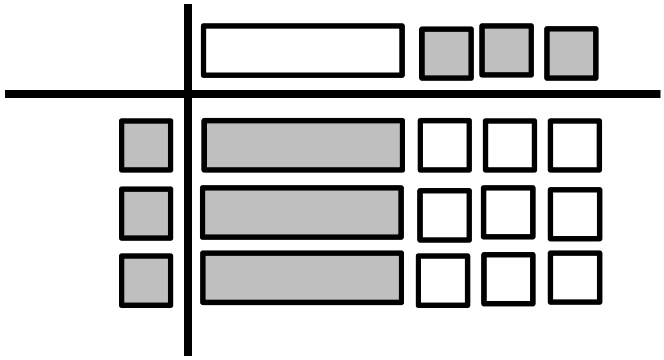


Method #2 (4 groups of $3x$)



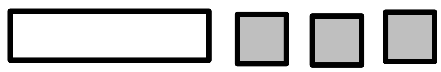
4. Example 4: have students try the following multiplication $-3(x-3)$

Method #1

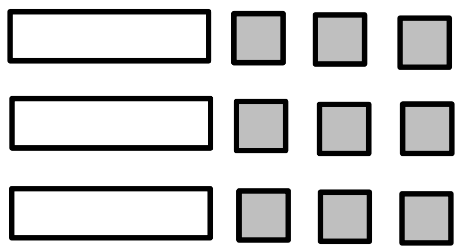


Method #2

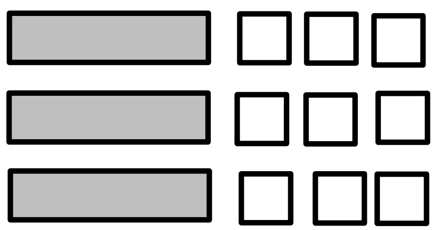
Draw $(x-3)$



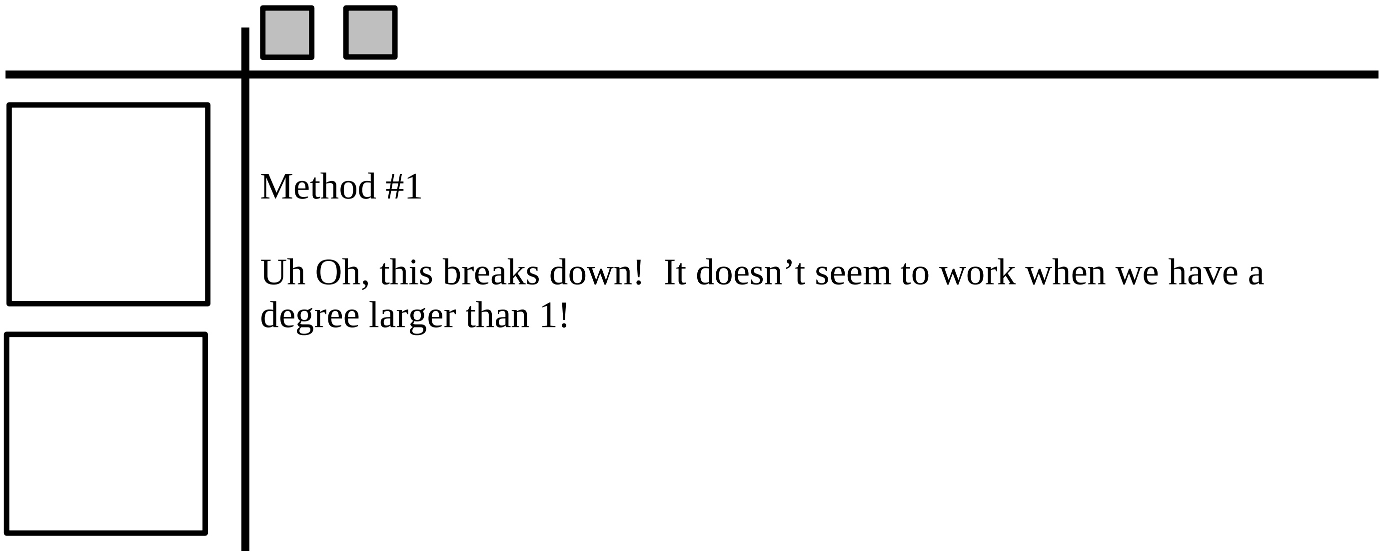
Multiply that by 3



Multiply by negative
Flip tiles



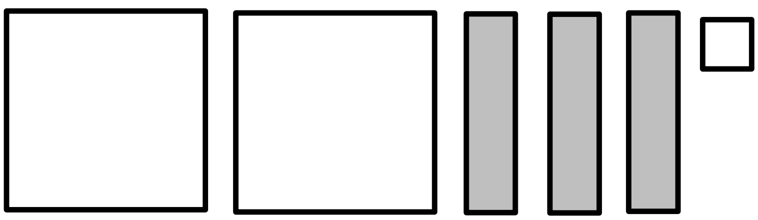
5. Example 5: Have students try a constant × trinomial. $-2(2x^2 - 3x + 1)$



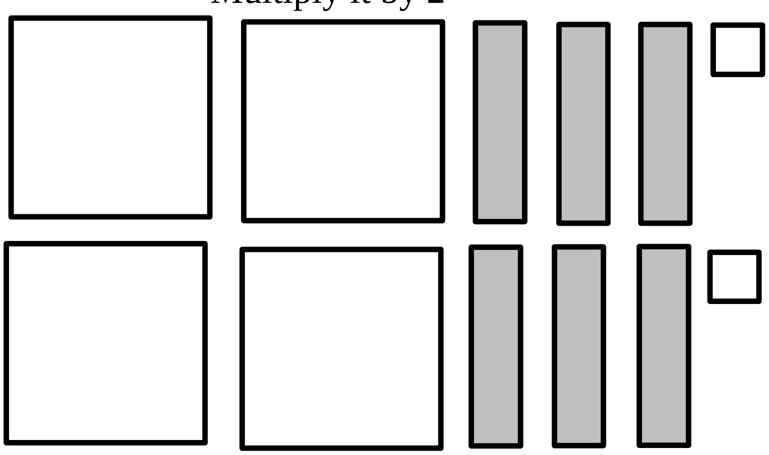
Method #1

Uh Oh, this breaks down! It doesn't seem to work when we have a degree larger than 1!

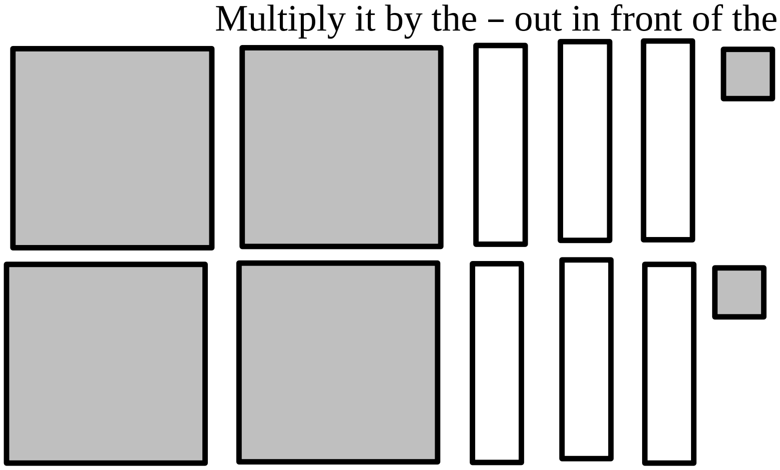
Method #2 – Model $2x^2 - 3x + 1$



Multiply it by 2



Multiply it by the – out in front of the 2



6. Discuss with students the importance of being able to do the math without the algebra tiles. How can we ensure we get the right answer without having to draw algebra tiles each time? Answer: Distribution, multiply the number out front through the bracket.

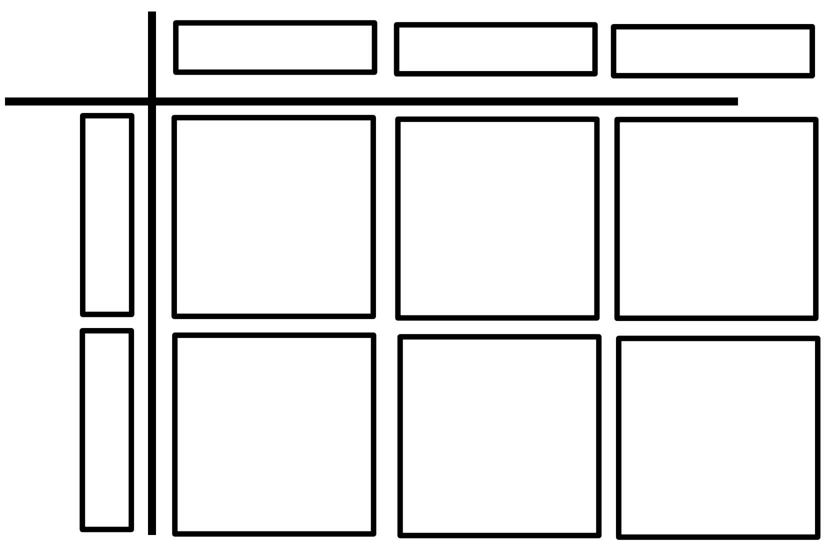
7. Introduce the distributive property.

8. Answer the previous questions using the distributive property.
   a. $(4)(3x)$ b. $-3(x-3)$ c. $-2(2x^2-3x+1)$

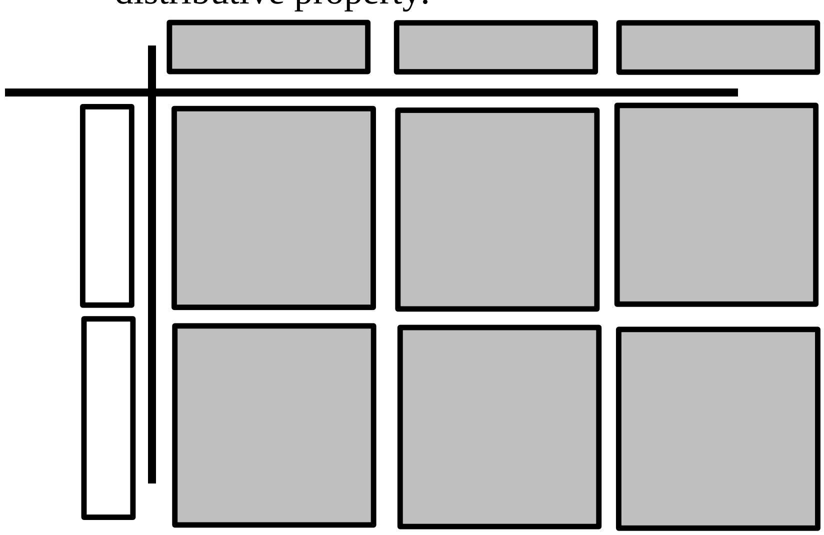
9. Example 6: discuss with students what happens when we multiply monomials $(5^3)(5^2)$. Students have already seen this type of question in the exponent unit.

10. Example 7: have students extend that knowledge to $(x^3)(x^2)$. We aren't able to model this because of the x^3.

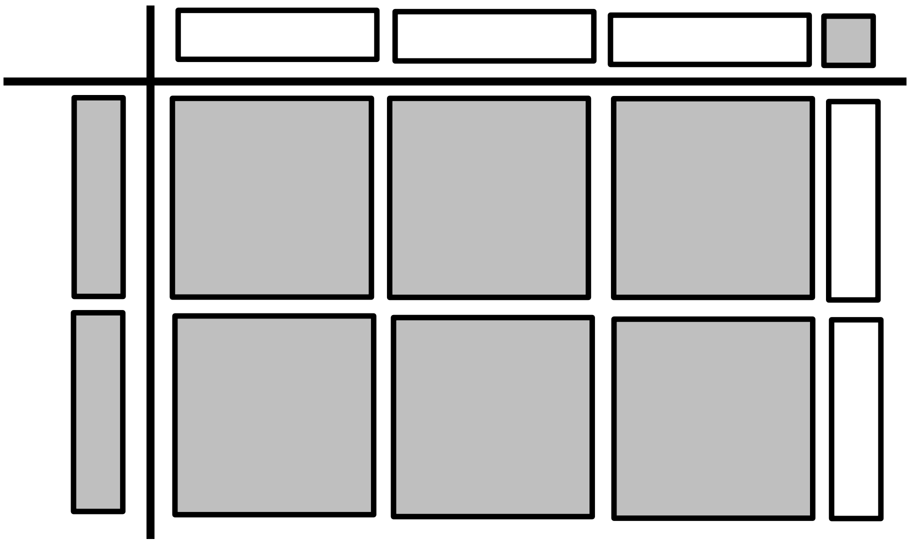
11. Example 8: Have students model $(3x)(2x)$ using the array model as well as the distributive property.



12. Example 9: have students model $(-3x)(2x)$ using the array model as well as the distributive property.



13. Example 10: discuss with students what will happen when there is a variable out front of the bracket. Focus on what happens when we multiply the x and x. It becomes x^2, why? Have students model $-2x(3x-1)$ using the array model as well as the distributive property.



14. Conduct the whiteboard activity **RN#5**

15. Assignment: Lesson 6

Multiplying Polynomials

A. What is Multiplication?

Example 1: Multiply the following integers

a. $(5)(3)$ b. $(-5)(3)$ c. $(5)(-3)$ d. $(-5)(-3)$

Example 2: Model the following multiplication 12×13

B. Multiplying a Polynomial by a Constant

How does multiplication of integers help us with multiplication of polynomials?

Example 3: Model the following multiplication using algebra tiles. $(4)(3x)$

Method #1

Method #2

What happens when we use negative values?

Example 4: Model the following multiplication using algebra tiles. $-3(x-3)$

Method #1

Method #2

What happens when we try to multiply a constant to a degree bigger than 1?

Example 5: $-2(2x^2-3x+1)$

Method #1

Method #2

How do we do the work without using algebra tiles?

Let’s answer all of the previous examples using the distributive property.

a. $(4)(3x)$ b. $-3(x-3)$ c. $-2(2x^2-3x+1)$

C. Multiplying a Polynomial by a Monomial

Let’s review one of our exponent rules.

Example 6: Multiply $(5^3)(5^2)$

Example 7: Multiply $(x^3)(x^2)$

Example 8: Model the following multiplication using algebra tiles. $(3x)(2x)$

Method #1

Method #2 – Distributive Property

Example 9: Multiply using the following methods. $(-3x)(2x)$

Method #1

Method #2 – Distributive Property

Example 10: Model the following multiplication using algebra tiles. $-2x(3x-1)$

Method #1

Method #2 – Distributive Property

Lesson 6: Whiteboard Activity **P9.4**

1. $5(x+3)$
2. $-3(-x-2)$
3. $-2(x+9)$
4. $3(-x+4)$
5. $6(x-10)$
6. $-7(-4x+5)$
7. $-2(7x-13)$
8. $5(-3x-3)$
9. $x(2x-3)$
10. $-3x(-4-7x)$
11. $2c(c+1)$
12. $6y(-2k+3y)$
13. $4t^2(3t^2-2t-1)$

Lesson 6: Assignment **P9.4**

1. Multiply

a. $4(6s)$ b. $(7s)(3)$ c. $-5(4d)$ d. $(-2t)(6)$

e. $5(4r-2)$ f. $-2(5y^2-2y+3)$ g. $(-x^2-x)(2)$

h. $(x^2+x)(-3)$ i. $-3(2x^2-4x+5)$ j. $10(2s^2-s+2)$

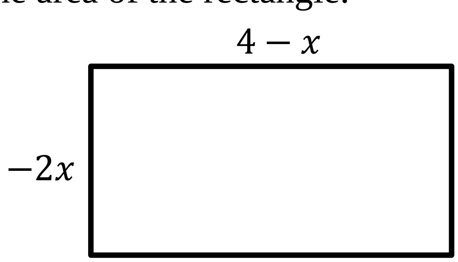
2. Multiply

a. $(3r)(2r)$ b. $(-t)(-6t)$ c. $(-5x)(2x)$

d. $6x(x-2)$ e. $-3w(2w+4)$ f. $2g(-g-1)$

g. $-y(-1+y)$ h. $4s(7r+1)$ i. $(-t)(8u-7t)$

3. Determine the area of the rectangle.



Lesson 7: Dividing Polynomials **P9.4**

To The Teacher:

The lesson starts with a discussion of what division with respect to integers is and how to represent it visually. By relying on their previous knowledge on division of integers, students will extend their knowledge to polynomial division. I recommend using Smartboard software, or magnetic whiteboard tiles to show how to divide polynomials rather than using sketching. Often times sketching on the board seems less dynamic than using interactive manipulatives. It is important for students to be working with algebra tiles at their desk, or using an interactive program on a tablet. As with most other lessons, it is best to solicit as much information from the students as possible.

You may want to review the two types of division (quotitive and partitive) before beginning the lesson. When dividing by a constant, you can think of $6x \div 2$ in two ways:

1. $6x$ divided into two equal groups (partitive – equal sharing)
2. $6x$ is divided into groups of 3 (quotitive)

This may be one of those lessons where you feel that the algebra tiles are just adding more steps and procedures to memorize than giving students a better understanding of division. In that case, do what feels best for you and your students.

RN #6

Representation

RN #2

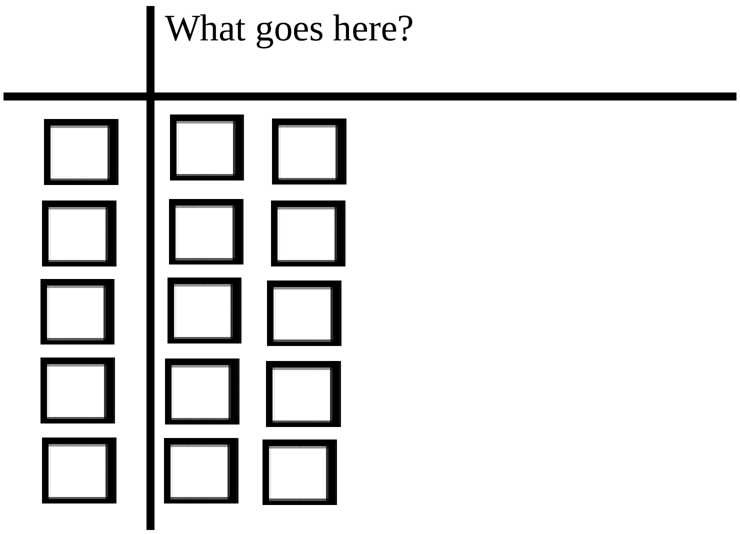
Prior Knowledge

RN #5

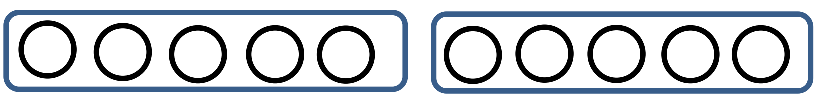
Formative Assessment

Lesson Process:

1. Begin the lesson by asking students if they know what division is.
2. Example 1: Check student’s previous understanding of division with integers.
3. Example 2: have students model the division of $10 \div 5$. **RN #6**

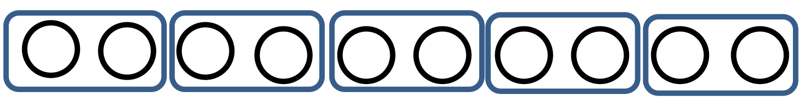


OR



Two groups of 5

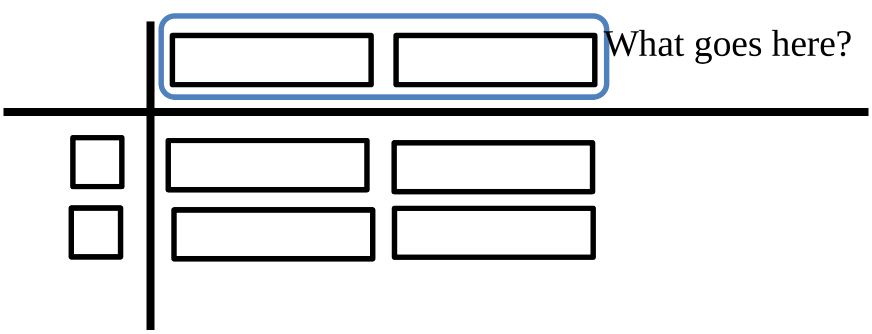
OR



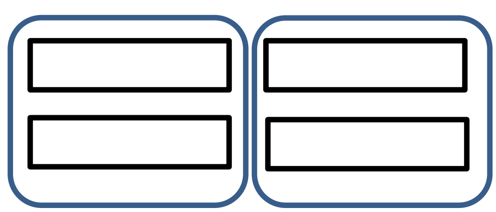
Two in each group

4. Example 3: Using the two methods of arrays and grouping, show how division of polynomials can be modelled.

Method #1



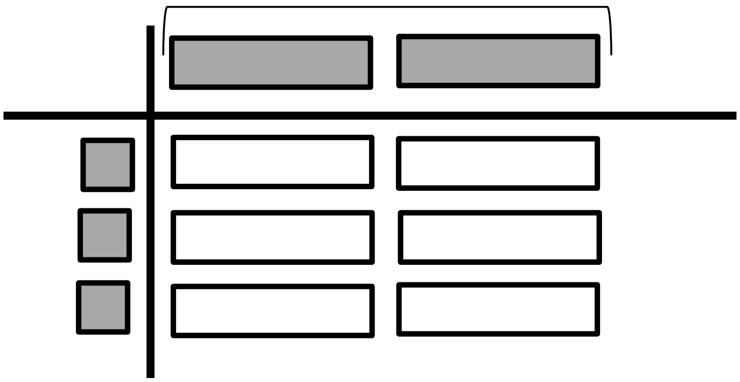
Method #2



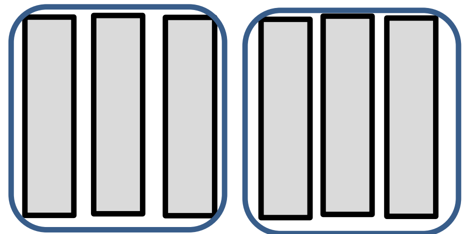
$2x$ in each group

5. Example 4: what happens when we divide by a negative number? Using the methods of arrays and grouping, show how division of polynomials can be modelled. $\frac{6x}{-3}$

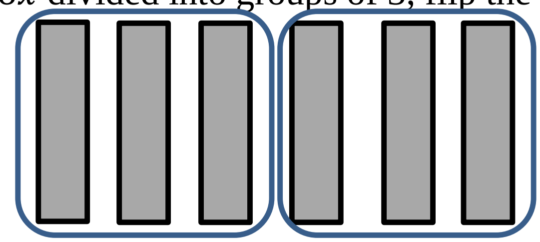
Method #1: answer is $-2x$



Method #2:

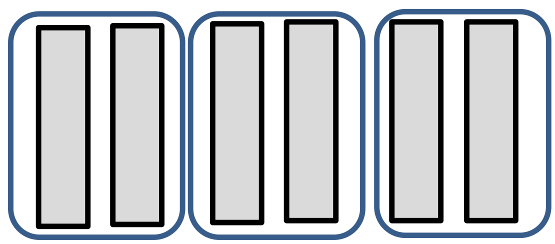


$6x$ divided into groups of 3, flip the tiles

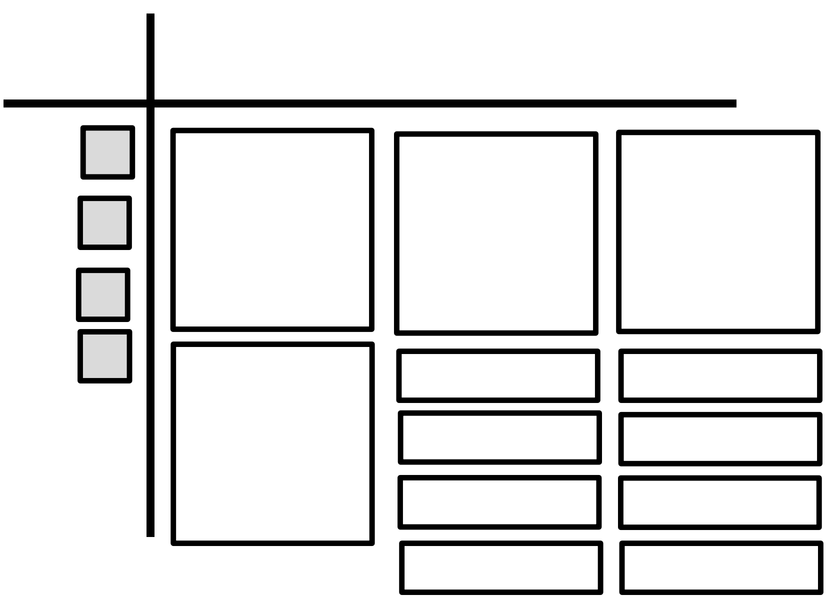


There are 2 groups of x, the answer is $-2x$

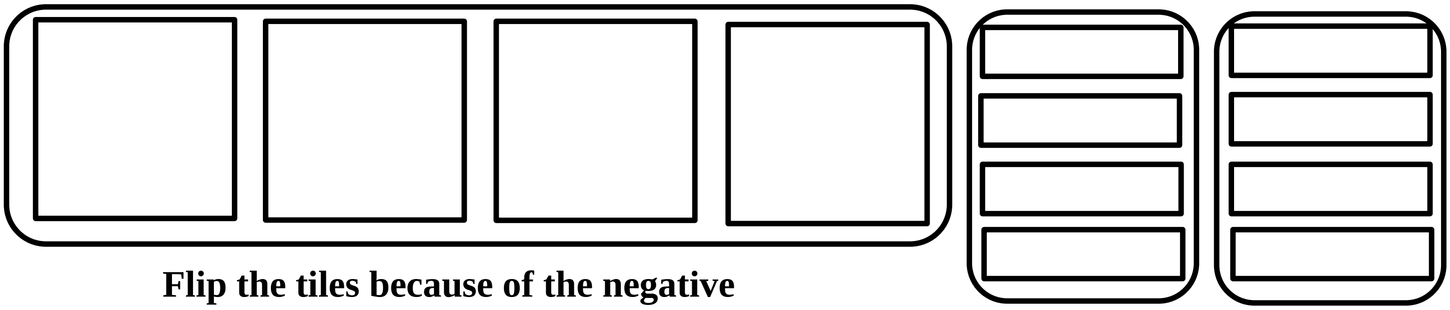
OR



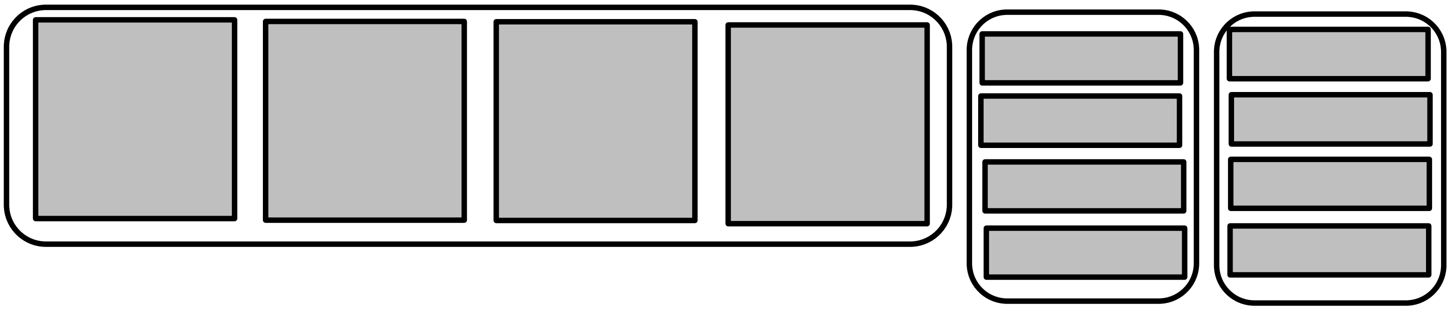
6. Example 5: this example investigates what happens when we divide a constant into a polynomial with a degree larger than 1. Students will see that the array model stops working at this point.
   Method #1



Method #2: (Quotitive) Divide the polynomial into groups of 4



Flip the tiles because of the negative



We have one group of $-x^2$ and two groups of $-x$, the answer is $-x^2 - 2x$

Method #2: (Partitive) divide them up into 4 equal groups. Each group would have one $-x^2$ and two $-x$'s

7. Example 6: review the exponent rule for dividing using integers.
8. Example 7: using the exponent rule for dividing, extend that knowledge to polynomial division.
9. Ask students how we can divide polynomials without algebra tiles. They should understand that they can subtract exponents. However, they will need to create mini-fractions.
10. Example 8: using student's new knowledge, answer the previous examples again, but this time without algebra tiles.
11. Example 9-11: using the array model, as well as mini-fractions and exponent rules, work with students to complete the examples.
12. Example 12: answer this example without using algebra tiles.
13. Whiteboard activity **RN #5**
14. Assignment: Lesson 7

Dividing Polynomials

A. What is Division?

Example 1: Divide the following integers

a. $(10) \div (5)$ b. $(-10) \div (5)$ c. $(10) \div (-5)$ d. $(-10) \div (-5)$

Example 2: Model the following division $10 \div 5$

B. Dividing a Polynomial by a Constant

How does division of integers help us with multiplication of polynomials?

Example 3: Model the following multiplication using algebra tiles. $(4x) \div (2)$

Method #1

Method #2

What happens when we divide with negative numbers?

Example 4: Model the following multiplication using algebra tiles. $\frac{6x}{-3}$

Method #1

Method #2

What happens when we try to divide a constant into a polynomial with a degree bigger than 1?

Example 5: $\frac{-4x^2+8x}{-4}$

Method #1

Method #2

C. Dividing a Polynomial by a Monomial

Let's review one of our exponent rules.

Example 6: Divide $\frac{5^3}{5^2}$

Example 7: Divide $\frac{x^3}{x^2}$

How do we do the work without using algebra tiles?

Example 8: Let's answer all of the previous examples using mini fractions.

a. $(4x) \div (2)$

b. $\frac{6x^2}{-3x}$

c. $\frac{-4x^2+8x}{-4}$

Example 9: Model and simplify symbolically $\frac{6x^2}{2x}$

Method #1

Method #2 – Mini Fractions and Exp. Rules

Example 10: Model and simplify symbolically $\frac{3g^2+9g}{3g}$

Method #1

Method #2 – Mini Fractions and Exp. Rules

Example 11: Model and simplify symbolically $\frac{32c^2-48c}{-4c}$

Method #1

Method #2 – Mini Fractions and Exp. Rules

Example 12: Divide $(16k^{11} - 32k^{10} + 8k^8 - 40k^4) \div (8k^8)$

Lesson 7: Whiteboard Activity **P9.4**

1. $\frac{60x}{-10}$
2. $\frac{12x}{4x}$
3. $\frac{-36x^2}{-12x}$
4. $\frac{-25n^2}{5n}$
5. $\frac{18u^2}{-6u^2}$
6. $\frac{-6x^2}{6x^2}$
7. $\frac{10x^2+6x}{2x}$
8. $\frac{5c^2-2c}{c}$
9. $\frac{24g^2+48g}{-4g}$
10. $\frac{24t^2-12t}{12t}$
11. $(7x^3 - 14x^2 - 7x) \div (-7x)$

Lesson 7: Assignment **P9.4**

1. Divide

a. $\frac{14x}{7}$ b. $\frac{-14x}{7}$ c. $\frac{-14x^2}{-7}$ d. $\frac{14x^2}{-7}$

e. $\frac{15x-6}{3}$ f. $\frac{36-16x}{-4}$ g. $\frac{-12x^2-8x}{2}$ h. $\frac{5x^2-10x}{-5}$

2. Divide

a. $\frac{6s^2}{-2s}$ b. . $\frac{-14x}{7x}$ c. $\frac{-14x^2}{-7x}$ d. $\frac{14x^2}{7x^2}$

e. $\frac{12m^2-6s+8m}{2}$ f. $\frac{15x^2-10x}{5x}$ g. $\frac{14v^2-21v}{7v}$ h. $\frac{-8a^2+8a}{-8a}$

i. $\frac{16f-f^2}{-f}$ j. $\frac{x^2-x}{x}$ k. $\frac{-12x^2-4x}{2x}$ l. $\frac{x^2}{-x^2}$

Research Notes

RN #1: Math-in-Context

What the Research Says:

Learning math-in-context means that students are working through a problem that they can connect to. It is a long held belief that if students can connect to the mathematics they are learning, they will be more engaged and understand the content better. The goal is an admirable one. When students see the connection between mathematics and the outside world, they see how the math works concretely rather than abstractly. As a result, students start to realize that you don't need to memorize math and it can be applied to their daily lives. You may have also heard the term "real-world" math to describe this effort. Unfortunately, there has not been overwhelming success when trying to achieve this goal. According to Boaler (1993) the effort to instill more context into mathematics curricula has primarily involved the "random insertion of contexts into assessment questions and classroom examples in an attempt to reflect real life demands and to make mathematics more motivating and interesting" (p. 13).

Often, math-in-context and real-world math are viewed as the same thing. I argue that they are not. Real-world math is the idea of linking mathematical content to situations outside of the classroom. This is very difficult to do well and as a result, has many downfalls. Often, the attempts to implement real-world questions lead to questions involving pseudo-context. Boaler (2008) calls this learning without reality, where students work on problems that "involve trains travelling toward each other on the same track, people paint houses at identical speeds all day long, water fills tubs at the same rate each minute, and people run around tracks at the same distance from the edge" (p. 51). She believes that in order for students to be successful at these questions, "they need to leave their common sense at the door" (Boaler, 2008, p. 51). Boaler (2008) argues that "if the ridiculous contexts were taken out of math books (traditional or reform) they would fall in size by more than half" (p. 55). This is where math-in-context differs. Math-in-context takes into account student's sense making ability and challenges them with questions that try to help students move from the abstractness nature of mathematics to a more concrete understanding.

When coming up with questions or activities for students that will put math-in-context, it is important to note that students do not perceive a school mathematics task as real, useful, or interesting just because we say it is. Wiliam (1988) states that a good problem has two crucial features:

1. Intent: A problem is not a problem if you don't want to solve it.
2. Obstacle: If you know what to do, then there is no problem.

What I Want to Say:

I may know what you are thinking, “I believe you, but rather than spewing theory at me, give me examples of what this looks like in the classroom!” and I don’t disagree. When reading through the literature, it can be frustrating when no examples of what it looks like in the classroom are given. When examples are included, they have often taken topics in the curriculum that are most easily put into context such as statistics. What about solving equations, polynomials, or dividing fractions? The literature tells us to put the math in context when their research isn’t put into context.

Taking a step past the initial gut reaction of “sure, but…” there are some important messages to take from this when considering problems you are presenting to students. For example, when a high school student is asked to create a budget or interpret a pay stub, those activities may be more of a barrier than the mathematics behind it. It is important to realize that for some students, they may make a connection because they have job. For others, it becomes a make-believe world where they don’t connect to the activity at all. As a result, students are so far removed from the situation that it becomes as meaningless to them as calculating the slope of a line. How often have I heard, “I will just pay someone to do my taxes”, or “my dad will do my budget”, or “there’s an app for that”.

Don’t get discouraged. You can’t please everyone all of the time. Students interact with the context of a task in many different and unexpected ways and this interaction is individual (Boaler, 1993). The best thing you can do is keep mathematics at their fingertips. It doesn’t always need to be a big idea, or one that students will immediately see and use outside the classroom. Gravenmeijer & Doorman (1999) believe that moving from “real-world” mathematics to “math in context” is seen as a “shift from a model of situated activity to a model for mathematical reasoning” (p. 111). It can be as simple as seeing fractions through a bag of skittles or by reading a book that introduces how quickly exponents grow. You also don’t need to put every single mathematics lesson into context for students. It would be a daunting and an almost impossible task to do. Do the best you can. Hopefully some of the ideas I have included will spark more ideas for your classroom.

Must Reads:

Boaler, J. (1993). The role of contexts in the mathematics classroom: Do they make mathematics more “real”? *For the Learning of Mathematics*, *13*, 12-17.

Boaler, J. (2008). What’s math got to do with it? Helping children learn to love their most hated subject: And why it is important for America. New York, NY: Viking.

References:

Boaler, J. (1993). The role of contexts in the mathematics classroom: Do they make mathematics more “real”? *For the Learning of Mathematics*, *13*, 12-17.

Boaler, J. (2008). What’s math got to do with it? Helping children learn to love their most hated subject: And why it is important for America. New York, NY: Viking.

Gravemeijer, K. Doorman, M. (1999). Context problems in realistic mathematics education: A calculus course as an example. *Educational Studies in Mathematics, 39*, 111-129.

Wiliam, D. (1988). *Open ends and open beginnings.* Retrieved from Dylan Wiliam’s website: http://www.dylanwiliam.org/Dylan_Wiliams_website/Papers.html

RN #2: Prior Knowledge

What the Research Says:

Accessing and assessing prior knowledge of students is integral when planning a lesson. Many concepts in the grade 9 curricula are part of the grade 8 and grade 7 curriculums. As a result, most students "should" enter the grade 9 classroom with prior knowledge of many of the concepts. But what is prior knowledge? Can we assume that just because students have had experience with the concepts, that they understand them and remember them in grade 9? Prior knowledge is the knowledge that students have learned prior to the lesson that they hold in their long term memory. Resnick (1983) states, "all learning depends on prior knowledge. Learners try to link new information to what they already know in order to interpret the new material in terms of an established schemata" (p. 478). But what happens if what they know is conceptually wrong, or they have gaps in what they remember? Myhill & Brackley (2004) assert that "asking questions which elicit children's prior knowledge can provide valuable information about children's current level of understanding, or even misconceptions that they may be harbouring" (p. 272). By doing this, the teacher can address any gaps or misconceptions the learners have with the content. How does one go about accessing student's prior knowledge in a purposeful way? Researchers argue that accessing prior knowledge goes beyond pre-tests and start-up questions, there needs to be active discourse between the students, and between students and teacher (Myhill & Brackley, 2004). Brainstorming and using a KWL chart (Know-Want to know-Learned) are a couple of techniques that can be used for accessing prior knowledge. Some of the techniques used to access prior knowledge can also be used as formative assessment strategies as well.

Brainstorming is a strategy that most teachers are familiar with. The teacher gives the students a word or topic and students provide as much information as they know about it. For example, you could put the word 'equation' in the centre of the board and have students create a concept map in print or electronically with all of the key words, phrases, aspects, and characteristics that they can remember.

KWL chart was first developed by Ogle (1986) as a reading strategy but it is easily adaptable to mathematics. It is a 3-column chart with the headings: know, want to know, and learned. Before the lesson starts, students identify what they know about the topic and what they want to know. After the lesson is complete, students fill in the last column with what was learned in the lesson.

Students' understanding builds and accumulates upon their prior knowledge. By accessing and assessing students' knowledge before a topic is taught, teachers will be able to tailor the lessons to better meet the students' needs.

What I Want To Say:

Although prior knowledge doesn't make up a large part of the research in the teaching and learning of mathematics, I felt it was important to include in my project. If you were to read through the Grade 9 Saskatchewan curriculum, there isn't a tremendous amount of topics. Rational numbers should only include order of operations (rather than all operations) and solving equations should begin at variables on both sides of the equation for example (rather than starting over at solving simple equations). However, in the past I have found that students don't always have a strong understanding or have a fragmented memory of the topics.

Accessing prior knowledge can be as simple as administering a pre-test or a few start-up questions before the lesson. From there, it is important to discuss with students their background on the topic and what they remember. The aforementioned brainstorming activity (concept map) and KWL chart are just a couple of starter ideas that can spark discussion. I have included a website in the "must read" section below that provides more strategies for accessing prior knowledge.

Must Reads:

http://www.classhelp.info/Biology/Strategies%20for%20Activating%20Prior%20Knowledge.pdf

References:

Myhill, D. & Brackley, M. (2004). Making connections: Teachers' use of children's prior knowledge in whole class discourse. *British Journal of Educational Studies*, *52*, 263-275.

Ogle, D. M. (1986). K-W-L: A teaching model that develops active reading of expository text. *Reading Teacher*, 39, 564-570.

Resnick, L. B. (1983). Mathematics and science learning: A new conception. *Science*, 220, 477-478.

RN #3: Learner Generated Examples

What the Research Says:

Examples play a large role in most mathematics classroom. It is hard to imagine a mathematics classroom without them. Teachers rely on examples for introducing new concepts, practicing a concept, and giving students the opportunity to try questions on their own. Typically, in many mathematics classrooms, the examples are often supplied by the teacher or resource. Learner generated examples (LGE) are just that, examples created by the learners. This follows a student-centered approach and requires the learner to explore and make connections between their prior knowledge and the new knowledge they have just acquired. LGEs require students to reach deeper into their understanding of a topic rather than just mimic the rules and procedures given to them by the teacher.

Watson and Mason (2005) have written one of the most current and applicable books on learner generated examples in mathematics. It is written for teachers, and as a result, there is more focus placed on the practical than the theory. Not only does it include prompts for teachers to implement learner generated examples, it discusses them in the context of real classroom situations. Watson and Mason (2005) state that learner generated examples effectively enable students to construct a deeper meaning or extend on their meaning of mathematical content. They feel that LGEs is "an entirely natural but underused strategy in mathematics lessons and that learning is greatly enhanced when learners are stimulated to create their own examples" (Watson and Mason, 2005, p. 32).

What I Want to Say:

I wasn't introduced to LGEs until I was well into my project. I think that they are an uncovered gem in mathematics teaching. They have a low entry point, so teachers can really incorporate them in almost every lesson. They can also become more elaborate as teachers become more comfortable and familiar with them. For example, in the polynomial unit, having students create their own polynomial given certain parameters gives them more understanding than just answering a chart where they need to state the degree, coefficient, etc. A more elaborate LGE is having students create their own equations. Can you imagine the deeper understanding students would develop if we asked them to create their own factorable trinomial? The discussion on how students created their trinomial would be fascinating. It would also reveal students who did not understand how factors are related to the final product of the trinomial.

Brian Crawley wrote his final masters project devoted to mathematics lessons using LGEs. I strongly suggest that if you are interested in knowing more about LGEs to acquire his project from the University of Saskatchewan. In appendix F, he supplies actual mathematics LGE lesson ideas within the context of Saskatchewan curriculum.

I have included a link to Watson and Mason's (2005) book. It is a fantastic read and provides some clarity on how LGEs look and feel in the classroom.

Must Reads:

Watson, A., & Mason, J. (2005). *Mathematics as a Constructive Activity: Learners Generating Examples.* Mahwah, NJ: Lawrence Erlbaum Associates, Inc. http://eduir.files.wordpress.com/2009/02/mathematics_as_a_constructive_activity__learners_generating_examples.pdf

Crawley, B. (2010). *Mathematics Lessons Using Learner-Generated Examples.* Unpublished manuscript, Department of Curriculum Studies, University of Saskatchewan, Saskatoon, Canada.

References:

Dahlberg, R. P. & Housman, D. L. (1997). Facilitating learning events through example generation. *Educational Studies in Mathematics, 33*, 283-299.

Watson, A., & Mason, J. (2005). *Mathematics as a Constructive Activity: Learners Generating Examples.* Mahwah, NJ: Lawrence Erlbaum Associates, Inc. http://eduir.files.wordpress.com/2009/02/mathematics_as_a_constructive_activity__learners_generating_examples.pdf

RN #4: Inquiry

What the Research Says:

If you are an educator in Saskatchewan, you have probably heard the word 'inquiry' a lot. It is included in each of the Saskatchewan mathematics curricula from grades 8-12 as a stand-alone section. The Saskatchewan Curriculum (2009) defines inquiry based learning as a philosophical approach which provides students the "opportunities to build knowledge, abilities, and inquiring habits of mind that lead to deeper understanding of their world and human experience" (p. 23). In order for this to be achieved, "well-formulated inquiry questions are broad in scope and rich in possibilities. They encourage students to explore, gather information, plan, analyze, interpret, synthesize, problem solve, take risks, create, conclude, document, reflect on learning, and develop new questions for further inquiry" (p. 24). Inquiry is not new, and the research on inquiry based learning is immense. Some of the first major contributors were John Dewey, Jean Piaget, Lev Vygotsky, and Jerome Bruner. Inquiry based learning falls under the umbrella of constructivism which believes that students need to construct their own knowledge of mathematics. Using an inquiry approach in mathematics helps students construct their own understanding of mathematical principles by being participants rather than recipients.

As with many initiatives in the province and in school divisions, teachers are inundated with theory and given very little on the side of practical. Because of this, teachers are often put off by the idea of inquiry and the perception of what it looks like in the classroom which then creates an "inquiry or bust" attitude. Banchi & Bell (2008) recognized that many teachers were not giving inquiry a chance in their classroom because of the idea that true inquiry was unattainable. As a result, they created four levels of inquiry; confirmation inquiry, structured inquiry, guided inquiry, and open inquiry. Here is a brief synopsis of each:

1) Confirmation Inquiry: In this level of inquiry, the question, procedure, and answer is known in advance. Students go through and prove why it is so. For example, having students model the operations of fractions pictorially would fit into this category.
2) Structured Inquiry: In this level of inquiry, the question, procedure, is known to students, but the answer is not. Many tasks in my project fall under this category. For example, when students do the paper folding exercise in the exponent section to understand the fast action of exponents. Another example in the exponent section is when the students work through the operations with exponents to generate their own algorithms.
3) Guided Inquiry: In this level of inquiry, the teacher provides the students with the question and students design the procedure and the resulting answer. This is often more successful when students have had multiple chances to work with structured inquiry questions.

4) Open Inquiry: In this level of inquiry, the students are to provide the question, procedure, and answer. This level of reasoning is the most difficult and is only attainable after students have had multiple opportunities to work through the first three levels of inquiry. In this project, I do not have any open inquiry tasks.

Inquiry Level	Question	Procedure	Solution
1. Confirmation Inquiry Students confirm a principle through an activity when the results are known in advance	✓	✓	✓
2. Structured Inquiry Students investigate a teacher presented question through a prescribed procedure	✓	✓	✗
3. Guided Inquiry Students investigate a teacher presented question using student designed/selected procedures	✓	✗	✗
4. Open Inquiry Students investigate questions that are student formulated through student designed/selected procedures	✗	✗	✗

(Banchi and Bell, 2008, p. 27)

Many teachers have been introduced to inquiry as 'open inquiry'. As a result, they discounted the approach because open inquiry in the mathematics classroom is very difficult to implement. Blair (2008) stated that full open inquiry in the mathematics classroom is incompatible with provincial or national curriculum. Teachers are required by law to stay within the guidelines of the curriculum. How are students supposed to have curiosities within the context of a document they don't even know? Banchi and Blair (2008) have created opportunities for teachers to include inquiry based learning in their classrooms by giving teachers permission to use more structure.

What I Want To Say:

When I first started creating and researching for this project, I struggled finding tasks that were considered true 'inquiry'. As much as I wanted to incorporate inquiry into my project, I felt that maybe it was too big of an ideal to work within the parameters of my project. Once I read Banchi and Blair's (2008) article, I felt that it gave me permission to assign the label of inquiry to tasks that didn't fit into my previous belief of inquiry. They look at the levels of inquiry through the lens of science, but much of what they discuss can be paralleled to mathematics. Unfortunately, many teachers are introduced to inquiry in the form of open inquiry with little to no practical examples of classroom tasks. As a result, teachers feel that inquiry can only work in a fairy tale classroom. One with a manageable number of students, plenty of resources, internet that works, inquisitive and motivated students, and an open curriculum. My hope is that by seeing some of the tasks that can be considered inquiry, it will give you more freedom to apply more inquiry based activities in your classroom without fear of 'doing it wrong'.

Must Reads:

Banchi, H. & Bell, R. (2008). The many levels if inquiry. *Science and Children. 46*, 26-29.

Blair, A. (2008). Inquiry teaching. *Mathematics Teaching Incorporating Micromath. 211*, 8-11.

References:

Banchi, H. & Bell, R. (2008). The many levels of inquiry. *Science and Children. 46*, 26-29.

Blair, A. (2008). Inquiry teaching. *Mathematics Teaching Incorporating Micromath. 211*, 8-11.

Ministry of Education (2009). Mathematics 9 [Curriculum Guide]. Regina: Ministry of Education. Retrieved from https://www.edonline.sk.ca/bbcswebdav/library/Curricula/English/Mathematics/Mathematics_9_2009.pdf

RN #5: Formative Assessment

Assessment plays a big role in mathematics classrooms. There are two main purposes of assessment, summative and formative. You may have heard of them under the labels of assessment *of* learning and assessment *for* learning respectfully.

1) Summative assessment (assessment of learning) is done at the end of a unit or course to record overall achievement in comparison to standards goals and criteria. It is regarded as a final destination point and "encapsulates all the evidence up to a given point" (Taras, 2005, p. 468). Summative assessments are designed to judge the extent of students' learning of the material in a course and are then usually assigned a grade.
2) Formative assessment (assessment for learning) is similar to summative assessment in that it makes a judgement according to standards, goals and criteria. Where it differs is that it isn't done with high stakes. "Formative assessment refers to frequent, interactive assessments of student progress and understanding to identify learning needs and adjust teaching appropriately" (Assessment for learning, 2008, p. 1). It serves two purposes; first, it gives the learner(s) immediate feedback on how they are doing and what they need to improve on, secondly, it provides the teacher feedback so they can adjust their lesson(s) appropriately.

Chances are if you are a teacher in the Saskatoon Public School Division, you have heard a lot about formative assessment and Dylan Wiliam. Black and Wiliam (1998) conducted a review of 250 research studies on the effectiveness of formative assessment. What they found was that increasing the use of formative assessment in classrooms produces significant improvements in students' learning. More importantly, good formative assessment has the "power to change the distribution of attainment" (Wiliam, 1999, p. 1). What that means is that while formative assessment improves all students learning, it seems to be even more beneficial for low achievers.

Wiliam and Thompson have laid out five key strategies for which to implement effective formative assessment:

1. Clarifying and sharing learning intentions and criteria for success;
2. Engineering effective classroom discussions and other learning tasks that elicit evidence of student understanding;
3. Providing feedback that moves learners forward;
4. Activating students as instructional resources for one another; and
5. Activating students as the owners of their own learning.

Moving past the theory and into the practical, Wiliam (2011) provides a plethora of ideas for putting formative assessment into practice. Some of his suggestions are hot-seat questioning, exit passes, mini whiteboards, peer evaluation of homework, end-of-topic questions, error classification, traffic lights, and red/green disks. These are just some of his ideas that he goes into detail in his book, Embedded Formative Assessment.

What I Want To Say:

How you decide to implement effective formative assessment in your classroom is up to you and how comfortable you feel with each strategy. Taking the entire grade 9 curriculum and looking at it through the lens of formative assessment is a task on its own. What I have done is include a few ideas that fall under the second key strategy and embed them in the grade 9 course. The two main methods I use fall under the title of all student response systems. They are mini-whiteboards and exit/entrance questions. Some of the other topics I have discussed and used in my project, such as discourse and cooperative work, also fall under the umbrella of formative assessment. Here is a little bit of background on the methods I have chosen for this project.

Mini Whiteboards: Wiliam has claimed that “mini whiteboards are the greatest development in education since the slate. The reason being is that “you can get an overall view of what the whole class thinks” (Gerard, 2010). Before I had white boards supplied by my school, I used white paper inside plastic sheet covers. The whiteboards I use now in my classroom are blank on one side and have a grid on the other. I implement them at the end of a topic or lesson to see if the students have understood the content they have just learned. The wait time is dependent on the question but I do time them using an online stopwatch. If there are a lot of students who answer incorrectly, or leave the whiteboard blank, I revisit the topic again. If students seem to have all of the correct answers, I assume that they have understood and I move on.

Entrance/Exit Question: Entrance and exit questions are given to students at the beginning or the end of class. They are typically one or two questions based on the lesson students have just learned. They give students instant feedback as to whether they are understanding the concept or if they need to do more work. They should not be included in the students overall grade, but rather used to tell the student and teacher if more work is needed on a concept.

Must Reads:

Wiliam, D. (2011). *Embedded formative assessment.* Bloomington, IN: Solution Tree Press

Wiliam, D. (2007). *Five “key strategies” for effective formative assessment*. Retrieved from the National Council of Teachers of Mathematics, Research Brief website: *http://www.nctm.org/uploadedFiles/Research_News_and_Advocacy/Research/Clips_and_Briefs/Research_brief_04_-_Five_Key%20Strategies.pdf*

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Black, P. & Wiliam, D. (1998). Assessment and classroom learning. *Assessment in Education, 5*, 7-74.

Gerard, G. (2010). The six secrets of a happy classroom [Online article]. Retrieved from http://www.independent.co.uk/news/education/schools/the-six-secrets-of-a-happy-classroom-2086855.html

Taras, M. (2005). Summative and formative: Some theoretical reflections. *British Journal of Educational Studies, 53,* 466-478.

Wiliam, D. (2011). *Embedded formative assessment.* Bloomington, IN: Solution Tree Press

Wiliam, D. (1999). *Formative assessment in mathematics, part I: rich questioning*. Retrieved from: http://eprints.ioe.ac.uk/1150/1/Wiliam1999Formativepart115.pdf

RN #6: Representation

Brahier (2009) defines representation as the "mathematical process by which students take a given problem and attempt to model it in a useful way that will enable them to solve a problem" (p. 25). Representation is an integral part of the teaching and learning of mathematics. The NCTM names representation as one of their 10 standards for school mathematics. In their definition of representation they feel that all mathematical programs from prekindergarten through grade 12 should enable students to: (1) create and use representations to organize, record, and communicate mathematical ideas; (2) select, apply, and translate among mathematical representations to solve problems; and (3) use representations to model and interpret physical, social, and mathematical phenomena (p. 67). Representation can be internal, in the minds of the person doing the mathematics, or externally, in the form of a picture drawn on a page (Pape & Tchoshanov, 2001). Although representation has always been part of the teaching and learning of mathematics, it hasn't been utilized as much as it should be.

In order for it to be successful, Pape and Tchoshanov (2001) state that representation must be embedded in the culture of the classroom rather than being a stand-alone activity, students must be given the opportunity to work in pairs or groups, and representations must be viewed as tools to explain an end product, rather than the being the product of an end task. Representations can be concrete (eg. algebra tiles), pictorial (eg. a drawing), or symbolic (eg. $5x + 2 = 12$). This is based on Jerome Bruner's theory of learning from the 1960s. Bruner's theory has led to the use of manipulatives in mathematics classrooms (Brahier, 2009). The two main processes of representation in my project are manipulatives and pictorial representation. I will discuss each briefly.

Pictorial Representation (Visualization)

Representation is well-represented (pun intended) in the Saskatchewan curriculum guides from grades 1 through 12. Pictorial representation refers to the pictures drawn by students to help clarify, enhance, or describe a situation. It is an important skill for students to refine and use throughout their mathematical career and into adulthood. Whether it is representing $\frac{1}{4}$ pictorially, drawing a scale diagram, or using squares to discover Pythagorean's theorem, many aspects of the grade 9 curriculum are able to incorporate pictorial representation.

Manipulatives

Manipulatives are "concrete tools used to create external representation of a mathematical idea" (Puchnar, Taylor, O'Donnell, & Fick, 2008, p. 314). There is ample research on the effective use of manipulatives in the classroom. Although many researchers advocate for the use of manipulatives, they advise that they must be used with caution (Ball, 1992, Clements, 1999, Puchnar et al., 2008, Uttal et al., 1997). There is a well-known quote by

Ball (1992), “although kinesthetic experiences can enhance perception and thinking, understanding does not travel through the fingertips and up the arm…mathematical ideas really do not reside in cardboard and plastic materials” (p. 47). What she is saying is that teachers cannot assume that students are attaining a deeper understanding by just using manipulatives. Many researchers have found that students often use manipulatives in a rote manner and see them as a means to an end (Clements, 1999, Puchnar et al., 2008, Pape & Tchoshanov, 2001). What they are failing to understand is how the manipulatives are actually linked to the mathematics. Uttal et al. (1997) found that “children focus more on the manipulatives as objects rather than on the relation of the objects to a concept” (p. 48). Often, we as teachers can see the relationship between the manipulative and the concept quite clearly. However, students don’t often see that relationship we intend for them to see, and we as teachers need to take more time to work with students so they can make connections between the manipulative and the concept. When students use manipulatives correctly, it can bring abstract symbols to life. However, “using manipulatives without ensuring that students fully understand their relation to the mathematical concepts being taught might be counter-productive…the time and effort spent mastering the manipulatives would be time and effort not devoted to learning the concepts in the first place.” (Uttal et al., 1997, p. 48).

What I Want To Say:

I firmly believe that representation is often a missing part and under-utilized area of mathematics in grades nine to twelve. I agree that there are times when it isn’t necessary or the representation is naturally part of the process (sketching graphs). However, there are other times when it can be incorporated to aid or enhance student’s understanding of mathematical concepts. For example, having students pictorially model the operations on fractions may help them understand why they need to find a common denominator when adding or subtracting. Also, implementing algebra tiles when discussing like terms may help students understand why x^2 can’t be added to x. I strongly suggest that teachers must be comfortable and practice the representation before implementing it into the classroom. In this way, teachers can help students attain an understanding of how the manipulative or representation is tied to the concept.

Must Reads:

Ball, D. L. (1992). Magical hopes: Manipulatives and the reform of mathematics education. *American Educator, 16*, 14-18, 46-47.

Clements, D. H. (1999). ‘Concrete’ manipulatives, concrete ideas. *Contemporary Issues in Early Childhood, 1*, 45-60.

Uttal, D. H., Scudder, K. V., Deloache, S. J. (1997). Maniupulatives as symbols: A new perspective on the use of concrete objects to teach mathematics. *Journal of Applied Developmental Psychology, 18,* 37-54.

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Clements, D. H. (1999). 'Concrete' manipulatives, concrete ideas. *Contemporary Issues in Early Childhood, 1*, 45-60.

Moyer, P. & Jones, M. G. (2004). Controlling choice: Teachers, students, and manipulatives in mathematics classrooms. *School Science and Mathematics*, *104,* 16-31

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA.

Puchner, L., Taylor, A., O'Donnell, B., & Fick, K. (2008). Teacher learning and mathematics manipulatives: A collective case study about teacher use of manipulatives in elementary and middle school mathematics lessons. *School Science and Mathematics, 108*, 313-325.

Thompson, P.W., & Lambdin, D. (1994). Concrete materials and teaching for mathematical understanding. *The Arithmetic Teacher*, *41*, 556-558.

Uttal, D. H., Scudder, K. V., Deloache, S. J. (1997). Maniupulatives as symbols: A new perspective on the use of concrete objects to teach mathematics. *Journal of Applied Developmental Psychology*, *18,* 37-54.

RN #7: Cooperative Learning

What the Research Says:

Cooperative learning is "understood to be learning that takes place in an environment where students in small groups share ideas and work collaboratively to complete academic tasks" (Davidson & Kroll, 1991, p. 362). Much of the research done on cooperative learning dates back to the 1980s and 1990s. Unfortunately, the theory behind cooperative learning did not find its way into practice in mathematics classrooms. Although cooperative learning has had a home in other subjects such as English and the sciences, high school mathematics has lagged behind. That is not to say that cooperative learning hasn't existed at all in mathematics classrooms, it is just usually in the form of students working through an assignment in pairs. Now that teachers are rethinking the way they have been teaching mathematics, cooperative learning has made a comeback.

Wiliam (2011) stated four reasons why cooperative learning works so well:

1. Motivation. Students help their peers learn because, in well-structured cooperative learning settings, it is in their own interests to do so, and so effort is increased.
2. Social Cohesion. Students help their peers because they care about the group, again leading to increased effort.
3. Personalization. Students learn more because their more able peers can engage with the particular difficulties a student is having
4. Cognitive elaboration. Those who provide help in group settings are forced to think through the ideas more clearly (p. 133).

Wiliam (2011) finds that there are key aspects of incorporating productive and successful cooperative learning in mathematics classrooms. First, all students need to participate, and second, the group task needs to be more elaborate than just working on a homework assignment (although it is still a useful tool to use in the classroom).

Often a teacher will go through an elaborate process to make sure the groups are created as fairly as possible, and there always seems to be one or two students who are not participating. Wiliam (2011) feels strongly that cooperative learning will only work if there are group goals and individual accountability. Brahier (2009) states that although the activity is done as a team, each student must have individual accountability in the form of a written journal, interview, or test. One method that Wiliam (2011) mentions is called 'secret student'. The idea is that one student is selected at random, but the identity of the secret student is hidden from the class. If at the end of the class the student has pulled their weight, then the entire class is rewarded. If the student has not pulled their weight than the entire class does not receive the award. Another form of this is to let students know that a random student from each group will be responsible for sharing their conclusion and that there is a no opting out policy. Cohen (1994) proposed that students be “trained for cooperation” (p. 26). Because the behaviour from students in a conventional classroom setting is completely different from the behaviour required in a small group, she recommends team-building activities before cooperative learning.

Choosing the right task can be tricky. Typically finding a task that has a low entry point for students to engage with but also a high ceiling for extensions if needed is important. The more you practice implementing tasks and cooperative learning, the more at home you will feel with the strategy.

Brahier (2009) lists 5 practical tips for implementing cooperative learning strategies.

1. Teachers should select the groups. Most often a group of 4 (1 high achiever, 2 average students, and 1 low achiever). Other times you may want to try similar ability groupings. Mixed gender is important as well.
2. Once learning teams are established, they should stay together for 4-6 weeks. Change the teams periodically because students need to work with others and may grow tired of working with the same students.
3. The process of changing to a cooperative environment can be difficult for some students and should be gradual. Give them time.
4. Regular practice will help you (the teacher) feel more comfortable.
5. Be prepared for chaos and disorder when you first introduce cooperative learning. Remember your role is to be a guide, not a dispenser of knowledge. (p. 200).

What I Want To Say:

I will admit, before this project I felt that having students work on their textbook assignment in small groups was cooperative learning (I still sort of feel they are learning cooperatively). But I see why the researchers are pushing us to go a little further with our implementation of cooperative learning in our classrooms. I have done some basic implementation of cooperative learning in this project where students are working together through tasks. By calling them student learning groups, hopefully students will understand that they are doing more than just sitting in a circle working on the same task. As well, having discussions with them about collaboration and on my expectations of how they should be working in small groups is important. Students should know that they are still individually accountable even though they are working in a group.

As for the creation of deep meaningful tasks that have low entry points and high ceilings, I have done my best, but I know that there is room for improvement. I hope that by giving you some tasks to implement in your classroom that cooperative learning will become (if it isn’t already) a more utilized strategy in your mathematics class.

References:

Brahier, D. J. (2009). *Teaching secondary and middle school mathematics*. Boston, MA: Pearson.

Davidson, N. & Kroll, D. L. (1991). An overview of research on cooperative learning related to mathematics. *Journal for Research in Mathematics Education*, *22*, 362-365.

Wiliam, D. (2011). *Embedded formative assessment.* Bloomington, IN: Solution Tree Press.

RN #8: Discourse in the Mathematics Classroom

What the Research Says:

Communication is an integral part of the mathematics classroom. So much so, the National Council of Teachers of Mathematics recognizes communication as one of their standards. “Through communication, ideas become objects of reflection, refinement, discussion, and amendment (NCTM 2000, p. 60). Discourse is a word often used to describe the “process through which groups of individuals communicate” (Sherin, 2002, p. 206). In an age where we are trying to transform our classrooms into a more inquiry-based, student focused environment, the first plan of action should be exploring and manipulating our classroom discourse model.

Wertsch (1991) defined two types of discourse: univocal and dialogic. Univocal is the type of discourse most often found in mathematics classrooms. This is where the questioning and feedback from the teacher is used to convey information to the students. When students respond to the teacher, they are usually answering a question that the teacher already knows the answer to. The teacher will acknowledge the students answer and either agree or elaborate further. The order of communication is usually teacher – student – teacher. In contrast, dialogic discourse involves give-and-take communication in which students actively construct meaning (Truxaw & DeFranco, 2007). Initially, the discourse may start out univocally, but when it moves past generating the correct answer, it becomes more dialogic in nature. Knuth & Peressini (2001) state the model of discourse used depends on the speaker’s intent. If it is to transfer meaning, it is more univocal in nature. If it is to generate new meaning, it is more dialogic in nature. However, all discourse to some degree, is both dialogic and univocal (Knuth & Peressini, 2001).

It is ideal to use both types of discourse in mathematics classroom. Typically, most mathematics classrooms use univocal discourse. To begin implementing more dialogic discourse in the classroom, there are three important teacher actions to consider: 1) The classroom environment, 2) The types of tasks chosen 3) The way teachers question their students (Groves & Doig, 2004). I will discuss the three briefly. I encourage you to read the articles in the “must reads” for more elaboration of each.

Establishing an appropriate classroom environment is integral for implementing more dialogic discourse (Groves & Doig, 2004). Classroom norms that value individual (and group) contributions to the solution process are needed in order for students to feel comfortable sharing their ideas (Groves & Doig, 2004). Encouragement of mistakes, sharing their ideas, risk-taking, and critical thinking are all important things to discuss with students before dialogic discourse can be successful. The expectation that there is a “no opting out” policy where all students are expected to contribute and explain their ideas are also important so all students are contributing to the classroom discussions.

Once the learning environment is understood and accepted, teachers need to "problematize the curriculum" (Groves & Doig, 2004, p. 500). This is where teachers take a standard piece of mathematics and transform it into something challenging and problematic, but still leaving it accessible to students. This can be as small as asking students to model fractional division, or as big as having them explore a linear pattern. Giving students a task where they can all enter and build from is important to making dialogic discourse a possibility.

Sherin (2002) found that by using three simple teacher questions, a structure for class discussions emerged. Those questions were 1) "What do you think about this?" 2) "Why?" and 3) "What do other folks think about that?" More specifically, she called them (a) idea generation (b) comparison and evaluation, and (c) filtering (p. 219). By implementing those three questions in a mathematics class, the dialogue in the class changed. Instead of focusing on right answers, students were generating ideas, sharing their different methods, and more importantly talking about math.

What I Want to Say:

It is pertinent to note that we are not trying to eliminate univocal discourse from mathematics classrooms in favour of dialogic discourse. Both types of discourse are essential for a well-balanced mathematics classroom. However, current mathematics classrooms typically use univocal discourse with little or no dialogic discourse. The good news is that implementing more dialogic discourse is an achievable goal if you are interested. Currently, I am in the process of using more dialogic discourse in my classroom. I have been following the process laid out by Groves & Doig (2004). Here is a brief synopsis of my experiences thus far with each of the 3 teacher actions.

1) Establishing classroom norms – I began the semester by showing "Kid Snippets: Math Class" from YouTube. It is a comedic video depicting a teacher trying to explain subtraction to a student who really doesn't understand. It is a great way to break the ice with students and discuss how we want to avoid situations such as that one. I then showed students an interview with Jo Boaler, a prominent researcher on the teaching and learning of mathematics. In the interview she discusses the importance of having a growth mind-set rather than a fixed mind-set. Students that maintain a growth mind-set while learning will be able to learn from their mistakes and as result, attain a high level of mathematics. Here is a list of discussion points you may want to address with students.

a. The importance of making mistakes
b. No opting out policy (you must answer when called upon)
c. Be respectful of your classmates answers. You can critically analyze student's

answers, but not the person.

d. Be prepared to work in groups and discuss mathematics

e. The importance of maintaining a growth mind-set

2) Problematize the curriculum – I haven't problematized the entire curriculum but there are a number of places in the student workbook where I took a standard mathematical procedure and attempted to make it more challenging. For example, having students model the operations on fractions, or challenge them by having them find patterns with respect to linear relations. Many times teachers shy away from this aspect because they think it needs to be a "shiny new" lesson when in fact, it doesn't.

3) The words I use to question the students – In order for me to feel comfortable with my transformation of implementing dialogic discourse, I started by first doing more univocal discourse. I began asking students more questions and getting them used to being more vocal in class. Once students became more comfortable with responding, and realizing that it wasn't a high stakes atmosphere, I began asking them "why". After students became used to validating their answers, I then asked the class (or another student specifically) what they thought of the student's response. Once students were comfortable with the level of communication in the class, I began challenging them with less algorithmic questions, and began challenging them with problems that really had them thinking.

In closing, the great thing about implementing more dialogic discourse is that it doesn't take a major revolution and it doesn't happen quickly. The opportunity to start small, and gradually make more changes as students become more comfortable, makes dialogic discourse a real possibility and trust me, it is worth it. To have students talking and discussing mathematics without fear of mistakes is a very worthy end result.

Must Reads:

Groves, S., & Doig, B. (2004). Progressive discourse in mathematics classes – the task of the teacher. In M. J. Hoines & A. B. Fuglestad (Ed.), *Proceedings of the Annual Meeting of the International Group for the Psychology of Mathematics Education (PME).* (pp. 495-502). Bergen, Norway.

Knuth, E., & Peressini, D. (2001). Unpacking the nature of discourse in mathematics classrooms. *Mathematics Teaching in the Middle School, 6,* 320-235.

Reference List:

Groves, S., & Doig, B. (2004). Progressive discourse in mathematics classes – the task of the teacher. In M. J. Hoines & A. B. Fuglestad (Ed.), *Proceedings of the Annual Meeting of the International Group for the Psychology of Mathematics Education (PME).* (pp. 495-502). Bergen, Norway.

Knuth, E., & Peressini, D. (2001). Unpacking the nature of discourse in mathematics classrooms. *Mathematics Teaching in the Middle School*, *6*, 320-235.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA.

Sherin, M. G. (2002). A balancing act: Developing a discourse community in mathematics classroom. *Journal of Mathematics Teacher Education*, *5*, 205-233.

Truxaw, M. P., & DeFranco, T. C. (2007). Lessons from Mr. Larson: An inductive model of teaching for orchestrating discourse. *Mathematics Teacher*, *101*, 268-272.

Wertsch, J. V. (1991). *Voices of the mind: A sociocultural approach to mediated action.* Cambridge, MA: Harvard University Press.

RN #9: Nix The Trix

Nix the Trix is a free e-book for teachers. It was put together by Tina Cardone. The book is a collection of the tricks that students use in mathematics classrooms, but have no idea why they work. What first started as a collegial conversation about the tricks they hate within the math department at her school, grew to include an online math community and culminated in the book Nix the Trix.

Cardone (2014) acknowledges that in order for students to drop the short-cuts, they will have to think and parents and tutors will have to readjust their expectations. After a very clever introduction, she gets into the tricks that students bring to the classroom. She then takes it a step further and offers some advice on how to teach students without using tricks.

There are two places in my project where I am attempting to 'nix the trix.' The first being the case of invert and multiply (multiply by the reciprocal) when dividing fractions, and the second being students' misconception of multiplication.

1. Dividing Fractions: When students are taught to divide fractions, they are sometimes told "don't ask why, just invert and multiply". There might be some teachers who have taken the time to develop the algorithm with their students, but honestly, pictorially representing division of fractions is quite difficult. However, with a little help, students should be able to pictorially represent division of fractions and discover the algorithm of multiplying by the reciprocal. Cardone (2014) provides two different methods for helping teachers move students past using the algorithm blindly. If you are not interested in the method I have presented out in the student workbook, take a look at her approaches. The first approach has students creating common denominators and then deriving the algorithm from there (which I did include a few examples of in the student workbook). The second approach takes the idea of multiplying by one to recognize the reciprocal nature of the multiplication.

2. Misconception of Multiplication: Students' misconception of multiplication is not part of Nix the Trix, but it fits quite well in this research section. Devlin (2007) brought it to the attention of many mathematics teachers that multiplication is not repeated addition. He acknowledges that "multiplication of natural numbers certainly gives the same result as repeated addition, but that does not make it the same" (Devlin, 2007, p. 1). Where it begins to fall apart is when students are asked to multiply numbers outside of the natural number system, such as fractions. For example, $\frac{1}{2} \times 3$ can be written as $\frac{1}{2} + \frac{1}{2} + \frac{1}{2}$, but when you are given $\frac{1}{2} \times \frac{1}{8}$, it is impossible to write this expression out using repeated addition. Devlin offers the idea that multiplication is scaling, rather than repeated addition. I am not suggesting that anyone should go to elementary teachers and discuss

that they should be teaching it differently, because honestly, I don't know how you introduce such a concept to young students without putting it into context of repeated addition. What I am purposing is that grade 9 teachers discuss with students how the idea of repeated addition worked for when they were younger, but now they are going to explore why this doesn't work when we move out of the natural number system.

What I Want To Say:

While being back at school to work on my classes for my masters, we were often reading articles that discussed the issue of mathematics being viewed as a series of arbitrary rules and procedures that were meant to be memorized and not understood (Schoenfeld, 1988, Ball, Hill, Bass, 2005, Ma, 1999). I feel strongly that 'tricks' need to be eliminated from mathematics classrooms. In order for students to understand mathematics conceptually, students need to be shown why algorithms work and why tricks may work well in one situation but not in all. There are places where algorithms are definitely needed, for example, dividing fractions. But the algorithms should only be introduced after students can concretely see the reasons behind the algorithms. There will always be some students who choose only to use the algorithm without understanding, but it is important for all students to have the opportunity to uncover the mathematics behind the algorithms. For a good bonus on a calculus exam, see if your grade 12 students can model $\frac{3}{4} \div \frac{7}{8}$.

Must Reads:

If you want to read more on this topic, or related topics with respect to teacher content knowledge in mathematics, any article by Deborah Ball is a safe bet.

Nix the Trix is a free e-book from the link provided.

Ball, D. L., Hill, H. C., & Bass, H. (2005). Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide? *American Educator*, *29*, 14-17, 20-22, 43-46.

Cardone, T. (2014). *Nix the trix* [E-Book Version]. Retrieved from http://www.teacherspayteachers.com/Product/Nix-The-Trix-A-Free-E-Book-about-Problems-with-Mathematical-Tricks-1009487

Devlin, K. (2008). It ain't no repeated addition. *Mathematical Association of America*. http://www.maa.org/external_archive/devlin/devlin_06_08.html

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Mahwah, MJ: Lawrence Erlbaum Associates.

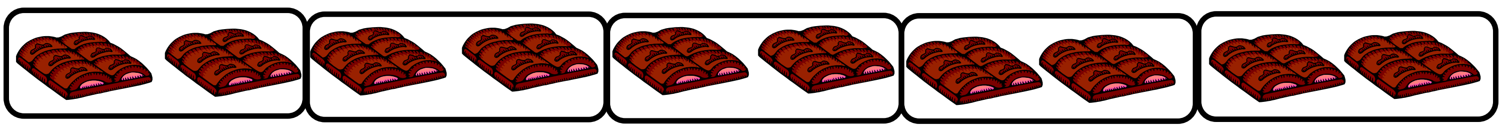
Schoenfeld, A. H. (1988). When good teaching leads to bad results: The disasters of "well-taught" mathematics courses. *Educational Psychologist*, *23*, 135-166.

RN #10: Quotative Division vs Partitive Division

What the Research Says:

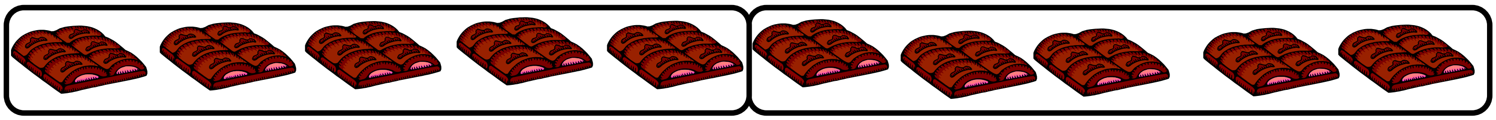
When students are asked to model a division statement such as $10 \div 5$, the model they draw depends on their interpretation of division. In mathematics, two models of division exist; partitive and quotitive. All division statements are seen as: $dividend \div divsor = quotient$. However, how students interpret the divisor and the quotient is what differs between the two division models.

In partitive division, also known as sharing, the number of groups is known but the size of the groups is unknown ie. (# of items) ÷ (# of groups) = (# items per group). For example, if we have 10 chocolate bars to share between 5 people, how many chocolate bars does each person get? Symbolically, the division statement is represented by $10 \div 5$. By drawing a visual representation of the situation, we have 10 chocolate bars being divided into 5 equal sized groups with the answer being 2 chocolate bars in each group.



Intuitively, students feel comfortable with this method because it involves the idea of equal sharing. This is the model that students are first introduced to in elementary school and is the model most adults and students fall back on when they are asked to assign a story or to model a division statement. Ball (1990) conducted a study of prospective elementary and secondary teachers' understanding of division and found that the majority of prospective teachers (17 out of 19) could calculate $1\frac{3}{4} \div \frac{1}{2}$ correctly, but only 5 out of the 19 were able to generate an acceptable story to represent the division statement. For those 5 prospective teachers that were successful, they found the task difficult and it took some time before they came up with an acceptable story. If we try to take $1\frac{3}{4} \div \frac{1}{2}$ and put it into context of chocolate bars using the partitive method, you have $1\frac{3}{4}$ chocolate bars to share with $\frac{1}{2}$ a person. It is obviously difficult to comprehend how we are going to share a chocolate bar with $\frac{1}{2}$ a person. Tirosh (2000) states that there are three constraints to partitive division: "(a) the divisor must be a whole number, (b) the divisor must be less than the dividend, (c) the quotient must be less than the dividend." (p. 7). The partitive model "seriously limits the children's and prospective teachers' abilities to correctly respond to division word problems involving fractions" (p. 7). This may be why students struggle with division of fractions. If students intuitively use a partitive method to divide, their method of understanding breaks down when they encounter fractional divisors. As a result, they turn to the invert and multiply algorithm, not really understanding why it works.

In quotitive division, also known as the multiplicative method, the number of items in the groups is known, but the number of groups is not known. ie. (# of items) ÷ (# of items per group) = (# of groups). For example, if we have 10 chocolate bars, how many people can we feed if each person gets 5 chocolate bars? As with the partitive model, the division statement is represented by $10 \div 5$. By drawing a visual representation of the situation, we have 10 chocolate bars being divided into groups containing exactly 5 chocolate bars, with the answer being 2 groups.



This is not the model that students intuitively fall back on when asked to pictorially represent or create a story about a division statement. However, the quotitive method is more powerful because it works in situations where the partitive method breaks down. In the case of $1\frac{3}{4} \div \frac{1}{2}$, it is much easier to model and find an appropriate story using the quotitive method. By asking "how many $\frac{1}{2}$s are in $1\frac{3}{4}$?" or "if I have a recipe that needs $\frac{1}{2}$ of a cup of sugar, how many batches can I make if I have $1\frac{3}{4}$ cups of sugar in the cupboard?" Essentially, you are asking how many of the divisors fit into the dividend.

Although the research advocates for students to use their intuition when first learning about division, it also suggests there comes a time when they need to extend their knowledge past partitive division to include an understanding of quotitive division. In doing this, students will have a stronger basis of understanding of what division means, especially when they are dividing fractions.

What I Want To Say:

There is plenty of research that looks at prospective teachers and students understanding of division with fractions. Most of the research points to the fact that although a large majority can calculate the correct answer, they have very little understanding of what division means and how it applies to fractions. When discussing division of fractions with students it is very important to consider the words you are using to describe the operation of division. Division is an operation that has multiple meanings and although it would be nice if division had a "single symbol, single calculation procedure, and a single set of descriptive words" it just isn't the case (Anghileri, 1995, p. 13). It is important for students and teachers to be flexible when it comes to understanding and modelling division. Some of the common phrases used by teachers in interpreting division (say 10 ÷ 5) include 10 divided by 5, 10 divided into 5, 10 shared by 5, 10 shared into 5 and how many 5s go into 10 (Anghileri, 1995),

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RN #11: Concept Attainment

What the Research Says:

Concept attainment is an instructional strategy devised by Bruner as a way to help students make generalizations on a concept. Most often a teacher will develop two sets of positive and negative examplars, say yes or no to each of them, and have students make generalizations on what rule the teacher is using. Most students enjoy this instructional method because it gives them the responsibility for constructing the definition. This method "promotes inductive thinking" (Brahier, 2009, p. 63). Concept attainment is a strategy that falls under the umbrella of structured inquiry.

What I Want to Say:

Most students enjoy looking for patterns. As a result, this strategy usually goes over really with students. One problem with concept attainment is if you have a diverse range of learners. Some students often find the rule quickly and want to share it before others have had a chance to discover it for themselves. I recommend having those students provide some more examples to the lists of yes and no categories. This way you will be able to tell if the students have figured out the rule without having them say it out loud and it will give other students in the class a little longer to try and find the pattern for themselves.

There is not a lot of information to be found on concept attainment. This is because it is a strategy and not a teaching theory. I felt that it was important to include in the research notes since it is easily transferred to virtually every concept in mathematics. It is important to pay special attention to the selection of the examples and non-examples for each activity since they are integral for students to understand. In the must reads, I included a Saskatoon Public School Division web site that describes concept attainment in detail.

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N9.1 Powers

Lesson 1: Introduction to Exponents **N9.1**

One Grain of Rice (adapted from illuminations)

In the book *One Grain of Rice* by Demi, the main character Rani cleverly tricks the raja into giving rice to the village. Use the story from the book to answer the questions below.

1. Estimate how many grains of rice you think Rani will have at the end of 30 days. ______
2. Use the chart below to record the number of grains of rice Rani would receive each day.

Day 1 ***1*** *Grain of rice*	*Day 2* ***2*** *Grains of rice*	*Day 3* *Grains of rice*	*Day 4* *Grains of rice*	*Day 5* *Grains of rice*	**Total days 1-5**
Day 6 *Grains of rice*	*Day 7* ***64*** *Grains of rice*	*Day 8* *Grains of rice*	*Day 9* *Grains of rice*	*Day 10* *Grains of rice*	**Total days 6-10**
Day 11 *Grains of rice*	*Day 12* *Grains of rice*	*Day 13* *Grains of rice*	*Day 14* *Grains of rice*	*Day 15* ***16,384*** *Grains of rice*	**Total days 11-15**
Day 16 *Grains of rice*	*Day 17* *Grains of rice*	*Day 18* *Grains of rice*	*Day 19* ***262,144*** *Grains of rice*	*Day 20* *Grains of rice*	**Total days 16-20**
Day 21 *Grains of rice*	*Day 22* *Grains of rice*	*Day 23* *Grains of rice*	*Day 24* *Grains of rice*	*Day 25* *Grains of rice*	**Total days 21-25**
Day 26 *Grains of rice*	*Day 27* *Grains of rice*	*Day 28* *Grains of rice*	*Day 29* *Grains of rice*	*Day 30* *Grains of rice*	**Total days 26-30**
					Total days 1-30

3. If the story continued, how can you determine how many grains of rice she would receive on day 31?

4. How can you determine how many grains of rice she would receive on day 65?

Bonus: Can you come up with a mathematical equation or expression that models this pattern?

A. Terminology

$$8^3$$

What does this mean? ____

Why would we needs this?____

Where do we see exponents?______________________________________

What is the difference between?	
8^3	8×3

Base:____

Exponent:____

Power:____

Example 1: List the base, exponent, and power for each of the following:

a) 12^4		b) 3^7		c) 2^8		d) 4^3	
Base:	_____	Base:	_____	Base:	_____	Base:	_____
Exponent:	_____	Exponent:	_____	Exponent:	_____	Exponent:	_____
Power:	_____	Power:	_____	Power:	_____	Power:	_____

Example 2: What happens if we swap the base and the exponent, do we get the same answer? Expand the following powers (don't evaluate).

a)

8^3 and 3^8	

Q: Which would you predict to be larger?

b)

6^4 and 4^6	

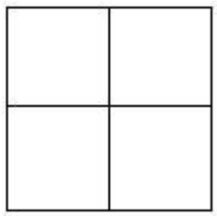
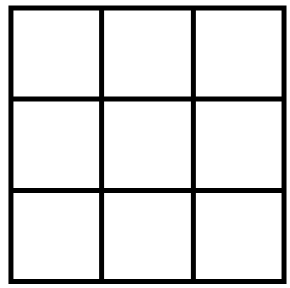
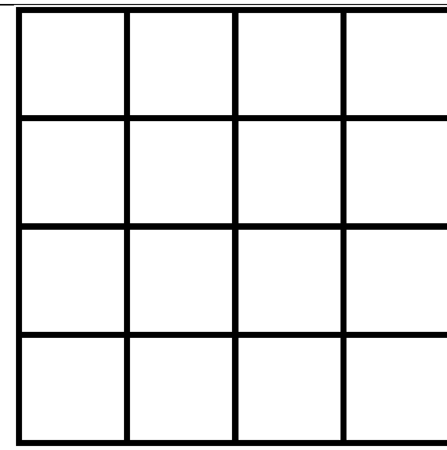
Q: Which would you predict to be larger?

Verify (check your answer with a calculator)

B. Common Exponents

A power with an integer base and exponent 2 is a **square number**.

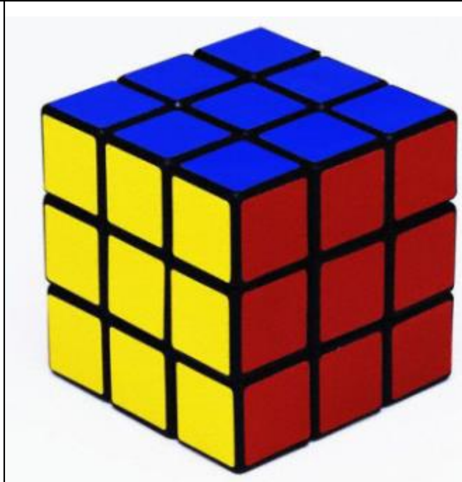
Why do you suppose they are called **SQUARES?**

2^2	3^2	4^2
		

Can you explain how the models given represent the powers 2^2, 3^2, and 4^2?

A power with an integer base and exponent 3 is a **cube number.**

Why do you suppose they are called **CUBES?**

2^3	3^3	4^3
		

Can you explain how the models given represent the powers 2^3, 3^3, and 4^3?

Example 3: Write as a power

a) $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$ b) 3×3 c) 6

Example 4: Write as repeated multiplication

a) 3^5 b) 2^7 c) 12^1 d) 11^5

Example 5: Calculate the volume of a cube that has side lengths of 6 cm.

Example 6: If the volume of a cube is 729, what are the side lengths?

Integers and Exponents

A. Find my Rule

Exponents are not limited to just positive bases, negative bases can have exponents too! See if you can spot a pattern?

A. $(-2)^1$

$(-2)^2$

$(-2)^3$

$(-2)^4$

$(-2)^5$

$(-2)^6$

B. $(3)^1$

$(3)^2$

$(3)^3$

$(3)^4$

$(3)^5$

$(3)^6$

C. $(-1)^1$

$(-1)^2$

$(-1)^3$

$(-1)^4$

$(-1)^5$

$(-1)^6$

D. $(1)^1$

$(1)^2$

$(1)^3$

$(1)^4$

$(1)^5$

$(1)^6$

What is the pattern?

Positive base to an odd exponent	
Positive base to an even exponent	
Negative base to an odd exponent	
Negative base to an even exponent	

B. The Importance of Brackets

Brackets don't always look like the ones above. There are many different ways that brackets are involved in a question.

Example 1: Evaluate

a) $(-3)^4$

b) $-(3)^4$

c) -3^4

d) (-3^4)

e) $-(-3)^4$

f) $(-3)^5$

g) $-(3)^5$

h) -3^5

i) $-(-3)^5$

Example 2: Predict whether each solution will be positive or negative, then evaluate **without** using repeated multiplication.

a) $(-2)^4$

b) $-(-5)^3$

c) $-(-5^3)$

d) $-(2^3)$

How do I use my calculator for larger powers?

Assignment: Lessons 1 & 2 **N9.1**

1. What is the base of each power?

a) -3^4 b) $(-3)^4$ c) $-(-8)^8$ d) 12^3

2. What is the exponent of each power?

a) -3^4 b) $(-3)^4$ c) $-(-8)^8$ d) 12^3

3. Use repeated multiplication to show why 6^5 is not the same as 5^6. Verify using a calculator which one is larger.

4. Complete the following table.

Power	Base	Exponent	Repeated Multiplication	Evaluate
3^3				
$(-10)^4$				
	-3	5		
			$1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1$	

5. Write each as a power, then evaluate.

a) 5×5 b) $6 \times 6 \times 6 \times 6 \times 6 \times 6$

c) $-(2 \times 2 \times 2 \times 2)$ d) $(-10) \times (-10) \times (-10)$

e) $(-3)(-3)(-3)(-3)$

f) $-(-7)(-7)(-7)$

g) $-(5)(5)(5)$

g) $(10)(10)(10)(10)(10)(10)$

6. For each power.

- Are the brackets needed?
- If you answer is yes, what purpose do the brackets serve?
- Evaluate

a) $(-3)^5$

b) $-(3)^5$

c) $-(-3)^5$

d) (-3^5)

7. Predict whether each solution will be positive or negative, then evaluate.

a) 4^3

b) 10^2

c) 2^4

d) 8^1

e) $(-3)^5$

f) -3^5

g) $(-3)^4$

h) -3^4

i) $-(-7)^2$

j) $-(-7)^3$

k) $(-5)^4$

l) -5^4

8. Write each number as a power with a base of 3.

a) 27

b) 2187

c) 81

d) 729

Powers of 10 and the Zero Exponent Law

A. Powers of 10

Name	Standard Form	Power	SI Prefix
Quintillion			
Quadrillion			
Trillion			
Billion			
Million			
Hundred Thousand			
Ten Thousand			
Thousand			
Hundred			
Ten			
One			
One Tenth			
One Hundredth			
One Thousandth			
One Ten Thousandth			
One Hundred Thousandth			
One Millionth			
One Trillionth			
One Quadrillionth			
One Quintillionth			

Calculator Experiment:

What happens when we have an exponent of 0? Using your calculator to experiment with different bases. We will share them as a class. Be adventurous with the numbers you choose.

Conclusion:____

B. Zero Exponent Law

A power with an integer base and an exponent of 0, is equal to 1.

* The base cannot be 0!
* $b^0 = 1, \ b \neq 0$

Terminology Refresher:

Repeated Multiplication	Expand	Power	Exponential Form	Standard Form	Evaluate
$4 \times 4 \times 4$	$4 \times 4 \times 4$	4^3	4^3	64	64

Example 1) Evaluate each power.

a) 5^0 b) 45^0 c) $(-3)^0$ d) -2^0

e) $(-2)^0$ f) $-(-2)^0$ g) 2245^0 h) -1000^0

Example 2) Write each number as a power of 10

a) 1 b) 100,000 c) One Billion d) -1

Example 3) Expand the following powers

a) 10^5 b) 10^8

Assignment: Lesson 3 **N9.1**

1. Evaluate the following powers.

a) $(-5)^0$ b) -3^0 c) 4^0 d) 1399^0

e) $-(-2)^0$ f) -2^0 g) 10^3 h) 10^7

2. Write each number as a power of 10

a) 10,000 b) Quadrillion c) 1 d) 100

3. Complete this table for powers of 10

Exponent	Power	Standard Form
7	10^7	10,000,000
6		
5		
4		
3		
2		
1		
0		

4. Arrange the following powers from least to greatest value: $1^{22}, 3^4,\ 4^3,\ 2^5,\ 7^2$

Order of Operations

A. Why do we Need an Order?

Example 1: Answer the following. How many different answers can we come up with?

$$7 - 1 \times 0 + 3 \div 3$$

Warning! If you calculate in the wrong order, you will get the wrong answer. What is the correct order?

B. What is the Order?

A long time ago people agreed to follow rules when doing calculations, and they are:

		D	A
B	E		
		M	S

Example 2:

a. $20 - (3 \times 2^3 - 5)$

b. $(5 + 2)^2 - 9 \times 3 + 2^3$

c. $4^2 + 2^6$

d. $(6 - 2)^2 - 3 \times 2$

e. $3(-4)^3$

f. $4^2 - 8 \div 2 - 2^3$

g. $-2(-15-4^2)+4(4+1)^3$

h. $5^2+(-5^2)$

i. $V=\frac{4}{3}\pi r^3$; the radius is 5

Example 3:

Three students got different answers when they evaluated this expression:

$$-5^2-2[27 \div (-9)]^3$$

Arshpreet's answer was 729, Jake's answer was 79, and Cooper's answer was 191.

a) Show the correct solution

b) Show and explain how the students who got the wrong answer may have evaluated. Where did each student go wrong? Provide feedback to the student so they don't make the same mistake again.

Task: Using the order of operations and four 4's, create the numbers 1 to 20.

0	11
1	12
2	13
3	14
4	15
5	16
6	17
7	18
8	19
9	20
10	

Assignment: Lesson 4 **N9.1**

1. Evaluate

a) $4^2 + 2$ b) $4^2 - 2$ c) $(4 + 2)^2$ d) $(4 - 2)^2$

e) $(2 - 4)^2$ f) $(2 - 4)^3$ g) $4^2 - 3^2$ h) $3^2 - 4^2$

2. Evaluate

a) $3^2 \times 5$ b) 3×2^3 c) $(3 \times 2)^3$ d) $4 \div 2^3$

e) $(8 \div 2)^3$ f) $8^2 \div 4^2$ g) $(-15 \div 3)^2$ h) $(-15 \div 3)^3$

3. Evaluate

a) $(18 \div 3^2 + 1)^3 - 3^2$ b) $5^3 - 2(5 - 2)^2$ c) $(12^2 + 5^3)^0 - 2[(-3)^3]$

4. Three students A, B, C did a question and got three different answers. You are the teacher and you need to find out who is correct AND where the other two students made their errors. Describe their error.

Student A

$$(-4)^2 - 3[(-9) \div 3]^2$$
$$(-4)^2 - 3[-3]^2$$
$$(-4)^2 + (9)^2$$
$$16 + 81$$
$$97$$

Is Student A correct? Circle their mistake(s) and give them feedback on where they went wrong.

Student B

$$(-4)^2 - 3[(-9) \div 3]^2$$
$$(-4)^2 - 3[-3]^2$$
$$16 - 3(9)$$
$$16 - 27$$
$$-11$$

Is Student B correct? Circle their mistake(s) and give them feedback on where they went wrong.

Student C

$$(-4)^2 - 3[(-9) \div 3]^2$$
$$(-4)^2 - 3[-3]^2$$
$$8 - 3[6]$$
$$8 - 18$$
$$-10$$

Is Student C correct? Circle their mistake(s) and give them feedback on where they went wrong.

Exponent Laws

A. What is an Exponent?

Before we begin, we must remember what an exponent means. An exponent is a shorter way to write ____.

For instance, 3^6 **can be thought of as repeatedly multiplying 3 by itself, 6 times, or** $3 \times 3 \times 3 \times 3 \times 3 \times 3$. We will use this notation to rewrite various expressions involving exponents.

It is important to keep in mind that there are other ways to simplify these expressions (including the calculator). But we are doing this to help us establish rules that are going to apply to algebraic expressions as well (which a calculator wouldn't be able to help us). Follow the first example and keep the same notation throughout the exercise. You should be able to see a pattern develop.

1. Complete the table using the idea of repeated multiplication. Then in the final row, use the pattern to write the general exponent rule.

Expression	**Repeated Multiplication**	**Simplified**
$3^3 \cdot 3^4$ or $3^3 \times 3^4$	$(3 \cdot 3 \cdot 3) \cdot (3 \cdot 3 \cdot 3 \cdot 3)$	3^7
$2^6 \cdot 2^3$ or $2^6 \times 2^3$		
$8^7 \cdot 8^3$ or $8^7 \times 8^3$		
$6^5 \cdot 6^3$ or $6^5 \times 6^3$		
Make up your own!		
$a^3 \cdot a^2$ or $a^3 \times a^2$		
$a^m \cdot a^n$ or $a^m \times a^n$		

Does this rule work for $5^3 \cdot 8^2$?

Rule: When the bases are the same…..

2. Complete the table using the idea of repeated multiplication. Then in the final row, use the pattern to write the general exponent rule.

Important: These problems also use the fact that $\frac{5}{5} = 1$. Anything divided by itself is always 1. $\frac{5 \cdot 3}{3} = 5$ since the 3's on the top and bottom can be thought of $\frac{3}{3}$ which is just 1.

Expression	**Repeated Multiplication**	**Simplified as a power**
$\frac{2^4}{2^3}$ or $2^4 \div 2^3$	$\frac{2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2}$	2^1
$\frac{10^6}{10^3}$ or $10^6 \div 10^3$		
$\frac{4^8}{4^3}$ or $4^8 \div 4^3$		
$\frac{7^9}{7^2}$ or $7^9 \div 7^2$		
$\frac{a^7}{a^3}$ or $a^7 \div a^3$		
$\frac{a^m}{a^n}$ or $a^m \div a^n$		
What happens when… $\frac{5^3}{5^3}$ Or $5^3 \div 5^3$		
What happens when… $\frac{0^4}{0^3}$ Or $0^4 \div 0^3$		

What happens when… $\frac{5^6}{4^3}$ or $5^6 \div 4^3$		

Rule: When the bases are the same…

EXCEPT

1. Complete the table using the idea of repeated multiplication. Then in the final row, use the pattern to write the general exponent rule.

You will need to expand what is inside the bracket first, then apply the outer exponent.

Expression	Repeated Multiplication	Simplified
$(4^2)^3$	$(4 \cdot 4)^3$ which is $(4 \cdot 4) \cdot (4 \cdot 4) \cdot (4 \cdot 4)$	4^6
$(6^3)^5$		
$(2^5)^3$		
$(8^4)^1$		
$(a^3)^4$		
$(a^m)^n$		

Rule:

1. Complete the table using the idea of repeated multiplication. Then in the final row, use the pattern to write the general exponent rule.

These problems also use the commutative property of multiplication that says the order in which I multiply two numbers does not matter. For example, $2 \times 3 = 3 \times 2$

Expression	Repeated Multiplication	Simplified
$(2 \cdot 3)^4$	$(2 \cdot 3) \cdot (2 \cdot 3) \cdot (2 \cdot 3) \cdot (2 \cdot 3)$ Which is.. $(2 \cdot 2 \cdot 2 \cdot 2) \cdot (3 \cdot 3 \cdot 3 \cdot 3)$	$2^4 \cdot 3^4$
$(5 \cdot 3)^3$		
$(4 \cdot 6)^2$		
$(7 \cdot 9)^5$		
$(a \cdot b)^3$		
$(a \cdot b)^m$		

Try BEDMAS with all the above and see if you get the same evaluated answer if you do the exponent rule. Are they the same?
Is $2^4 \cdot 3^4 = 6^4$?

Note: When we get into algebra $a \cdot b$ is just ab.

Rule:

3. Complete the table using the idea of repeated multiplication. Then in the final row, use the pattern to write the general exponent rule.

Use the idea from exponent rule #2 to complete the second column. To complete the third column, remember what you get when you divide a number by iself $\frac{5}{5}$ =?

Expression	**Work it out using exponent rule #2**	**Work it out using what you know about a number divided by itself.**
$\frac{6^3}{6^3}$ or $6^3 \div 6^3$	6^{3-3} 6^0 _______	$\frac{216}{216}$ Which is just 1
$\frac{10^4}{10^4}$ or $10^4 \div 10^4$		
$\frac{8^6}{8^6}$ or $8^6 \div 8^6$		
$\frac{3^5}{3^5}$ or $3^5 \div 3^5$		
$\frac{a^4}{a^4}$ or $a^4 \div a^4$		
$\frac{a^m}{a^m}$ or $a^m \div a^m$		

Rule: Anything to the power of 0 is ______.

Exponent Rules

- You can apply the exponent rules to help simplify expressions.

1. You can simplify a product of powers with the same base by adding exponent.

$(a^m)(a^n) = a^{m+n}$ Or $a^m \times a^n = a^{m+n}$	$(4^7)(4^3)$

2. You can simplify a quotient of powers with the same base by subtracting exponents.

$\frac{a^m}{a^n} = a^{m-n}, \qquad m > n$ OR $a^m \div a^n = a^{m-n}$	$\frac{4^7}{4^3}$

3. You can simplify a power that is raised to an exponent by multiplying the exponents.

$(a^m)^n = a^{mn}$	$(4^6)^3$

4. When a product is raised to an exponent you can rewrite each number in the product with the same exponent.

$(ab)^m = a^m b^m$	$(5 \cdot 3)^6$

5. When the exponent of a power is 0, the value of the power is 1 if the base is not 0.

$a^0 = 1, \qquad a \neq 0$	12^0

6. When a quotient is raised to an exponent, you can rewrite each number in the quotient with the same exponent.

$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	$\left(\frac{3}{4}\right)^4$

Example 1: Simplify and evaluate (Law 1 & 2)

Together	On Your Own
a) $(2^3)(2^8)$	b) $(-7)^9(-7)^5$
c) $(-3)^5 \div (-3)^2$	d) $(-7)^9 \div (-7)^5$
c) $12^7 \div 12^5 \div 12^2$	d) $3^2 \times 3^4 \times 3^5$
e) $(-10)^4[(-10)^6 \div (-10)^4]$	f) $-2^2(2^3 \div 2^1)$

Example 2: Write as a power and evaluate. (Laws 3&4)

a) $[(-3)^5]^2$

b) $-(4^2)^3$

c) $(9^2)^4$

d) $(5^3)^5$

e) $(8^2)^6$

Example 3: Why is the value of $[(-4)^3]^2$ positive and the value of $[(-4)^3]^3$ negative?

Example 4: For each expression below, evaluate in two different ways:

i. follow order of operations
ii. use the exponent laws first

a) $(5 \times 3)^4$

b) $[(-3) \times 5]^3$

c) $[15 \div (-3)]^2$

d) $-(2 \times 4)^2$

e) $\left(\frac{144}{12}\right)^3$

f) $\left(\frac{1}{4}\right)^3$

Example 5: Simplify and evaluate.

Together	On Your Own
a) $2(3)^4$	b) $-4(-2)^2$
c) $[(10^2)^4 \div (10^3)^2]$	d) $(7^4 \times 7^3)^0$
e) $2^3 - 2^0 \times 2^2 + 2^2$	f) $\frac{3^2 \times 3^8}{3^3 \times 3^6}$

Assignment: Lesson 5 **N9.1**

Write each product as a single power (if possible).

a) $3^5 \times 3^8$

b) $2^0 \times 2^0$

c) $10^3 \times 10^8$

d) $(-7)^6 \times (-7)^3$

e) $-6^3 \times (-6)^4$

f) $12^7 \times 12^{12}$

2. Write each quotient as a single power (if possible).

a) $3^8 \div 3^5$

b) $12^{15} \div 12^6$

c) $(-1)^5 \div (-1)^0$

d) $\frac{8^9}{8^5}$

e) $\frac{(-6)^8}{(-6)^2}$

f) $\frac{3^4}{3^4}$

3. Express as a single and evaluate.

a) $5^3 \times 5^7 \div 5^4$

b) $(-3)^{12} \div (-3)^3 \times (-3)^8$

c) $4^0 \times 4^8 \div 4^7$

d) $\frac{4^7 \times 4^3}{4^4 \times 4^2}$

Express as a single power (if possible) and then evaluate.

a) $2^3 \times 2^6 \div 2^9$

b) $(-5)^8 \div (-5)^4 \times (-5)^3$

c) $\frac{6^3 \times 6^5}{6^2 \times 6^4}$

d) $2^2 - 2^0 \times 2 + 2^3$

e) $(-2)^6 \div (-2)^5 \times (-2)^5 \div (-2)^3$

f) $(8^3 \times 8^7) \div (8^5 \div 8^2)$

5. Here is a student's work. Please correct their work and provide feedback on any errors.

a) $3^4 \times 3^2$

3^8

b) $5^3 \times 2^3$

10^6

c) $(-3)^8 \div (-3)^4$

$(-3)^4$

d) $\frac{4^2 \times 4^4}{4^2 \times 4^1}$

4^2

6. Write the following expression as a power raised to an exponent.

$$(3 \times 3) \times (3 \times 3) \times (3 \times 3) \times (3 \times 3)$$

7. Simplify and evaluate.

a) $(4^2 \times 4^3)^2 - (4^4 \div 4^2)^2$

b) $(2^3 \div 2^2)^3$

8. Find and correct any errors in each solution.

a) $(4^3 \times 2^2)^2 = (8^5)^2$
$= 8^{10}$
$= 1\ 073\ 741\ 824$

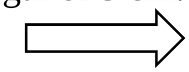
b) $[(-10)^3]^4 = (-10)^7$
$= -10\ 000\ 000$

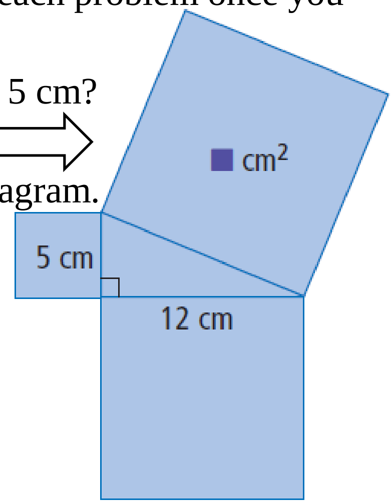
c) $(2^2 + 2^3)^2 = (2^5)^2$
$= 2^{10}$
$= 1024$

Assignment: Lesson 6 N9.1

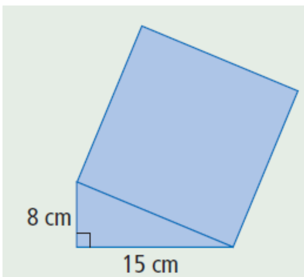
1. The surface area, SA, of a sphere can be calculated using the formula $SA = 4 \times \pi \times r \times r$, where r is the radius. Rewrite the formula using powers and no multiplication signs. Identify the coefficient, variable, and exponent in your formula.

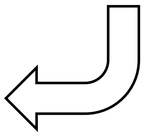
2. Write an exponential expression to solve each problem. Solve each problem once you have established an exponential expression.
   a. What is the surface area of a cube with an edge length of 5 cm?
   b. Find the missing side length of this right triangle.



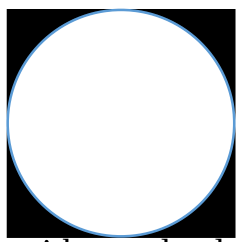


3. Find the area of the square attached to the hypotenuse in this diagram.





4. A circle is inscribed in a square with a side length of 3 cm. What is the area of the shaded region?



5. What is the volume of a cube with an edge length of 7 cm? Write an exponential expression to solve the problem.

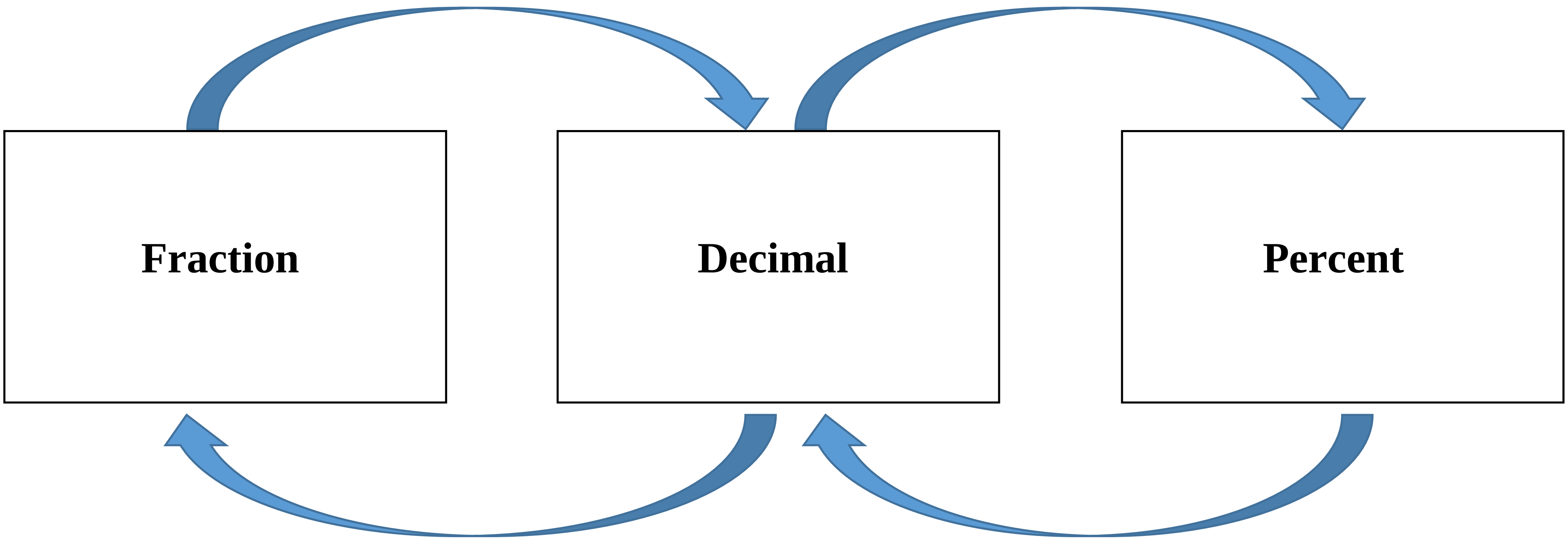
6. Which is larger, the area of a square with a side length of 14 cm or the surface area of a cube with an edge length of 6 cm? Show your work. Write an exponential expression to solve the problem;.

7. The number 10^{100} is known as a googol.
   a. Research where the term googol originated. Why do you think that the founders of Google$_{TM}$ used that name for their search engine?
   b. How many zeros would follow the 1if you wrote 10^{100} as a whole number?
   c. If you were able to continue writing zeros without stopping, how long would it take you to write a googol as a whole number? Explain why you believe your answer is correct. You will need to be prepared to share your answer and reasoning to the whole class.

N9.2 Rational Numbers

Lesson 1: What is a Rational Number? **N9.2**

What is a Rational Number?



What is a Rational Number?

A. Rational Number

Rational Number: ____

Provide some examples of rational numbers:

B. Background Essentials

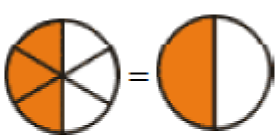
1. Equivalent:

____

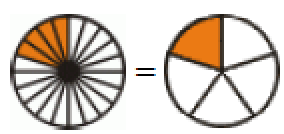
Provide some examples of rational numbers that are equivalent:

Example 1: Write down the simplification process

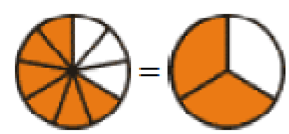
a.



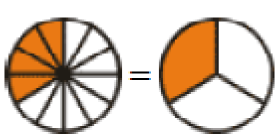
b.



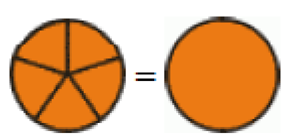
c.



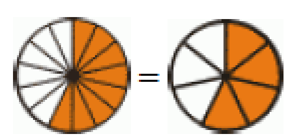
d.



e.

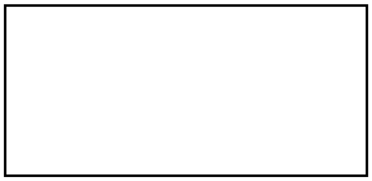


f.

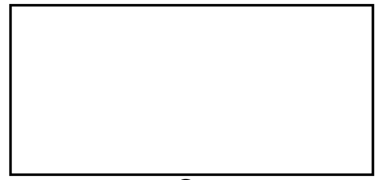


Example 2: Depict the following fractions using the following pictures.

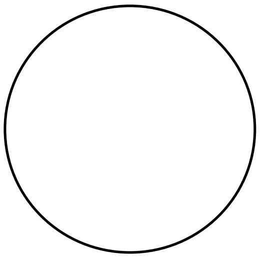
$$\frac{1}{2}$$



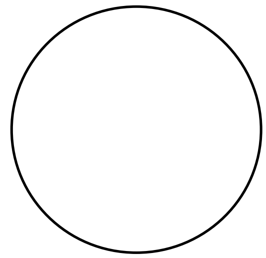
$$\frac{6}{12}$$



$$\frac{2}{3}$$



$$\frac{8}{12}$$



$$\frac{3}{4}$$

$$\frac{9}{12}$$

2. Fractions

   a. Numerator and Denominator

$$\frac{a}{b}$$

   b. Reducing Fractions/Enlarging Fractions

Reducing Fractions	**Enlarging Fractions**
• Some fractions can be reduced. They are another fraction in disguise. • Find a number that divides into both the numerator and denominator. It has to be the SAME for both. • It is customary to write fractions with the negative sign in the numerator or out in front of the fraction (if it is negative).	• You can enlarge fractions. • Pick any number and multiply it to both the numerator and denominator. It has to be the SAME for both. • It is customary to write fractions with the negative sign in the numerator or out in front of the fraction (if it is negative).

Example 3: Practice reducing fractions!

a. $\frac{5}{5}$

b. $\frac{15}{30}$

c. $\frac{36}{72}$

d. $\frac{125}{200}$

e. $-\left(-\frac{3}{10}\right)$

f. $\frac{10}{-40}$

g. $\frac{-3}{-4}$

h. $-\frac{-4}{9}$

Example 4: Some of the following fractions CANNOT be simplified, cross them out. Some of the following fractions CAN be simplified, simplify those.

$$\frac{2}{3} \quad \frac{2}{6} \quad \frac{6}{12} \quad \frac{6}{13} \quad \frac{7}{12} \quad \frac{11}{12} \quad \frac{11}{22} \quad \frac{9}{21} \quad \frac{9}{20} \quad \frac{6}{13} \quad \frac{14}{27} \quad \frac{14}{28} \quad \frac{14}{29} \quad \frac{8}{21} \quad \frac{8}{15} \quad \frac{8}{16} \quad \frac{8}{22}$$

Example 5: Identify equivalent rational numbers from the following list.

$$\frac{12}{4} \quad \frac{-8}{4} \quad \frac{-9}{-3} \quad -\frac{4}{2} \quad \frac{4}{-2} \quad \frac{12}{3} \quad -\left(\frac{4}{-1}\right) \quad -\left(\frac{-4}{-2}\right)$$

Example 6: For each rational number, write two fractions that represent the same number.

a. $\frac{1}{2}$

b. $\frac{8}{9}$

c. $\frac{-3}{4}$

Comparing Rational Numbers

Example 1: Which fraction is larger?

$$\frac{4}{9} \qquad \frac{3}{7}$$

Method 1:	Method 2:

Example 2: Compare and order the following rational numbers. Record the numbers on a number line.

$$-1.5 \qquad \frac{4}{5} \qquad \frac{9}{10} \qquad -0.\overline{3} \qquad -\frac{9}{10}$$

Solution:

Method 1:	Method 2:
Depict on a Number Line:	

On Your own #1:

Compare and order the following rational numbers from least to greatest. Record the numbers on a number line

$$1\frac{2}{5} \qquad -0.8 \qquad 2.1 \qquad -1 \qquad \frac{4}{5} \qquad -1.\overline{3}$$

______ ______ ______ ______ ______ ______

Example 3: Which fraction is greater, $-\frac{7}{8}$ or $-\frac{8}{10}$?

Method 1:	Method 2:

On Your Own #2:

Which fraction is smaller, $-\frac{5}{6}$ or $-\frac{9}{11}$?

Example 4: Identify a **fraction** between -0.5 and -0.6.

Example 5: For each pair of fractions, find one that is in between them. Make sure your answer is in fraction form, there is more than one answer.

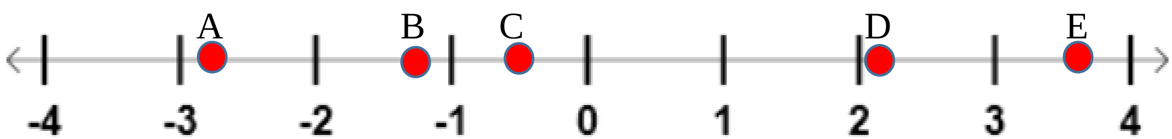
a. $\frac{1}{6} < \frac{}{} < \frac{5}{9}$

b. $\frac{3}{8} < \frac{}{} < \frac{1}{2}$

1. Identify equal rational numbers in the list that follows. Write you answer in the same way as the example from the notes.

$$\frac{15}{3} \qquad \frac{-12}{4} \qquad -\frac{9}{3} \qquad -\left(\frac{-20}{4}\right) \qquad -\left(\frac{-6}{-2}\right) \qquad \frac{-10}{-2}$$

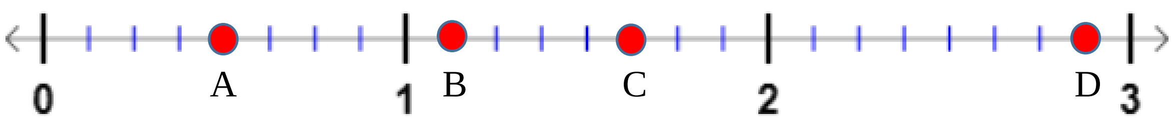
2. Match each rational number to a point on the number line.



a. $\frac{18}{5}$ b. -0.5 c. $2\frac{1}{5}$

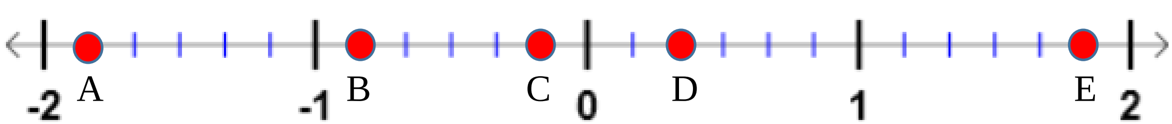
d. $-\frac{14}{5}$ e. $-\frac{4}{3}$

3. Write the rational number represented by each letter on the number line, as a decimal.



A ________ B ________ C ________ D ________

4. Write the rational number represented by each letter on the number line, as a fraction. Ensure that all fractions are reduced.
5.

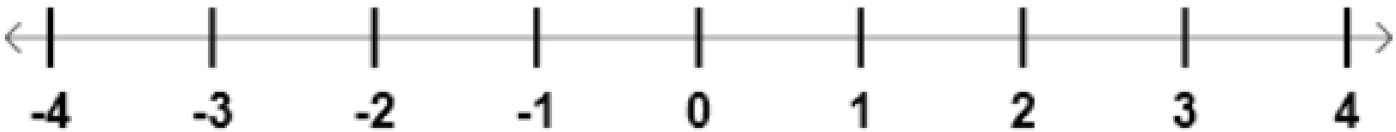


A ________ B ________ C ________ D ________ E ________

6. Place each number on the same number line. Be as accurate as possible. Label the dot.

a. -2.8 b. -1.7 c. $2\frac{2}{7}$ d. $-\frac{11}{3}$

e. $\frac{5}{6}$ f. 2.5 g. $\frac{2}{5}$ h. $-\frac{5}{8}$



7. Compare the following numbers and arrange in increasing order. Use the original numbers from the question in your answer.

$$\frac{3}{2}, -\frac{5}{4}, -1.5, -\frac{1}{3}, 0.9, 0.92, -0.\overline{6}$$

8. For each pair of fractions, find one that is in between them. Any fraction will do!

a. $\frac{2}{3} < - < \frac{3}{4}$ b. $\frac{1}{7} < - < \frac{1}{3}$

c. $\frac{4}{12} < - < \frac{4}{11}$ d. $\frac{1}{2} < - < \frac{7}{10}$

9. Identify a rational numbers (in the same form) between each pair of numbers.

a. 1.2, 1.3 (as a decimal) b. $\frac{3}{4}, \frac{4}{5}$ (as a fraction) c. $\frac{19}{21}, \frac{20}{21}$ (as a fraction)

Adding & Subtracting Rational Numbers

A. Adding and Subtracting Fractions

1. Modelling: model the following expression pictorially

$$\frac{5}{8}+\frac{3}{4}$$

2. Mathematically: add the following expression numerically

$$\frac{5}{8}+\frac{3}{4}$$

We need the denominators of each fraction to be the same in order to add. Although you can add some fractions intuitively, there are others that require manipulation in order to add or subtract. LCD (lowest common denominator). There are times when the question can't be modelled because you aren't able to represent a negative fraction pictorially.

Example 1: Evaluate, give your answer in lowest terms.

a) $-\frac{5}{6}+\left(-\frac{3}{4}\right)$

b) $\frac{4}{5}-\frac{7}{8}$

c) $2\frac{4}{5}-\frac{1}{4}$

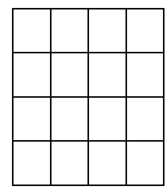
d) $-\frac{5}{11}+\frac{1}{8}$

c) $-3\frac{2}{3} + 2\frac{3}{4}$

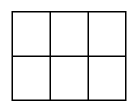
d) $-\frac{7}{8} - (-2)$

e) Assume that the entire figure shown represents 1 unit. Shade the appropriately to show the addition of :

$$\frac{5}{8} + \frac{1}{16}$$



$$\frac{1}{2} + \frac{1}{3}$$



B. Adding and Subtracting Decimals

A strong skill to have is estimation. When working with money it is always handy to estimate the cost of a number of items or a restaurant bill to make sure you have enough money before you order.

Example 2: Estimate and calculate.

a) $3.22 + (-5.75)$

b) $-4.65 - (-8.97)$

Lesson 4: Assignment **N9.2**

1. Evaluate, leaving your answer in lowest terms.

a. $\frac{6}{9}+\frac{2}{9}$ b. $\frac{1}{4}+\frac{1}{4}$ c. $\frac{3}{4}-\frac{7}{4}$ d. $-\frac{1}{4}-\frac{3}{4}$ e. $\frac{7}{15}-\frac{2}{15}$

2. State the lowest common denominator that would be used to evaluate the following questions. **Do not evaluate!**

a. $\frac{2}{3}+\frac{1}{2}$ b. $\frac{5}{8}+\frac{2}{3}$ c. $-\frac{3}{4}-\frac{1}{2}$ d. $\frac{4}{5}-\frac{3}{4}$ e. $\frac{7}{15}-\frac{3}{5}$

3. Evaluate, leaving your answer in lowest terms.

a. $\frac{4}{9}-\frac{4}{6}$ b. $-\frac{9}{20}-\frac{3}{20}$ c. $-\frac{2}{9}+\frac{5}{4}$

d. $\frac{3}{25}+\frac{7}{10}$ e. $\frac{3}{8}-\frac{4}{5}$ f. $\frac{5}{6}+\frac{2}{3}-\frac{1}{2}$

g. $-\frac{3}{4}+\left(-\frac{7}{8}\right)$ h. $-\frac{1}{2}+\frac{2}{3}$ i. $\frac{9}{4}-\frac{4}{5}$

j. $2\frac{3}{5}+1\frac{1}{2}$

k. $\frac{1}{9}-\frac{1}{6}$

l. $-\frac{1}{6}-\frac{1}{4}$

m. $\frac{5}{6}-\left(-\frac{1}{8}\right)$

n. $-\frac{1}{10}-\left(-\frac{1}{12}\right)$

o. $\frac{1}{6}+\frac{3}{8}$

p. $-\frac{5}{6}+\frac{5}{8}$

q. $\frac{3}{10}-\frac{1}{4}$

4. Estimate and then evaluate.

a. $-5.3+2.4$

b. $6.55+(-3.62)$

c. $-0.228+(-14.8)$

d. $-32.55+14.76$

e. $50.25+120.56$

f. $-9.45+3.89$

5. Jessie practices running, and he is told to use 10 minutes for a warm-up and 10 minutes for a cool down. In total he only has 1 hour to complete the warm up, workout, and cool down.

a. What fraction of the total time is the warm up?

b. What fraction of the total time is the workout?

c. What fraction of the total time is the cool down?

5. Here is a monthly budget for a student. Write a mathematical expression that shows the calculation. What does your final answer mean?

Item	Income	Expense
Money from work	\$825.75	
Rent		\$625
Food		\$200
Car Payment		\$175.87
Insurance		\$144.23
Student Loan	\$350	
Gas		\$200

a. Expression and answer:

b. What does that number mean?

6. Below is a chart with some Saskatoon temperatures from the past year.

Date	Temperature on that Day	Average Temp for that Day	Difference between column 2 and column 1	What does it mean?
January 31st, 2013	-36.2℃	-19.7℃		
March 19th, 2013	-27.1℃	-8.8℃		
August 25th, 2013	33.8℃	23.3℃		
January 15th, 2013	4.4℃	-10.9℃		

Multiplying Rational Numbers

1. What is multiplication?____

Example 1: What is 2×3? Write out what this expression means in words as well as pictorially.

In Words:	Pictorially/Model:

2. Multiplication of an Integer and a Fraction: $a\left(\frac{b}{c}\right)$ or $\left(\frac{b}{c}\right)c$

Example 2: Write out what each of the following expressions means in words as well as pictorially

a) $5\left(\frac{1}{2}\right)$ or $5 \times \frac{1}{2}$

In Words:	Pictorially/Model:

b) $4\left(\frac{2}{3}\right)$ or $4 \times \frac{2}{3}$

In Words:	Pictorially/Model:

c) $(3)\frac{7}{8}$ or $3 \times \frac{7}{8}$

In Words:	Pictorially/Model:

d) $\left(\frac{1}{2}\right) \times 5$

In Words:	Pictorially/Model:

e) $\left(\frac{2}{3}\right) \times 4$

In Words:	Pictorially/Model:

f) $\left(\frac{7}{8}\right) \times 3$

In Words:	Pictorially/Model:

Do you notice a pattern?

$5 \times \frac{1}{2}$	$4 \times \frac{2}{3}$	$3 \times \frac{7}{8}$	Algorithm:
$\frac{1}{2} \times 5$	$\frac{2}{3} \times 4$	$\frac{7}{8} \times 3$	

3. Multiplication of a Fraction and a Fraction: $\left(\frac{a}{b}\right)\left(\frac{c}{d}\right)$

Example 3: Write out what each of the following expressions means in words as well as pictorially

a) $\frac{1}{2} \times \frac{3}{4}$

In Words:	Pictorially/Model:

b) $\left(\frac{3}{4}\right)\left(\frac{1}{2}\right)$

In Words:	Pictorially:

c) $\left(\frac{2}{3}\right)\left(\frac{3}{4}\right)$

In Words:	Pictorially:

d) $\left(\frac{3}{4}\right)\left(\frac{2}{3}\right)$

In Words:	Pictorially:

e) $\frac{3}{5} \times \frac{7}{4}$

In Words:	Pictorially:

f) $\frac{7}{4} \times \frac{3}{5}$

In Words:	Pictorially:

Do you notice a pattern?

$\frac{1}{2} \times \frac{3}{4}$	$\left(\frac{3}{4}\right)\left(\frac{1}{2}\right)$	$\left(\frac{2}{3}\right)\left(\frac{3}{4}\right)$	$\left(\frac{3}{4}\right)\left(\frac{2}{3}\right)$
$\frac{3}{5} \times \frac{7}{4}$	$\frac{3}{5} \times \frac{7}{4}$	Algorithm:	

Example 4: Calculate using the algorithm. All answers need to be in reduced form.

a. $\left(\frac{2}{5}\right)\left(\frac{9}{4}\right)$

b. $\left(\frac{5}{6}\right)\left(-1\frac{1}{2}\right)$

c. $(-10)\left(\frac{3}{7}\right)$

d. $\left(-\frac{1}{4}\right)\left(-\frac{1}{2}\right)$

4. Reducing BEFORE Multiplying (only do this if you are wanting to save time)

Example 5: Find the product of $\left(\frac{15}{14}\right)\left(\frac{7}{65}\right)$. Make sure you reduce your answer.

Reduce AFTER Multiplying	Reduce BEFORE Multiplying

Example 6: Find each of the following products, writing your answers in lowest terms. Be sure to reduce first, then multiply.

a. $\left(\frac{5}{16}\right)\left(-\frac{8}{20}\right)$

b. $\left(\frac{12}{56}\right)\left(\frac{16}{24}\right)$

c. $\left(3\,\frac{3}{5}\right)\left(-2\,\frac{7}{9}\right)$

d. $\left(5\,\frac{1}{3}\right)\left(-2\,\frac{1}{4}\right)$

Lesson 5: Assignment **N9.2**

1. Calculate the following using a model.

a. $\frac{3}{4} \times \frac{7}{8}$

b. $\frac{5}{8} \times \frac{1}{2}$

c. $1\frac{3}{4} \times \frac{2}{3}$

2. Calculate. Write all answers in reduced form.

a. $\frac{9}{11} \times \frac{5}{6}$

b. $\frac{2}{5} \times \frac{1}{2}$

c. $\left(-\frac{6}{13}\right)\left(-\frac{1}{4}\right)$

d. $\left(\frac{8}{9}\right)\left(\frac{1}{6}\right)$

e. $\frac{2}{7} \times \frac{7}{4}$

f. $-\frac{5}{6} \times \frac{6}{5}$

g. $\left(\frac{9}{10}\right)\left(\frac{13}{15}\right)$

h. $\frac{9}{2} \times 5$

i. $\frac{10}{11} \times 1\frac{1}{2}$

j. $\left(\frac{7}{4}\right)\left(-1\frac{2}{3}\right)$

l. $\left(\frac{7}{2}\right)\left(\frac{6}{5}\right)\left(\frac{11}{6}\right)$

m. $\left(\frac{12}{5}\right)\left(\frac{15}{11}\right)(11)$

n. $2\frac{1}{3} \times 1\frac{1}{8}$

o. $\left(\frac{1}{12}\right)\left(4\frac{1}{2}\right)\left(\frac{5}{9}\right)$

p. $\left(\frac{2}{7}\right)\left(\frac{1}{4}\right)\left(1\frac{2}{3}\right)$

3. In everyday speech, *in a jiffy* means in a very short time. In science, a specific value sometimes assigned to a jiffy is $\frac{1}{100}s$. You just told someone you would be there "*in a jiffy*" but it really took you 5 minutes. How many jiffies was that?

4. Li and Ray shared a vegetarian pizza and a Hawaiian pizza of the same size. The vegetarian pizza was cut into eight equal slices. The Hawaiian pizza was cut into six equal slices. Li ate two slices of the vegetarian pizza and one slice of the Hawaiian pizza. Ray ate two slices of the Hawaiian pizza and one slice of the vegetarian pizza.

a) Who ate more pizza?

b) How much more did that person eat?

c) How much pizza was left over?

Lesson 6: Dividing Fractions **N9.2**

1. What is division? ____

Example 1: Write a story that would work for the following expressions. Create a model that demonstrates how that division works.

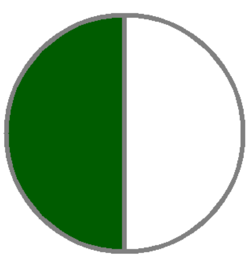
$5 \div 10$	$10 \div 5$
Story:	Story:
Model:	Model:

2. Division of a Fraction by a Whole Number $\frac{a}{b} \div c$

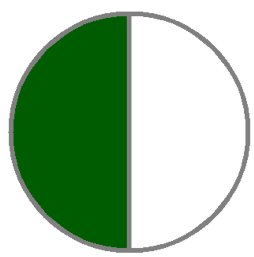
When you divide by a whole number, think of it as dividing something between people. The following pictures show how much pie is left (shaded region). If you are dividing equally between people, how much does each person get? **Write a division expression for each**.

Example 2:

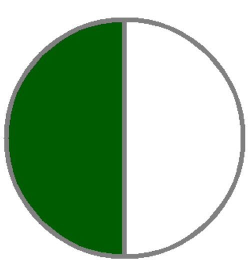
a. Divide between two people



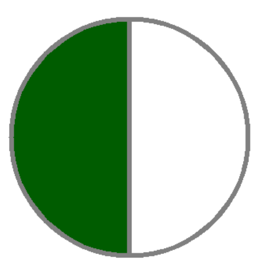
b. Divide between three people



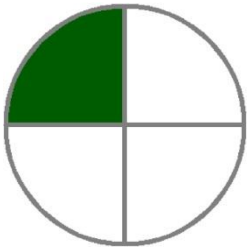
c. Divide between four people



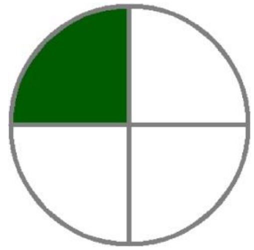
d. Divide between 5 people



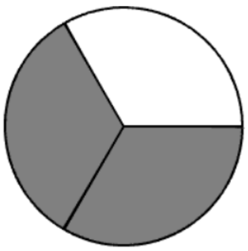
e. Divide between two people



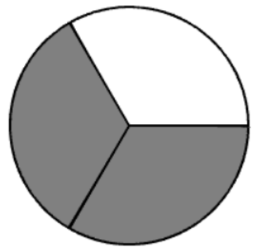
f. Divide between three people



g. Divide between two people



h. Divide between four people



Can you spot a pattern?

$\frac{1}{2} \div 2$	$\frac{1}{2} \div 3$	$\frac{1}{2} \div 4$	$\frac{1}{2} \div 5$	Algorithm:
$\frac{1}{4} \div 2$	$\frac{1}{4} \div 3$	$\frac{2}{3} \div 2$	$\frac{2}{3} \div 4$	

Example 3: Based on your observations, try the following. Draw a model for question d.

a. $\frac{1}{5} \div 2$ b. $\frac{1}{3} \div 3$ c. $\frac{2}{3} \div 2$ d. $\frac{4}{5} \div 4$

3. Division of a Whole Number by a Fraction:

$$c \div \frac{a}{b}$$

When you have a fraction as the second value, it is hard to imagine a fraction of a person. You can also think of division as "how many of the second number fits into the first?"

Example 4: Using the idea of rectangles or pies, calculate the following

a. $1 \div \frac{1}{4}$ b. $2 \div \frac{1}{3}$ c. $3 \div \frac{1}{2}$

d. $2 \div \frac{1}{4}$ e. $2 \div \frac{2}{5}$

Can you spot a pattern?

Example 5: Based on your observations, try the following. Check by drawing a picture that represents the question and answer.

$2 \div \frac{1}{5}$ b. $3 \div \frac{1}{3}$ c. $2 \div \frac{2}{3}$ d. $4 \div \frac{4}{5}$

4. Division of a Fraction by a Fraction

A. Bigger Value ÷ Smaller Value

Example 6: Write a story that would work for the following expressions. Create a model that demonstrates the division statement.

a. $\frac{1}{2} \div \frac{1}{4}$

Story:	Model:

b. $\frac{7}{8} \div \frac{2}{3}$

Story:	Model:

Finding a common denominator is a very useful skill for division of fractions. If you can put both fractions into the same increments, it makes it a lot easier to model. Let's try the last example with a common denominator.

$$\frac{7}{8} \div \frac{2}{3}$$

Model:

Example 7: Model the following expressions with rectangles and evaluate.

a. $\frac{3}{4} \div \frac{1}{8}$ b. $\frac{2}{3} \div \frac{1}{2}$ c. $\frac{5}{4} \div \frac{2}{3}$

B. Smaller Value ÷ Larger Value

Example 8: What happens when we reverse the numbers? Use the idea of rectangles or pies to calculate the following.

a. $\frac{1}{8} \div \frac{3}{4}$ b. $\frac{2}{3} \div \frac{5}{4}$ c. $\frac{1}{2} \div \frac{2}{3}$

Can you spot a pattern?

$\frac{3}{4} \div \frac{1}{8}$	$\frac{2}{3} \div \frac{5}{4}$	$\frac{2}{3} \div \frac{1}{2}$
$\frac{1}{8} \div \frac{3}{4}$	$\frac{2}{3} \div \frac{5}{4}$	$\frac{1}{2} \div \frac{2}{3}$

What is the pattern?

Algorithm

Some division statements are hard to model (numbers are just too big or they have negative values). So an algorithm is used. This way you can compute the answer without having to draw a model.

Example 9: Let's add in some mixed numbers and negative signs. You will not be able to model negative fractions, so use the division algorithm.

a. $-1\frac{2}{3} \div \frac{5}{6}$

b. $-\frac{2}{3} \div \left(-2\frac{2}{5}\right)$

Example 10: Sara has $2\frac{1}{2}$ cups of chocolate chips to make cookies. The recipe uses $\frac{1}{3}$ cup of chips in each batch. How many batches of cookies can Sara make?

a) Model the problem.

b) Write an expression and show how to solve the problem.

Lesson 6: Assignment **N9.2**

1. For the following division statements
   a. Write out what each statement means in words
   b. Model and answer the statement WITHOUT using the division algorithm

a. $2 \div \frac{2}{3}$

b. $\frac{2}{3} \div 2$

c. $\frac{3}{4} \div \frac{1}{2}$

d. $\frac{1}{2} \div \frac{3}{4}$

2. Simplify, leaving your answers in lowest terms and as improper fractions, if applicable. You may invert and multiply OR find a common denominator and divide.

a. $\frac{3}{5} \div \frac{7}{8}$

b. $\frac{1}{8} \div \frac{1}{2}$

c. $\frac{9}{10} \div \frac{7}{5}$

d. $\frac{7}{10} \div \left(-\frac{4}{9}\right)$

e. $-\frac{5}{8} \div \frac{3}{4}$

f. $-\frac{1}{5} \div \left(-\frac{8}{15}\right)$

g. $-\frac{2}{3} \div \frac{5}{7}$

h. $\left(-1\frac{7}{8}\right) \div \left(-1\frac{1}{3}\right)$

i. $2\frac{2}{3} \div \frac{3}{4}$

j. $-5 \div \frac{2}{3}$

k. $\frac{2}{3} \div 5$

l. $24 \div \frac{3}{2}$

3. Sanjay works for the SPCA and has to buy food for the dogs. She bought $6\frac{1}{2}$ pounds of dog food. She feeds each dog about one-third of a pound. How many dogs can she feed with one bag?

   a) Model the problem.

   b) Write an expression and show how to solve the problem.

B E D M A S

Example 1: Simplify $-\frac{9}{4} \times \left(-\frac{10}{21}\right) \div \left(\frac{45}{7}\right)$

Example 2: Simplify $\frac{2}{3} + \left(-\frac{1}{4}\right) - \left(\frac{-5}{6}\right)$

Example 3: Simplify $\left(-\frac{5}{6} + \frac{2}{3}\right) \times \left(\frac{3}{4}\right) \div \left(-\frac{5}{6}\right)$

1. Simplify

a. $-\frac{2}{3}+\frac{1}{4}-\left(\frac{-5}{6}\right)$

b. $\frac{3}{2}-\left(\frac{3}{8}\right)-\frac{3}{4}$

c. $-\frac{7}{2}+1\frac{1}{3}-\left(-\frac{5}{6}\right)$

d. $\frac{5}{9}+\frac{2}{3}+\left(-\frac{7}{6}\right)$

2. Simplify

a. $\left(\frac{4}{9}\right)\times\left(\frac{-21}{-32}\right)\times\left(\frac{-3}{14}\right)$

b. $\left(\frac{3}{4}\right)\left(\frac{8}{5}\right)\left(\frac{20}{9}\right)$

c. $\left(-\frac{4}{9}\right)\div\left(\frac{5}{6}\right)\times\frac{3}{10}$

d. $\left(\frac{15}{8}\right)\div\left(\frac{25}{16}\right)\div\left(-\frac{6}{5}\right)$

3. Simplify

a. $\left(\frac{5}{6}+\frac{2}{3}\right)\times\frac{4}{9}$

b. $\frac{7}{8}\left[\frac{4}{3}-\left(-\frac{5}{18}\right)\right]$

c. $\frac{4}{5}\times\left[\frac{3}{8}+\left(-\frac{4}{7}\right)\right]$

d. $\left(-\frac{5}{6}+\frac{2}{3}\right)\times\frac{3}{4}\div\left(-\frac{5}{6}\right)$

e. $\frac{3}{5}+\left(-\frac{2}{3}\right)\times\left[-\frac{3}{4}\div\left(-\frac{1}{2}\right)\right]$

f. $\left[\frac{7}{12}\div(-14)\right]-\frac{3}{8}\times\frac{5}{3}$

4. Which student correctly answered the following question? Find the student's mistake and give him feedback on his/her error.

$$\frac{3}{4} - \left(-\frac{1}{2}\right)\left(\frac{5}{8}\right) \div \left[\left(-\frac{1}{4}\right)\left(-\frac{1}{4}\right)\right]$$

Student #1

$$\frac{3}{4} - \left(-\frac{1}{2}\right)\left(\frac{5}{8}\right) \div \left[\left(-\frac{1}{4}\right)\left(-\frac{1}{4}\right)\right]$$

$$\frac{3}{4} - \left(\frac{-5}{16}\right) \div \frac{1}{16}$$

$$\frac{3}{4} + \frac{5}{16} \div \frac{1}{16}$$

$$\frac{17}{16} \div \frac{1}{16}$$

$$17$$

Student #2

$$\frac{3}{4} - \left(-\frac{1}{2}\right)\left(\frac{5}{8}\right) \div \left[\left(-\frac{1}{4}\right)\left(-\frac{1}{4}\right)\right]$$

$$\frac{3}{4} - \left(\frac{-5}{16}\right) \div \frac{1}{16}$$

$$\frac{3}{4} - \left(\frac{-5}{16}\right) \times \frac{16}{1}$$

$$\frac{3}{4} - (-5)$$

$$\frac{3}{4} + 5$$

$$\frac{23}{4}$$

Working With Decimals

A. Addition & Subtraction

What do you remember?

$0.28 + 3.2 + 2.339$		
$0.6 - 0.22$		

B. Multiplication

What do you remember?

$(0.03)(2.5)$		

C. Division

What do you remember?

$\frac{0.64}{0.002}$		

D. Order of Operations with Decimals

Example 1: $(2.3)(-.003) - (2.1)^2$

Example 2: $4 \div 0.2 - 8(0.25)$

Example 3: Find the error in the student's work. Give the student feedback on their error.

$$0.24 \div (-0.6) \div 2(12)$$
$$-0.4 \div 2(12)$$
$$-0.2(12)$$
$$-2.4$$

Correctly answer the question here:

1. Evaluate. Do not use a calculator.
   a. $2.2 - 4.32 - 6.5 + 3.45$
   b. $2.4 - (-5.5) \times (0.3)$
   c. $(-12.3) \div (-0.3) - 2.5 \div (-0.5)$
   d. $2.5 - 6.2 + 7.8 \div 0.2$

2. Evaluate using a calculator.
   a. $[-3.8 + (-0.9)] \times [7.2 - 4.7]$
   b. $\frac{79.12}{9.2}(-2.18 + 5.27)$
   c. $(-4.91) \times (-3.78) + \left(\frac{50.827}{-6.85}\right)$
   d. $(5.4)(-0.07) - (1.2)^2 \div (-0.3)$

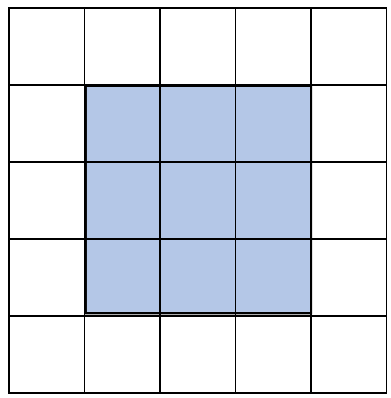
N9.3 Square Roots

Lesson 1: What is a Square Root?

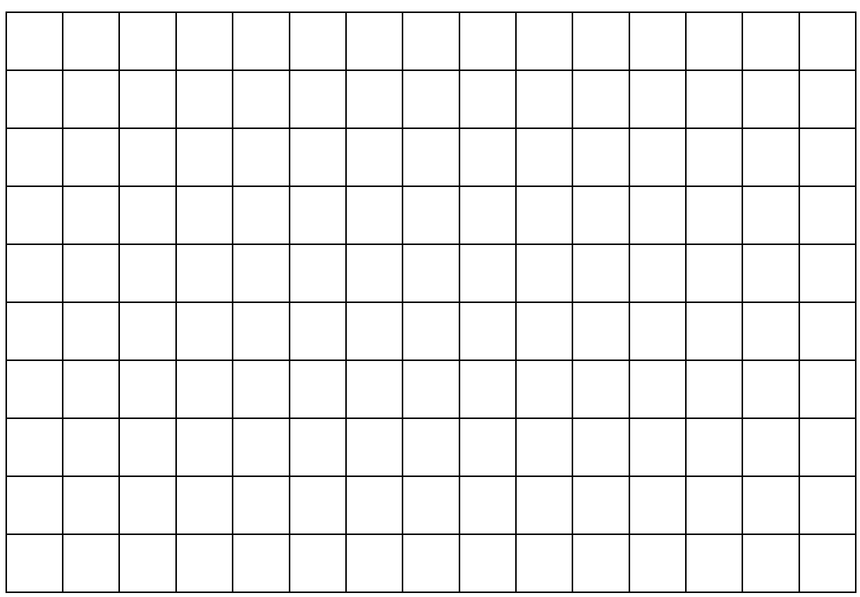
Terminology & Review

A. Square Roots & Terminology

Example 1: Explain how the shaded are in the diagram represents $\sqrt{9}$. What else can you tell me about the diagram?



Example 2: Draw a diagram that represents $\sqrt{16}$



Example 3: On the grid above, draw a diagram that represents an approximation of $\sqrt{12}$

1. Perfect Square: ____

____

2. Square Root: ____

____

3. Non-perfect Square: ____

____

4. Surface Area: ____

____

5. Pythagorean Theorem: ____

____

6. Rational Number: ____

____

B. Square Roots of Perfect Squares

Example 4: Complete the following chart.

Square Root	Square of the Number	Perfect Square
1	1^2	1
2	2^2	
3		
4		
5		
6		36
7		
8		
9		
10		
11		
12		
$\frac{5}{8}$		
1.2		

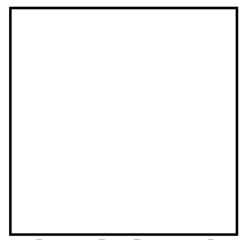
The last two are new questions for you!

D. Square Roots and Surface Area

Square roots and the surface area of a square are related. Let's investigate!

Example 5: Find the area of the **square** below. Show how you got your answer numerically AND pictorially.

4cm



Example 6: Construct a square with side lengths of 6 cm. Calculate the surface area. Display your answer numerically AND pictorially.

Example 7: Calculate $\sqrt{36}$

How are square roots and the surface area of a square related?

Example 8: Provide 4 examples of perfect squares not already listed in the chart from example 3. How did you create them (you don't need to draw them)?

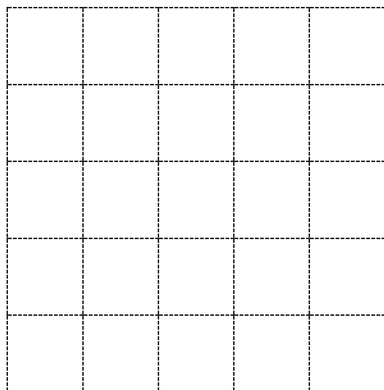
Square Roots of Fractions and Decimals

A. Square Roots of Fractions

Some fractions can also be perfect squares. Up until now, you have worked primarily with positive whole numbers (positive integers). If we can represent the area using squares, then it is a perfect square.

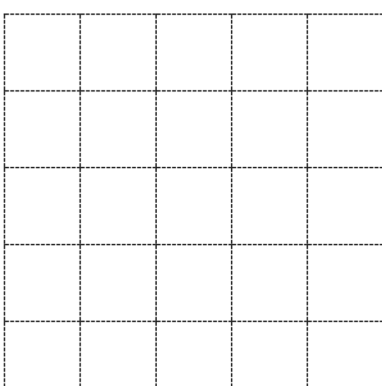
Example 1: a. Using a rectangular model, draw what $\frac{3}{4}$ looks like.

b. Using the grid below, model what $\left(\frac{3}{4}\right)^2$ looks like as a square.

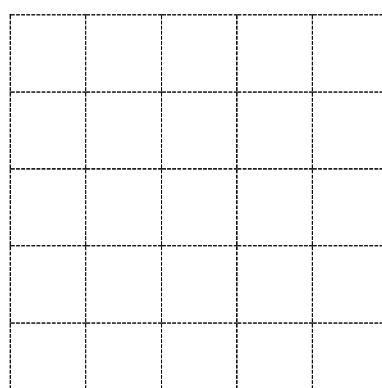


c. What is $\left(\frac{3}{4}\right)^2$? How is that represented in the above diagram?

Example 2: Model $\left(\frac{2}{3}\right)^2$



Example 3: Model $\left(\frac{1}{2}\right)^2$



When squaring fractions, how can we get the result without having to model the fraction?

____

____

$$\left(\frac{a}{b}\right)^2 =$$

Now that you know how to SQUARE a fraction, let's figure out how to determine if a fraction is a PERFECT SQUARE.

Example 4: Circle the following fractions that ARE perfect squares

$\frac{4}{9}$ $\frac{2}{3}$ $\frac{9}{16}$ $\frac{1}{2}$ $\frac{6}{10}$ $\frac{8}{50}$ $\frac{36}{49}$ $\frac{24}{16}$ $\frac{1}{25}$ $\frac{144}{225}$ $\frac{36}{99}$

How do we determine if a fraction is a perfect square?

____

Example 5: Calculate (if possible)

a. $\sqrt{\frac{16}{49}}$ b. $\sqrt{\frac{12}{25}}$ c. $\sqrt{\frac{1}{81}}$ d. $\sqrt{\frac{16}{144}}$

B. Square Roots and Decimals

Did you know that decimals can be perfect squares too? How can we build a perfect square with decimals? Let's square a few decimals and find out!

Example 6: Calculate the following

a. $(0.2)^2$ b. $(1.6)^2$ c. $(1.2)^2$ d. $(.005)^2$

Example 7: Create your own

a. b. c. d.

Conceptual Learning Task:

Perfect Square:	Non-Perfect Square:

How do we recognize a decimal that may be a **perfect** square?

Example 8: Find the square root of the following

a. $\sqrt{400}$

b. $\sqrt{40}$

c. $\sqrt{4}$

d. $\sqrt{0.4}$

e. $\sqrt{0.04}$

f. $\sqrt{0.004}$

g. $\sqrt{0.0004}$

All of the above questions contain a 4 (which is considered to be a perfect square). Why aren't all of the questions above perfect squares then?

___

___

How are we going to be able to recognize a decimal that is a perfect square without a calculator?

___

___

Example 9: Which decimal is a perfect square, 8.1 or 0.81?

Assignment: Lesson 2 **N9.3**

1. Which decimals are perfect squares? You ARE NOT allowed to use a calculator! Circle the ones that are perfect squares.

a. 0.18 b. 0.4 c. 3.6 d. 0.36 e. 1.25 f. 2.25 g. 0.225

2. Which fractions are perfect squares? You ARE NOT allowed to use a calculator! Circle the ones that are perfect squares.

a. $\sqrt{\frac{1}{63}}$ b. $\sqrt{\frac{25}{49}}$ c. $\sqrt{\frac{8}{81}}$ d. $\sqrt{\frac{16}{25}}$ e. $\sqrt{\frac{144}{225}}$ f. $\sqrt{\frac{72}{50}}$ g. $\sqrt{\frac{100}{169}}$

3. Calculate the side length of each square from its given area.

a. $900\ m^2$ b. $0.09\ cm^2$

4. Which of the following ARE perfect squares?
   a. For those that are, find the square root.
   b. Explain why the others are NOT perfect squares.

900 90 9 0.9 0.09 0.009 0.0009

5. Calculate the square root of the following WITHOUT A CALCULATOR.

a. $\sqrt{\frac{36}{100}}$ b. $\sqrt{\frac{49}{144}}$ c. $\sqrt{\frac{1}{81}}$ d. $\sqrt{\frac{9}{25}}$

6. Calculate the square root of the following WITHOUT A CALCULATOR.

a. 0.01 b. 0.25 c. 1.69 d. 0.04

Square Roots of Non-Perfect Squares

A. What is a Non-Perfect Square Root?

Perfect Squares		
Whole Numbers	Fractions	Decimals
Non-Perfect Squares		
Whole Numbers	Fractions	Decimals

What about repeating decimals, are they perfect or non-perfect squares? i.e. $0.\overline{3}$

B. Estimating Non-Perfect Squares of Whole Numbers, Decimals, and Fractions

i. Whole Numbers

We need to use benchmarks to find estimates of non-perfect squares. The benchmarks will be two perfect squares that sandwich your number.

Example 1: Without using a calculator, estimate the value of $\sqrt{30}$

Example 2: Without using a calculator, estimate the value of $\sqrt{18}$

Example 3: Between which two perfect squares would you place each value? Don't use a calculator.

Root	$\sqrt{0}$	$\sqrt{1}$	$\sqrt{4}$	$\sqrt{9}$	$\sqrt{16}$	$\sqrt{25}$	$\sqrt{36}$	$\sqrt{49}$
Square	0	1	2	3	4	5	6	7

a. $\sqrt{6}$ b. $\sqrt{14}$ c. $\sqrt{27}$ d. $\sqrt{45}$

ii. Decimals

Let's use the idea of benchmarks from above and extend it to decimals.

Example 4: Without using a calculator, estimate the value of $\sqrt{13.7}$

Example 5: Without using a calculator, estimate the value of $\sqrt{1.52}$

iii. Fractions

Let's use the idea of benchmarks again to help us with fractions.

Example 6: Without using a calculator, **estimate** the value of $\sqrt{\frac{10}{47}}$

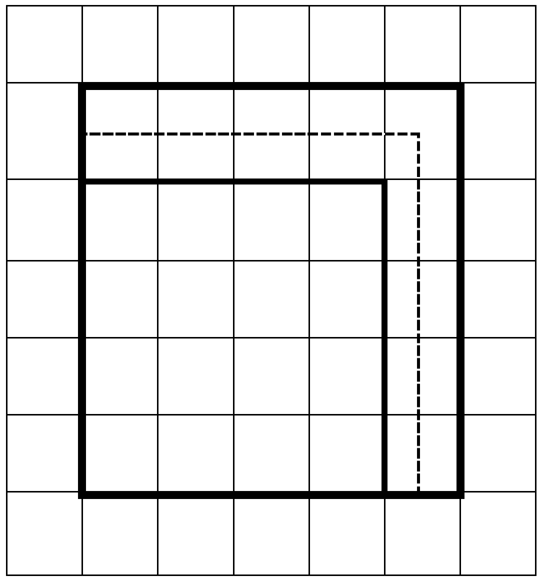
Example 7: Using a calculator, **calculate** the value of $\sqrt{\frac{10}{47}}$

Example 8: Without using a calculator, **estimate** the value of $\sqrt{\frac{30}{12}}$

Example 9: Using a calculator, **calculate** the value of $\sqrt{\frac{30}{12}}$

Assignment: Lesson 3 **N9.3**

1. Use the diagram to identify a rational number with a square root between 4 and 5 (dashed line). Using the diagram, what is the square of that number? Mark on the diagram how you found it. Check with a calculator.



2. Identify the rational number which has a square root of

a. 0.22 b. 0.5 c. $\frac{5}{8}$ d. $\frac{1}{2}$ e. $\frac{1}{6}$

3. Show using benchmarks how you would estimate the square of 4.3.

4. Show using benchmarks how you would estimate the square root of 4.3.

5. What makes question #3 and #4 different because they look the same? Did I accidently type out the same question twice, what is going on here?

6. What would be the area of a square that has side lengths of $5.2m$?

7. Estimate each square root to one decimal place. Then, calculate it to the specified number of decimal places.

	Estimate to 1 decimal place (nearest tenth)	Calculate to 1 decimal place (nearest tenth)
a. $\sqrt{42}$		
b. $\sqrt{2.5}$		
c. $\sqrt{0.96}$		
d. $\sqrt{0.82}$		

8. If the area of the square is $1.96\ m^2$, what are the side lengths? Round your answer to two decimal places (nearest hundredth).

9. Use any strategy (besides a calculator) to estimate the value of each square root. You must explain and write out the strategy you used.

   a. $\sqrt{6.8}$ b. $\sqrt{\frac{8}{32}}$ c. $\sqrt{\frac{60}{27}}$

10. Calculate the value of each square root to 2 decimal places (nearest hundredth)

   a. $\sqrt{6.8}$ b. $\sqrt{\frac{8}{32}}$ c. $\sqrt{\frac{60}{27}}$

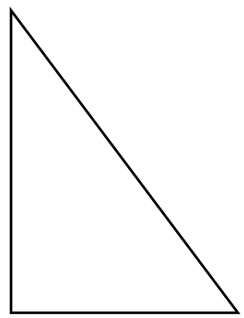
What does this mean?

Applications of Square Roots

A. Pythagorean Theorem

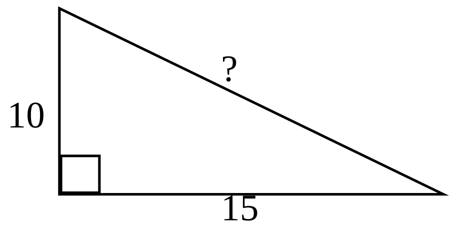
Pythagorean Theorem is a rule which states that, for any right triangle, the **area of the square** on the hypotenuse is equal to the **sum of the area of the squares** on the other two sides (legs).

What does that look like?

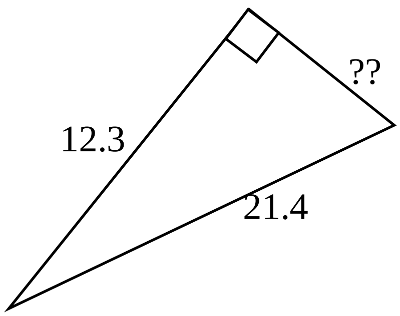


Pythagorean Theorem:____________________

Example 1: Use the Pythagorean Theorem to calculate the missing side of the right triangle.



Example 2: Use the Pythagorean Theorem to calculate the missing side of the right triangle.



B. When Would I use Pythagorean Theorem?

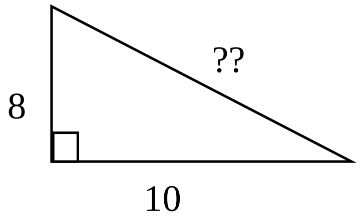
Example 3: Your TV just died and you need to purchase a new one. The spot on your entertainment system will fit a TV that has a width of 55 inches and a height of 33 inches. What is the maximum size of TV you can buy? Note: All TV's are listed by their diagonal size. Ex: A 32" TV has a diagonal of 32".

Example 4: A 20 foot ladder is leaning against a wall. If the base of the ladder is 2.5 feet away from the wall, how far does the ladder reach up the wall?

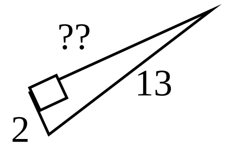
Assignment: Lesson 4 **N9.3**

1. Calculate the missing sides from each of the triangles. Show all your work, including the Pythagorean Theorem.

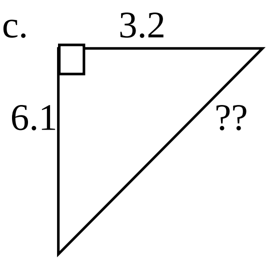
a.



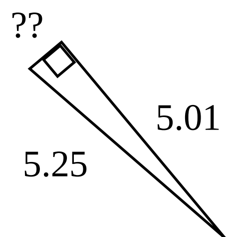
b.



c.



d.



2. If a 22 foot ladder is 3 feet from the base of the house. How far does the ladder reach up to the top of the house?

3. If you need to buy a ladder to reach up 18 feet and it needs to be placed at least 2.5 feet from the base of the house, what size of ladder do you need?

4. The length of the hypotenuse on an isosceles right triangle is 15. What are the lengths of the legs?

5. You are wanting to put a flat screen TV above your fireplace at home. You measured the dimensions to be:

Width:	44.70 inches
Height:	26.00 inches

   a. What is the maximum size of television you can get? Round your answer to two decimal places.

   b. Most TV's list themselves as 30" or 60". They don't have decimal places. Without decimals, what is the largest size of TV you can fit in space? Does it make sense to round your answer up or down?

P9.4 Polynomials

Lesson 1: Terminology **P9.4**

Translating English to Math and Math to English

Math	English
$+$	
$-$	
X	
$\div$	
$=$	
x, y, z, a, b, c,…	
$5x$	
$p-9$	
$r+3$	
$5x-10=15$	
x^2	

Translating English to Math and Math to English

A. Translate the following from "math" to English.

1. $+$
2. -5
3. $x-2$
4. $2c+6$
5. $(-3)(p)$
6. $3r=21$
7. $5=2v-8$

B. Translate from English to "math."

1. A certain number.
2. Three plus a number.
3. Five times an unknown number.
4. Six minus a number.
5. Increase a number by ten.
6. A number divided by 5 is equal to 8
7. A number plus 7 is equal to a different number.

C. Find the mathematical statement associated with each of the following situations.

1. A friend of mine is three years older than I am.
2. The skateboard costs five dollars less than I thought it would.
3. Three bags of apples cost twelve dollars.
4. The cost to order 5 books plus seven dollars of shipping is seventy-eight dollars.

Terminology & Like Terms

1. Term: ____

Number of Terms	Examples
1	
2	
3	
4	

2. Polynomial: ____

   a. Monomial: ____

   b. Binomial: ____

   c. Trinomial: ____

Example 1: Classify the following polynomials.

a. 5 ____________________

b. $2x^3 - 2x + 4$ ____________________

c. $5x^2y^3$ ____________________

d. $2x$ ____________________

e. $9a^2 - 5b$ ____________________

f. a ____________________

3. Degree of a Term: ____

Degree of a Polynomial: ____

Example 2: State the degree of the following polynomials.

a. 5	b. $2x^3 - 2x + 4$	c. $5x^2y^3$
____________	____________	____________
d. $2x$	e. $9a^2 - 5b$	f. a
____________	____________	____________
g.	h.	i.
____________	____________	____________

4. Coefficient: ____

____

Example 4: Circle the coefficient(s) in the following polynomials. List the numbers in the space provided.

a. 5	b. $2x^3 - 2x + 4$	c. $5x^2y^3$
____________	____________	____________
d. $2x$	e. $9a^2 - 5b$	f. a
____________	____________	____________

5. Constant Term: ____

____

Example 5: Circle the constant term in the following polynomials.

a. 5	b. $2x^3 - 2x + 4$	c. $5x^2y^3$
____________	____________	____________
d. $2x$	e. $9a^2 - 5b$	f. a
____________	____________	____________

6. Variable: ____

Example 6: Circle the Variable(s) in the following polynomials.

a. 5 b. $2x^3 - 2x + 4$ c. $5x^2y^3$

________________   ________________   ________________

d. $2x$ e. $9a^2 - 5b$ f. a

________________   ________________   ________________

7. Like Terms: ____

____

Example 7: Circle the polynomials that are like terms with $-3x^2$

$-3x$ x^2 $2x^2y$ $12x^2$ $4ab$ $-7x^2$ $-x^2$

Example 8: Circle the polynomials that are like terms with $4ab$

$-ab$ $2a^2b$ $4a^2b^2$ $4a$ $2ab$ $4b$ $-6ab$

Example 9: Circle the polynomials that are like terms with $5x$

$-3x$ x $2x^2$ $12x^3$ $4x$ $-7x^1$ $-x^2$

Example 10: List 5 other like terms to $2y^2$

__________   __________   __________   __________   __________

Example 11: List 5 other like terms to $-5xy$

__________   __________   __________   __________   __________

Example 12: Given $3x^2 + 5x - 6xy + y - 12$ answer each of the following

a. What is/are the constant(s)?

b. What is/are the variable(s)?

c. Is this an expression or an equation?

d. Write out each term.

e. How many terms are there?

f. Can we classify this as a trinomial, binomial, or monomial? Why?

g. Identify the coefficient(s)

h. Which term(s) have the highest degree?

i. What is the degree of this polynomial?

j. Write this polynomial in ascending order.

Example 13: Complete the table below.

Polynomial	Number of Terms	Type of Polynomial	Degree of Polynomial	Constant	Variable(s)	Numerical Coefficient(s)
$x^2 + 3x + 1$						
$3y^2 - 4m^3$						
$2a^2b^3c^4$						
		Trinomial	5			
	4				m,n,p	

Lesson 2: Group Task (Assignment) **P9.4**

1. Write 3 expressions which are NOT polynomials

a. ____________________ b. ____________________ c. ____________________

2. Give an example of the following polynomials

   a. Trinomial; degree 3 ____________________

   b. Monomial; degree 4 ____________________

   c. Binomial; degree 1 ____________________

3. For the polynomial $5x^2 - 3x - 2$ mark the statements as true or false. Be prepared to state why.

   a. The degree of the polynomial is 3. ___________

   b. The degree of the polynomial is 2 ___________

   c. The coefficient of x^2 is 5 ___________

   d. The coefficient of x is 3 ___________

   e. The constant term is -2 ___________

4. Justify the following statements with examples or counter examples:

   a. We can have a trinomial having a degree 7 ______________________

   b. The degree of a binomial cannot be more than 2 ______________________

   c. A monomial must have a degree of 1 ______________________

5. Complete the entries $-4x^2 + 2xy - y^2 + 4$

   a. Coefficient of x^2 is … ___________

   b. Coefficient of y^2 is … ___________

   c. Degree of the polynomial is … ___________

   d. Constant term … ___________

   e. Number of terms … ___________

Lesson 2: Assignment **P9.4**

1. Complete the following chart.

Polynomial	Number of Terms	Type of Polynomial	Degree of Polynomial	Constant	Variable(s)	Numerical Coefficient(s)
x^2y^3z						
$2x^3 + 3r^3 + 2$						
$2a^2 + 4$						
		Trinomial	3			
	5				w, x, y, z	

2. Write a polynomial that satisfies all of the following statements.

- Is a trinomial
- Has two variables
- Degree of 3
- Has a constant term

3. For each expression, identify the number of terms and whether the expression is a monomial, binomial, trinomial, or polynomial.

a. $9x^2 - 3x + 6$ b. -3 c. $2x - 3$ d. $5r^2 - 3rg + 8dr - g^2$

4. From the list, which terms are like $-7x^2$?

$7x^2 \quad 7x \quad 6x^2 \quad -7 \quad -5 \quad -7x \quad -3x^2$

5. From the list, which terms are like $5x$?

$5x^2 \quad 4x \quad 3 \quad -8x \quad -5x \quad 9x^2 \quad 5 \quad x \quad -2$

Modelling Polynomials with Algebra Tiles

A. Review of Terminology

Example 1: A polynomial must be one term or the sum or difference of terms whose variables have POSITIVE WHOLE number exponents. An expression that contains a term with a variable in the denominator such as $\frac{3}{n}$, or the square root of a variable, such as $\sqrt{p}$ is NOT a polynomial. Which are polynomials? (WHY or WHY NOT?)

a) $5 + 6x$ b) $\frac{1}{x^2} + \frac{5}{2x} - 1$ c) $\frac{1}{3}x$ d) $5\sqrt{x}$ e) 11

Example 2: Name the coefficient, variable, constant, and degree of each monomial

	$-2x^2$	f	$3x^2 + 2x - 10$	$9f^3 - 8f^2 + 12$
Degree				
Variable				
Coefficient				
Constant				

In your own words, what is a like term? How can you tell?

Example 3: Circle all the terms below that are like terms with $3x$.

$2x$ x^2 $3y$ x xy $3xy$ p^2 x^3 $8x$ $2308x$

Example 4: Write three terms that are like $5xy^2$.

_______________ _______________ _______________

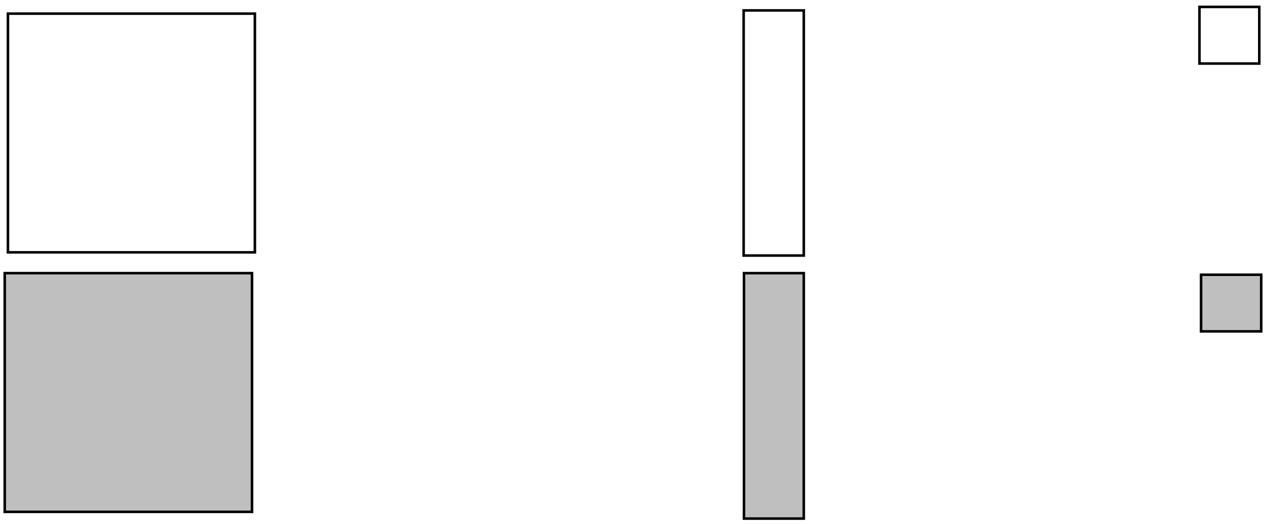
Example 5: We know that $-8m$ and $7m$ are like terms.

a. What does the -8 in $-8m$ tell us? b. What does the 7 in $7m$ tell us?

c. Explain why these are like terms.

B. Algebra Tiles

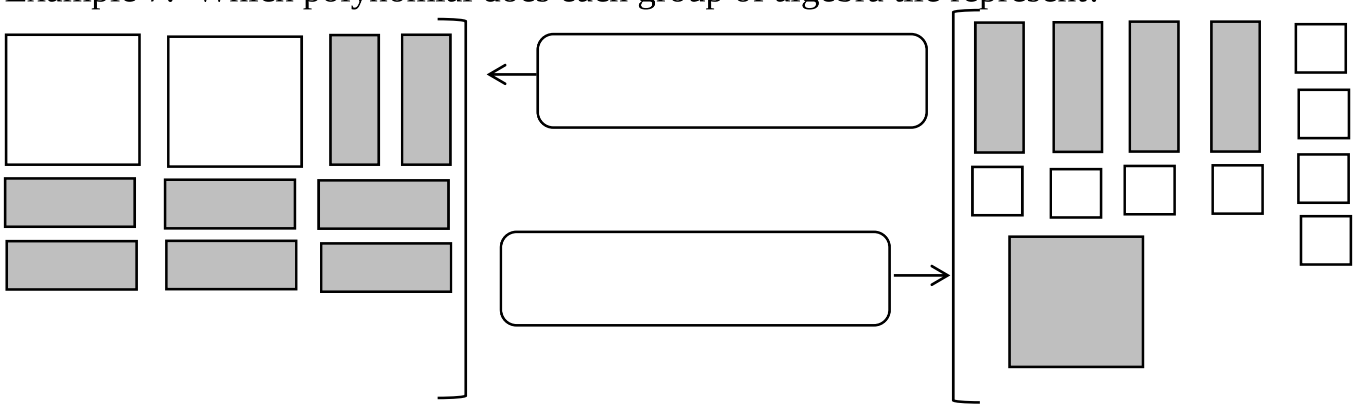
- When we are first working with polynomials, we often use algebra tiles to help us with like terms.
- Unfortunately, they only work for polynomials with one variable up to a degree of 2.
- You may see a variety of colours when working with algebra tiles.
- When drawing your own algebra tiles, shade for negative and leave white for positive.



Example 6: Use algebra tiles to model each polynomial.

a. $-3x^2$ b. $4b^2 - b + 3$ c. $6a - 3$

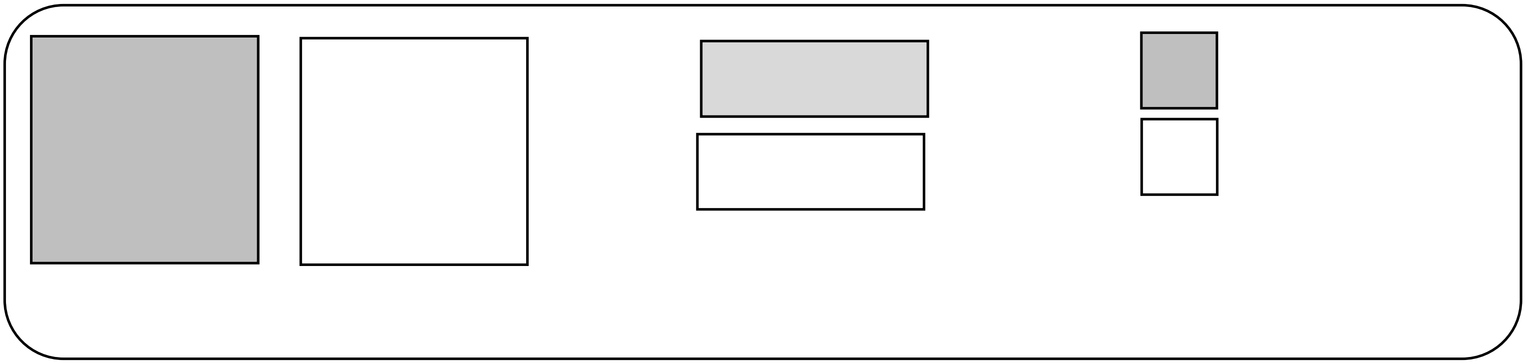
Example 7: Which polynomial does each group of algebra tile represent?



C. Zero Pair

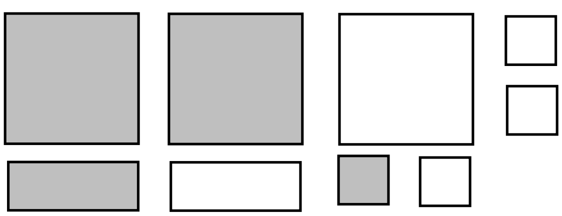
Zero Pair: ____

Examples of Algebra Tile Zero Pairs

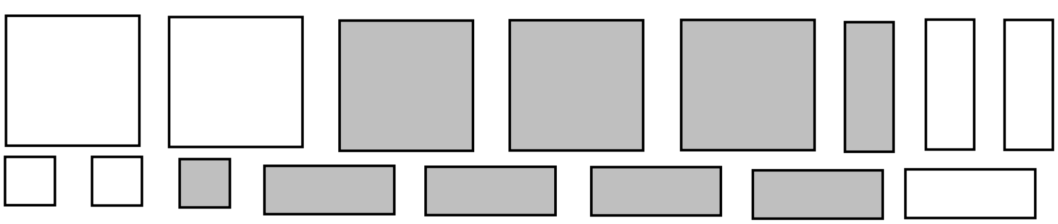


Examples of **Numerical** Zero Pairs | Examples of **Polynomial** Zero Pairs

Example 8: Write the **simplified** polynomial represented by the following algebra tiles.



Example 9: Write the **simplified** polynomial represented by the following algebra tiles.



D. Using Algebra Tiles for Combining Like Terms

Algebra tiles that are the same size and shape are **like terms**.
Example 10: Use algebra tiles AND the algebraic method to simplify the following polynomials.

a. Simplify $4n^2 - 1 - 3n - 3 + 5n - 2n^2$

Tile Model	Algebraic Method

b. Simplify $14x^2 - 11 + 30x + 3 + 15x - 25x^2$

Tile Model	Algebraic Method

c. Simplify $3xy - y^2 - 5x^2 + 3xy - x - 4y^2$

~~Tile Model~~	Algebraic Method
Why can't we use tiles for this polynomial?	

Lesson 3: Assignment **P9.4**

1. Which of the following expressions are polynomials? For the ones that are not, circle the problem area(s) of each polynomial.

   a. $6 + 3d$ b. $\sqrt{3} + x$ c. $\sqrt{x} + 3$

   d. $\frac{1}{x^2} - \frac{2}{x} + 5$ e. $\frac{2}{3}x^2 + 4x$ f. 0

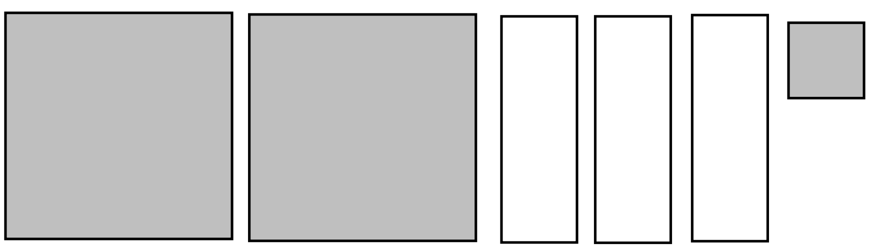
2. Use algebra tiles to model each polynomial (if possible). Sketch the tiles. Shade in for negatives.

   a. $5x^2 - 2x + 1$ b. $2b^2 + 3$ c. $-3f$

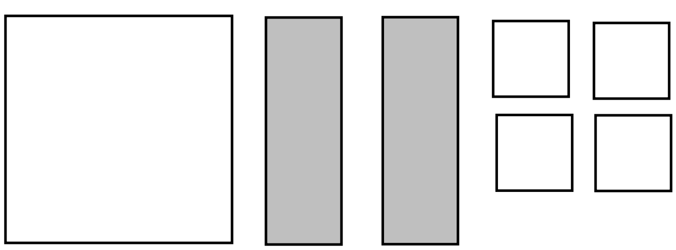
   d. 2 e. $-2d^2 + d - 2$ f. $5xy - 6zx$

3. Which polynomial does each collection of algebra tiles represent?

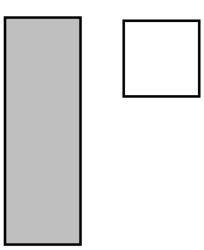
   a.



   b.



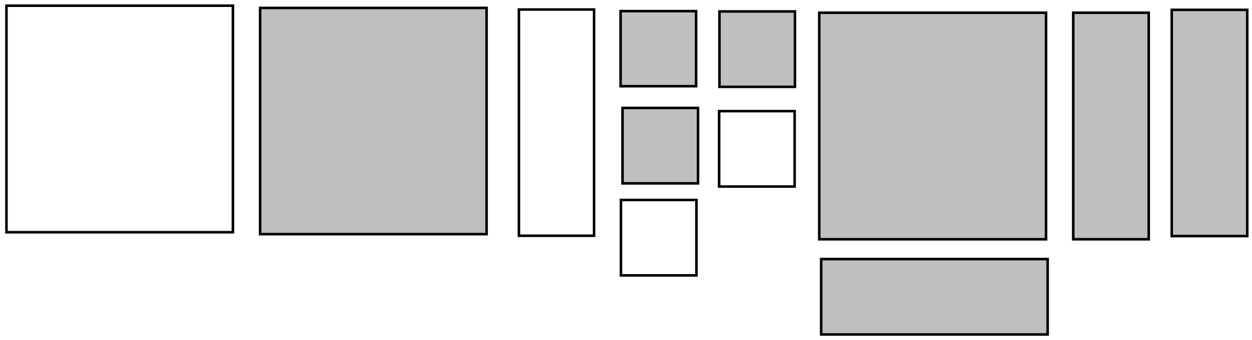
   c.

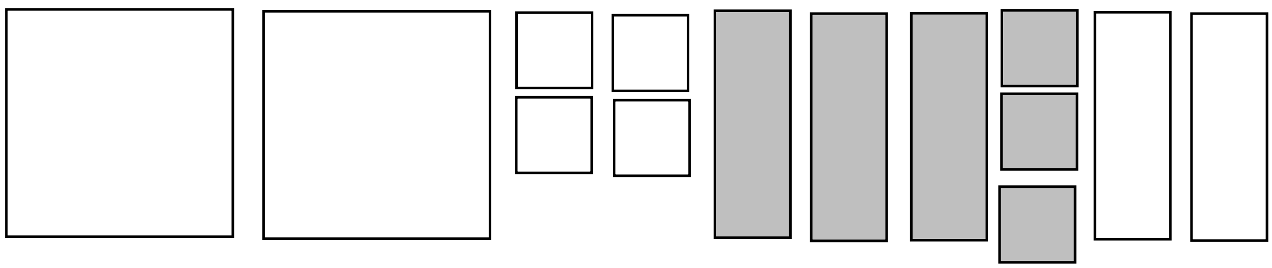


4. From the list, which terms are like $-2x$? (circle all that apply)

$$-2x^2 \quad -2 \quad 8x \quad x \quad -11x^2 \quad 2x \quad 2x^2$$

5. Write the ***simplified*** polynomial represented by the following algebra tiles.

a. 

b. 

6. Use algebra tiles and **simplify the polynomials**.

a. $3a + 5 + 4a + 2$

b. $2x^2 - 3x + 5x + 6x^2$

c. $2b^2 - 3b + 5 - 3b^2 + 4b + 2$

d. $-2a^2 - 3a^2 + 7a - 2 - 2a - 4$

7. Simplify each polynomial **without** algebra tiles.

a. $5x - 9 + x^2 - 3x + 3$

b. $3x^2 + 9x - 2 + 2x^2 - 3x + 3$

c. $6x^2 - 3x + 4x - 6 - 10x^2 + 5x - 3$

d. $6r - 4r - 3 - 6$

e. $2x^2 + 5y - x^2 - 3y + 1$

f. $4r^2 + rs - 7 + s^2 + 6rs - 12$

g. $3x^2 - 7 + 2x - 3x^2 + 7 - 2x$

h. $r + s - 1 - 6rs + 5s + 2r - 8$

Adding Polynomials

In order to add polynomials, you need to combine like terms. We are going to develop a strategy to add polynomials with and without using algebra tiles.

Example 1: Add the following trinomials together.

$$2x^2 - 3x + 4 \text{ and } -4x^2 - x + 2$$

Model	Algebraic

Brackets are used to group polynomials. When there is a "friendly" addition sign between the brackets, you can distribute (multiply) the $+$ sign through the second bracket. But we all know that multiplying a $+$ sign through doesn't change anything.

Example 2: Add the following integers

a. $(5) + (3)$ b. $(-5) + (-3)$ c. $(-5) + (3)$ d. $(5) + (-3)$

Polynomials follow the same rule as integers.

Example 3: Add $(2r - 3) + (2r^2 - r - 1)$

Example 4: Add the following polynomials

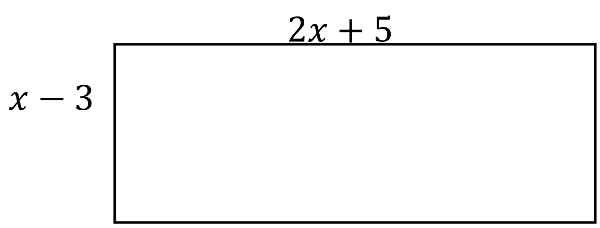
a. $(4x - 6) + (-8x + 11)$

b. $(-x - 7) + (5 - 2x)$

c. $(6k^2 - 2k) + (3 - k + 2k^2)$

d. $(-x^2 + 3 - 7x) + (7 + x^2 + 10x)$

Example 5: Find the perimeter of the following rectangle.



Example 6: A student added $(2x^2 - 3x + 5)$ and $(-3x^2 - x - 1)$ as follows.

$$(2x^2 - 3x + 5) + (-3x^2 - x - 1)$$

$$2x^2 - 3x + 5 - 3x^2 - x - 1$$

$$2x^2 - 3x^2 - 3x - x + 5 - 1$$

$$-x^2 - 3x + 4$$

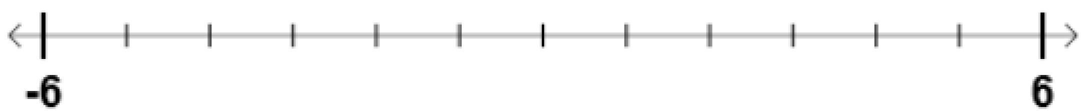
Is the student's work correct?
If not, find the student's mistake and provide feedback so they can recognize their error.

Subtracting Polynomials

Before we start subtracting algebraically, we should review what it means to subtract.

Example 1: Subtract

Method 1: Number Line



a. $(5) - (3)$ b. $(-5) - (3)$ c. $(5) - (-3)$ d. $(-5) - (-3)$

Method 2: Algebra Tiles

a. $(5) - (3)$ b. $(-5) - (3)$ c. $(5) - (-3)$ d. $(-5) - (-3)$

Method 3: Numerically (add the opposite). Adding the opposite is the EXACT same as subtracting! You can represent any subtraction statement as the addition of the opposite. Let's experiment with some subtraction statements and convert them into addition statements. a. $(5) - (3)$ b. $(-5) - (3)$ c. $(5) - (-3)$ d. $(-5) - (-3)$
Method 4: Numerically (distribution). The subtraction sign is representing a -1 that can be multiplied to the value(s) behind it. a. $(5) - (3)$ b. $(-5) - (3)$ c. $(5) - (-3)$ d. $(-5) - (-3)$

We can apply the same strategies when working with polynomials. Let's practice all of the methods.

Example 2: Subtract the following statements

a. $(5x - 2) - (2x + 4)$

Algebra Tiles	Method 3: Add the Opposite	Method 4: Distribute the Sign
$(5x - 2) - (2x + 4)$	$(5x - 2) - (2x + 4)$	$(5x - 2) - (2x + 4)$

b. $(6x^2 - 2x + 3) - (-x^2 + 4x - 2)$

Algebra Tiles	Method 3: Add the Opposite	Method 4: Distribute the Sign
$(6x^2 - 2x + 3) - (-x^2 + 4x - 2)$	$(6x^2 - 2x + 3) - (-x^2 + 4x - 2)$	$(6x^2 - 2x + 3) - (-x^2 + 4x - 2)$

c. $(-2x^2 + 7x - 2) - (3x^3 + 2x - 1)$ Use your own method

d. $(-x^2 + 4x - 2) - (4x^2 + 2x - 5)$

e. $(14y^2 + 3xy + x^2 - 5) - (3xy + 4x^2 + 12y^2 - 10)$

Lesson 4 & 5: Assignment **P9.4**

1. Use algebra tiles to model each difference. Record your answer.

a. $(5x+4)-(6x+7)$

b. $(-5x^2-2x+3)-(-2x^2-3x-1)$

2. Subtract the following without using algebra tiles.

a. $(7x+12)-(4+3x)$

b. $(8u^2-3u)-(u)$

c. $(2x^2+7)-(7x^2-2)$

d. $(7x^2+13x-8)-(17x^2-4x+6)$

e. $(4a+12)-(-2a-2)$

f. $(-3x^2+2x-3)-(5x^2-10x+7)$

g. $(-5x^2-17x+1)-(-3x^2+12x-10)$

h. $(-5x^2-11x+7)-(-8x^2+9x-10)$

i. $(-5a+12b-5c-4d)-(-2a+7b-6c-4d)$

3. Add or subtract as indicated.

a. $(5x - 9) + (-3x - 10)$

b. $(7x - 10) - (2x + 3)$

c. $(-2x^2 - 3x + 4) - (x^2 + 3x + 4)$

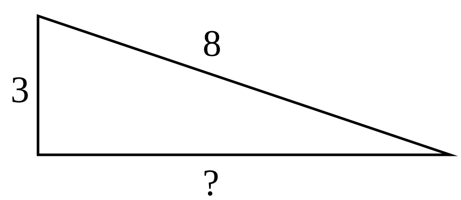
d. $(5x^2 - 3x + 10) + (-5x^2 + 3x - 10)$

e. $(10x - 6) - (10x - 6)$

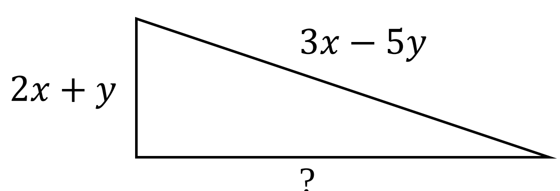
f. $(-3g^2 + 2gh + h^2) + (6gh - 3h^2 + 5g^2)$

g. $(xy - 3y + 2x^2 + 5y^2 - x) - (y^2 + 9x - 4y - 2xy + 12x^2)$

4. If the perimeter of the triangle below is 16, find the length of the missing side.



5. If the perimeter of the triangle below is $7x + 2y$ find the length of the missing side.



Multiplying Polynomials

A. What is Multiplication?

Example 1: Multiply the following integers

a. $(5)(3)$ b. $(-5)(3)$ c. $(5)(-3)$ d. $(-5)(-3)$

Example 2: Model the following multiplication 12×13

B. Multiplying a Polynomial by a Constant

How does multiplication of integers help us with multiplication of polynomials?

Example 3: Model the following multiplication using algebra tiles. $(4)(3x)$

Method #1

Method #2

What happens when we use negative values?

Example 4: Model the following multiplication using algebra tiles. $-3(x-3)$

Method #1

Method #2

What happens when we try to multiply a constant to a degree bigger than 1?

Example 5: $-2(2x^2-3x+1)$

Method #1

Method #2

How do we do the work without using algebra tiles?

Let's answer all of the previous examples using the distributive property.

a. $(4)(3x)$ b. $-3(x-3)$ c. $-2(2x^2-3x+1)$

C. Multiplying a Polynomial by a Monomial

Let's review one of our exponent rules.

Example 6: Multiply $(5^3)(5^2)$

Example 7: Multiply $(x^3)(x^2)$

Example 8: Model the following multiplication using algebra tiles. $(3x)(2x)$

Method #1

Method #2 – Distributive Property

Example 9: Multiply using the following methods. $(-3x)(2x)$

Method #1

Method #2 – Distributive Property

Example 10: Model the following multiplication using algebra tiles. $-2x(3x-1)$

Method #1

Method #2 – Distributive Property

1. Multiply

a. $4(6s)$ b. $(7s)(3)$ c. $-5(4d)$ d. $(-2t)(6)$

e. $5(4r-2)$ f. $-2(5y^2-2y+3)$ g. $(-x^2-x)(2)$

h. $(x^2+x)(-3)$ i. $-3(2x^2-4x+5)$ j. $10(2s^2-s+2)$

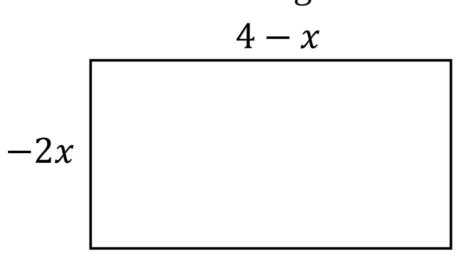
2. Multiply

a. $(3r)(2r)$ b. $(-t)(-6t)$ c. $(-5x)(2x)$

d. $6x(x-2)$ e. $-3w(2w+4)$ f. $2g(-g-1)$

g. $-y(-1+y)$ h. $4s(7r+1)$ i. $(-t)(8u-7t)$

3. Determine the area of the rectangle.



Dividing Polynomials

A. What is Division?

Example 1: Divide the following integers

a. $(10) \div (5)$ b. $(-10) \div (5)$ c. $(10) \div (-5)$ d. $(-10) \div (-5)$

Example 2: Model the following division $10 \div 5$

B. Dividing a Polynomial by a Constant

How does division of integers help us with multiplication of polynomials?

Example 3: Model the following multiplication using algebra tiles. $(4x) \div (2)$

Method #1

Method #2

What happens when we divide with negative numbers?

Example 4: Model the following multiplication using algebra tiles. $\frac{6x}{-3}$

Method #1

Method #2

What happens when we try to divide a constant into a polynomial with a degree bigger than 1?

Example 5: $\frac{-4x^2+8x}{-4}$

Method #1

Method #2

C. Dividing a Polynomial by a Monomial

Let's review one of our exponent rules.

Example 6: Divide $\frac{5^3}{5^2}$

Example 7: Divide $\frac{x^3}{x^2}$

How do we do the work without using algebra tiles?

____

____

____

Example 8: Let's answer all of the previous examples using mini fractions.

a. $(4x) \div (2)$

b. $\frac{6x^2}{-3x}$

c. $\frac{-4x^2+8x}{-4}$

Example 9: Model and simplify symbolically $\frac{6x^2}{2x}$

Method #1

Method #2 – Mini Fractions and Exp. Rules

Example 10: Model and simplify symbolically $\frac{3g^2+9g}{3g}$

Method #1

Method #2 – Mini Fractions and Exp. Rules

Example 11: Model and simplify symbolically $\frac{32c^2-48c}{-4c}$

Method #1

Method #2 – Mini Fractions and Exp. Rules

Example 12: Divide $(16k^{11} - 32k^{10} + 8k^8 - 40k^4) \div (8k^8)$

1. Divide

a. $\frac{14x}{7}$ b. $\frac{-14x}{7}$ c. $\frac{-14x^2}{-7}$ d. $\frac{14x^2}{-7}$

e. $\frac{15x-6}{3}$ f. $\frac{36-16x}{-4}$ g. $\frac{-12x^2-8x}{2}$ h. $\frac{5x^2-10x}{-5}$

2. Divide

a. $\frac{6s^2}{-2s}$ b. . $\frac{-14x}{7x}$ c. $\frac{-14x^2}{-7x}$ d. $\frac{14x^2}{7x^2}$

e. $\frac{12m^2-6s+8m}{2}$ f. $\frac{15x^2-10x}{5x}$ g. $\frac{14v^2-21v}{7v}$ h. $\frac{-8a^2+8a}{-8a}$

i. $\frac{16f-f^2}{-f}$ j. $\frac{x^2-x}{x}$ k. $\frac{-12x^2-4x}{2x}$ l. $\frac{x^2}{-x^2}$

P9.2 Linear Equations

Lesson 1: What is an Equation? **P9.2**

What is an Equation?

A. What is an equation?

Example 1: Circle the "equation(s)" in each of the circles. Explain why some of them are NOT equations.

$x + 5 = 12$	$x - 5 = 8$
$2 + x = 12$	$3 + x$

$5x = 10$	$2x + 2$
$9x = 18$	$-12x = 24$

$2x - 9 = 11x$	$x = 5x + 12$
$5x - 2 + 3x$	$3x + 4 = 2 + x$

$y = 3x + 1$	$P = 2l + 2w$
$F = \frac{3}{2}C - 32$	$3x + y$

Example 2: Answer True (T) or False (F). Be prepared to justify your answer.

a. Every equation has exactly two sides. ________

b. Every equation has exactly one equal sign. ________

c. Every equation has exactly one variable ________

Example 3: Answer the following with a sentence.

d. What is equality in mathematics?

e. What does the following mean in mathematics? $-3x + 5 = -4$

f. What is an inverse operation?

g. What is something in your life where you have to undo the steps to get back to the beginning?

B. Wrapping and Unwrapping Numbers

a. Let's try wrapping a number and then unwrapping it!

Operations	Number	Operations	Number
Choose a Number			

Let's Reverse the Process

The ENTIRE purpose of solving an equation is to discover the number that has been disguised with operations!

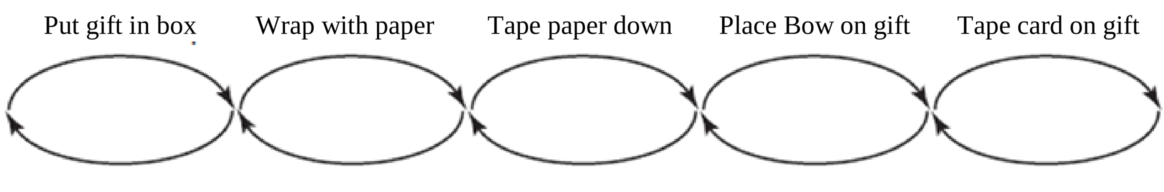
In order to UNDO an operation, you must use its ____________________________.

b. What are the four main operations that we use when solving equations? Can you state their inverses?

4 Main Oper- ations

Operations	Inverse Operation

c. The top row of the arrow diagram shows the steps to wrap a gift. What steps are needed to unwrap the gift?



d. Let's look at how someone constructed an equation and the process by which to deconstruct the number.

Start Equation $\boxed{x = -2} \underset{\longleftarrow}{\xrightarrow{\text{Multiply by 4}}} \boxed{4x = -8} \underset{\longleftarrow}{\xrightarrow{\text{Add 3}}} \boxed{4x + 3 = -5}$ End Equation

Solving Simple Equations

In order to solve equations, you need to use the inverse operations in order to discover what the variable actually equals.

Your goal is to separate the variables and numbers. Isolate the variable!

A. Solving Equations Using Addition and/or Subtraction

1. $x+3=-10$	Check:	2. $5=w-12$	Check:
3. $-7=x+17$	Check:	4. $18+x=2$	Check:

B. Solving Equations Using Division

1. $2x=14$	Check:	2. $-9e=-72$	Check:
3. $-60=15x$	Check:	3. $-6x=18$	Check:

Note: Variables can be on any side of the equation
Variables can be any letter (not just x)

C. Solving Equations Using Multiplication

1. $\frac{1}{2}x = 12$	Check:	2. $\frac{e}{5} = -1$	Check:
3. $\frac{x}{-6} = -4$	Check:	4. $\frac{1}{4}x = -2$	Check:

D. What Happens When The Variable Is Negative?

1. $-x = 10$	Check:	2. $20 - x = 2$	Check:
3. $-3 = -x + 15$	Check:	4. $-7 = -12 - q$	Check:

Solve the following equations. You need to show all of your steps!

1. $x + 8 = 11$

2. $a - 9 = 1$

3. $3 + x = 1$

4. $-8 + e = -8$

5. $12 = 6g$

7. $-4d = -24$

8. $-x + 8 = 14$

9. $\frac{d}{-5} = -4$

10. $3a = 2$

11. $3 = \frac{a}{-10}$

12. $32 = 8x$

14. $-7 = z + 1$

15. $20 = d - 5$

16. $-9 = -4 + k$

17. $x + 9 = 1$

Solving Equations with Variables on One Side

- The equality sign in an equation separates the equation into a **left-hand side** and a **right-hand side**. The variable can appear on either side.
- Your goal is to get the variable on one side and numbers on the other. The variable has to be number free (actually there is an invisible 1). When you do this, you will discover what the variable really represents.
- Here are some tips
    - Simplify each side first by collecting like terms
    - Undo subtraction and addition first
    - Undo division and multiplication second

Example 1: Solve the following equations and check your solutions.

a. $2x - 7 = -29$	b. $6 - 4t = 54$	c. $100 = -30 - 10y$
Check:	Check:	Check:
d. $-12 = -15a + 8$	e. $3x + 2x + x - 17 = 6$	f. $-23 = 2w - 2 + 4w + 11$
Check:	Check:	Check:

Example 2: Solve the following equations

a. $5w - 6 - 8w = 9$	b. $-8 = w + 5w - 10w - 20$	c. $3x - 8 + 5x - 4 = 8$

Example 3: Error Analysis

Find the ONE error in each of the following solutions

1. $4a - 13 = -31$

$4a - 13 + 13 = -31 - 13$ Add 13 to both sides

$4a = -44$ Simplify

$\frac{4a}{4} = \frac{-44}{4}$ Divide by 4

$a = -11$ Simplify

2. $-23 = 17 - 4x$

$-23 - 17 = 17 - 4x - 17$ Subtract 17 from both sides

$-40 = -4x$ Simplify

$\frac{-40}{4} = \frac{-4x}{4}$ Divide by 4

$-10 = x$ Simplify

3. $3x - 9 - 5x + 4 = -7$

$-2x - 13 = -7$ Combine Like Terms

$-2x - 13 + 13 = -7 + 13$ Add 13 to both sides

$-2x = 6$ Simplify

$\frac{-2x}{-2} = \frac{6}{-2}$ Divide by -2

$x = -3$ Simplify

Lesson 3: Assignment **P9.2**

Solve the following equations. For questions #1-6, code your steps. You do not need to code the remainder of the questions, but you do need to show all of your steps.

1. $3t + 10 = 28$

2. $-3x - 42 = 15$

3. $6x - 50 = 46$

4. $8x - 12 = 8$

5. $13 = -2x - 7$

6. $7x - 21 = -28$

7. $-20 = 40 + 5n$

8. $19 = -5 + 8m$

9. $8m - 3 = 9$

10. $-6 = 3m + 2m + m$

11. $3y - 2y + y = 5 - 11$

12. $12b + 14 - 3b = 16$

13. $7a - 35 = 0$

14. $12x - 2 = 24$

15. $3 = 5t - 11 + 14$

Solving Equations with Variables on Both Sides

- Collect like terms before you start moving terms around
- Choose a side where you will collect the variable (x's) and the other side will then have the constants (#'s without variables).
- Use addition or subtraction to move things to the appropriate sides.
- Use division or multiplication to finally isolate the variable.

Example 1:

a. $2x + 7 = 5x - 5$ Check:	b. $16 - 8x = 30 - 4x$ Check:
c. $-12 - 2x - 3 = -7x + 10x$ Check:	d. $5x - 7 - 3x + 16 = 4x - 11 - 8x + 5$ Check:

Example 2: your turn!

a. $3x - 1 = 7x + 11$ Check:	b. $x - 15 = -4x$ Check:
c. $2x - x + 3x = 9 - x - 14$ Check:	d. $2x - 9 + x = 10 - 5x - 3$ Check:

In order to move forward you need to articulate where you are having problems. Combining like terms, when to add, when to subtract, where to start. Please write any problems you having below.

____

____

Lesson 4: Assignment **P9.2**

Solve the following equations. Remember to show all of your steps and to reduce any fractional answers to lowest terms.

1. $3x - 5 = x + 11$

2. $-5 + 3x = x + 5$

3. $-x - 7 = -3x + 7$

4. $13x = -2x + 30$

5. $3x - 5 = -4x + 2$

6. $3x + 12 = -4x + 12$

7. $9x - 7 = 5x + 13$

8. $4m - 15 - 6m = 13 + m + m$

9. $2x - 5 = 5x + 6 - 4x$

10. $-12y = 65 + y$

11. $4x - 6x + 4 = 5x + 10 - 3x + 3$

12. $3 - 2e = -5e - 42$

13. $9y + 13 + 8y + 7 = 2y$

14. $5k + 1 + 4k + 1 = -10 - k + 11 + 6k$

Solving Equations Containing Parentheses

- Use the distributive property and then proceed as normal!
- Collect like terms
- Choose a side where you will collect the variable (x's) and the other side will have the constants (#'s without variables).
- Use addition or subtraction to move things to the appropriate sides.
- Use division or multiplication to finally isolate the variable.

Example 1: Solve the following equations and check your solution.

1. $8(3x + 10) = 28x - 14 - 4x$ Check:	2. $-2(x - 5) - (2x - 4) = 7(-3 + 3x)$ Check:
3. $6(2x + 8) = 2(x - 1)$ Check:	4. $-9 - (9x - 6) = 3(4x + 6)$ Check:

Lesson 5: Assignment **P9.2**

Solve the following equations. For questions #1-6, code your steps. You do not need to code the remainder of the questions, but you do need to show all of your steps.

1. $-35 = 5(2x - 3)$

2. $4(-2x - 3) = 36$

3. $-3(3x + 3) = 18$

4. $-24 = -3(-2x - 4)$

5. $2(3y - 3) = 0$

6. $-9(1 + r) - 4 = -2 + 2r$

7. $-6n - 7(3n - 5) = 116$

7. $-2(4v + 5) = -2 - 10v$

8. $10(3 + 3n) = 9 + 9n$

9. $4(5x - 3) = 7(2x + 3)$

10. $x + 2(x + 1) - 5(x - 3) + 3 = 0$

11. $3(m - 3) + 7(m + 3) = 16 - 4(7 - m)$

12. $10k - (2k - 8) - (2k - 3) = -4$

13. $5(2 + 3b) = 15 - (b - 7)$

Solving Equations Containing Fractions

A. Review of Past Lessons

Example 1: How many terms are in the following equations? Circle them.

1. $3x + 4 - 2x + 5 = -9x - 2 + 1 - x$ ___ terms

2. $5(2 + 4b) = 18 + 5b - 2(8 + 10b)$ ___ terms

3. $2x + 5(x - 1) = x - (3x - 1) - 2(x + 4)$ ___ terms

4. $\frac{1}{2}x - \frac{3}{4}(x + 2) = \frac{9}{10}(-3x + 1)$ ___ terms

5. $12 - \frac{1}{2}x + 3(5x + 1) = -\frac{1}{8}(x - 1) + x + 5$ ___ terms

Example 2: Answer the following questions

1. Translate $\frac{x}{2}$ into English ____

2. Translate $\frac{1}{2}x$ into English ______________________________________

3. How will you undo that type of operation in an equation? _______________

Regular Equation	The ENTIRE equation 3 times the original
4. $3x - 4 = 7$	5. $9x - 12 = 21$

What do we notice about the solutions?

____

C. Equations Involving Fractions

Fractions in equations make it a lot more difficult to solve. Fortunately for us, there is a technique to remove the fractions from the equation! We can use the laws of equality to eliminate fractions and work with just whole numbers! We use the…

LOWEST COMMON MULTIPLE

- Find the first number that all of the denominators will be able to divide into.
- Multiply the ENTIRE equation by that number (LCM).

Example 1: Solve the following equations

a. $\frac{2}{5}x = -16$ LCM = ______ Terms = ______	b. $\frac{3}{4}x = -15$ LCM = ______ Terms = ______
c. $\frac{7}{8}x - \frac{5}{4} = \frac{1}{2}x + \frac{2}{3}$ LCM = ______ Terms = ______	d. $\frac{5}{6}x - \frac{2}{3} = \frac{1}{2}x + \frac{1}{3}$ LCM = ______ Terms = ______
e. $\frac{2}{3}x + \frac{3}{4}(35 - x) = 25$ LCM = ______ Terms = ______	

Lesson 6: Assignment **P9.2**

Solve the following equations. For questions #1-6, code your steps. You do not need to code the remainder of the questions, but you do need to show all of your steps. State the LCM for each question.

1. $\frac{x}{3} = 5$ LCM = ______

2. $\frac{x}{7} - 1 = -6$ LCM = ______

3. $\frac{1}{4}b + \frac{1}{2}b = 3$ LCM = ______

4. $-\frac{2}{7}x = 6$ LCM = ______

5. $2y - \frac{3}{5} = \frac{1}{2}$ LCM = ______

6. $\frac{1}{4} + \frac{1}{2}t = 4$ LCM = ______

7. $\frac{1}{4}x + x = -3 + \frac{1}{2}x$ LCM = ______

8. $m + \frac{2}{3} = \frac{1}{4}m - 1$ LCM = ______

9. $\frac{1}{5}m + \frac{2}{3} - 2 = m - \frac{2}{5}$ LCM = ______

10. $\frac{1}{4}(3c + 5) - \frac{1}{2}(2c + 3) = \frac{1}{2}$ LCM = ______

Solving Equations Containing Decimals

Good news! All of the same rules from our previous lessons apply when solving equations with decimals in them. There are a few techniques you can use to come up with the solution. Let's look at them.

$$0.4x + 4 = 9$$

Solve by multiplying by 10's, 100's, 1000's etc. to remove decimals and make whole numbers	Solve by leaving decimals alone (calculator recommended!)
$0.4x + 4 = 9$	$0.4x + 4 = 9$

Example 1: Solve the following equations

a. $-6.3n = -8.19$	b. $\frac{x}{1.2} = -7$
c. $0.4x + 3.9 = 5.78$	d. $2.25(x - 4) = x + 3.28$

e. $0.3x - 2.4 = 0.36 + .52x$	f. $3.5x + 0.8 = 18.24 - 5.9x$

Example 2: What way do you prefer to solve equations involving decimals?

a. Multiplying by 10, 100, etc.
b. Leaving the Decimals Alone
c. I like to use both ways

Lesson 7: Assignment **P9.2**

Show all of your steps. If necessary, round to 3 decimal places.

1. $-2.8 = n + 1.3$

2. $-1.5x = -2.55$

3. $-4.84 = -1.3k + 2.7$

4. $0.72 = 0.4(x + 1.4)$

5. $-0.5x - 3.69 = x - 1.9 - 2.39$

6. $3.5(1 + 4s) = 24.5$

7. $6(9f + 8.5) = 18.5$

8. $-2x - 4 + 5.5x = 17.5$

9. $8v + 3 + 9v = -26.5$

10. $-8.5k + 4.5 + 4 = 15.5$

11. $-24.5 - 4.5c = -9(3c + 7)$

P9.3 Linear Inequalities

Lesson 1: Review of Solving Equations **P9.3**

Solving Equations Review

Type #1: Basic Equations; solving by using inverse operations.	Type #2: Solving equations with brackets.
1. What is the inverse of additions?	1. Solve $5(x+3) = -35 - 2(3x-3)$
2. What is the inverse of subtraction?	2. Solve $-2(4x+5) = 54$
3. What is the inverse of multiplication?	
4. What is the inverse of division?	
5. Solve $2x = 14$	Type #3: Solving equations with variables on both sides. 1. $-12 - 2x - 3 = -7x + 10x$
6. Solve $x - 5 = 10$	2. $-2x - 16 = 28 - 6x$
7. Solve $8 + p = -12$	Type #4: Solving equations with fractions. 1. $\frac{5x}{9} - \frac{7}{18} = \frac{3x}{2} + \frac{11}{6}$
8. Solve $2n + 8 = 22$	
9. Solve $5 - 6x = -31$	2. $\frac{3}{5}x - 4 = \frac{2}{3}x + 1$

Assignment: Lesson 1 **P9.3**

Solve the equations

1. $6 + 2x = -7 + x$

2. $8w - 12.8 = 6w$

3. $-12a = 15 - 15a$

4. $13 - 3q = 4 - 2q$

5. $4(g + 5) = 5(g - 3)$

6. $-3 = 5x + 3(-2x - 5)$

7. $\frac{x}{3} - 3 = 3$

8. $-4 = -\frac{7x}{6} + \frac{4}{6}$

9. $\frac{x}{4} + \frac{7}{4} = \frac{5}{6}$

10. $2 - \frac{x}{24} = \frac{5x}{24} + 1$

11. $\frac{x}{3}+\frac{x}{4}=x-\frac{1}{6}$

12. $5=2-\frac{x}{2}$

13. $-9-x=-3x+3$

14. $-4(-3x+2)=16$

15. $6x-7(-x+3)=44$

16. $49=7x-7(2x+3)$

17. $-8z+11=-10-5.5z$

18. $\frac{2}{5}(m+4)=\frac{1}{5}(3m+9)$

19. $36=4(-2x+3)$

20. $5x+2(-2x-4)=4$

What is an Inequality?

A. The Inequality Symbols

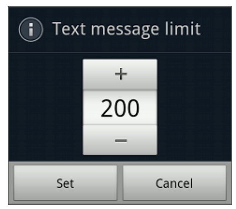
$<$	$\leq$	$>$	$\geq$
________	________	________	________

What is an inequality?

Example 1: Define a variable and write an inequality for each situation.

a. 

____

b. 

____

c. 

____

d. 

____

Difference between an EQUATION and an INEQUALITY

EQUATION	INEQUALITY
Has ________ solution. Example: $h + 3 = 5$	Has ________ solutions Example: $h + 3 < 5$

Example 2: Is each number a solution of the inequality $x \le -2$?

a. -10 b. 10 c. 0 d. -2.1 e. -1.9 f. -2

How do I read inequalities?

$x < -2$	$-2 < x$	$r \ge 5$	$5 \ge r$

How can I read an inequality?

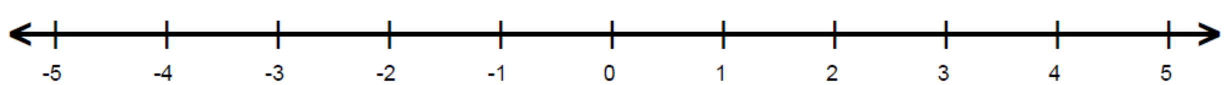
____

____

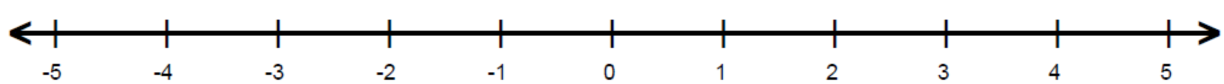
Introduction of the number line – What you need to know

a. Open dot vs. Closed dot

Graph $x < -2$



Graph $x \le -2$

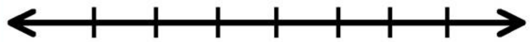


When do you use a closed dot?____

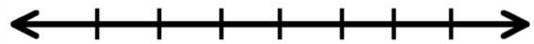
When do you use an open dot?____

Example 3: Graph each inequality on a number line. Make sure you have 4 numbers represented on your line that are solutions of the inequality.

a. $x > -8$

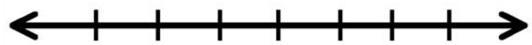


b. $3 < r$



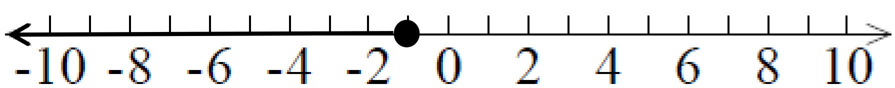
c. $w \leq -4$

d. $7 \leq x$

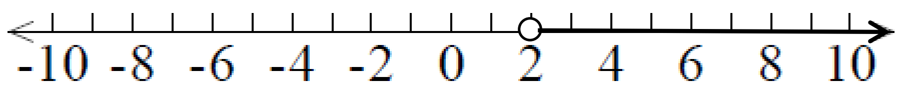


Example 4: State what inequality is represented by each graph

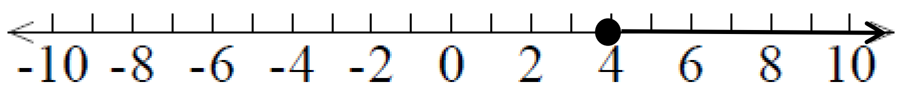
a. ______________________



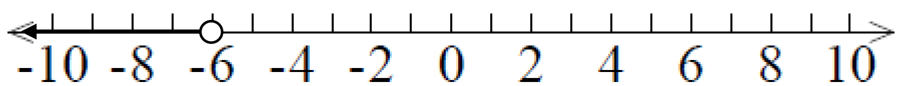
b. ______________________



c. ______________________



d. ______________________



Assignment: Lesson 2 **P9.3**

1. Is each inequality true or false?
   a. $3 < 5$
   b. $-3 < -5$
   c. $-3 \leq -3$
   d. $3.01 < 3$
   e. $-3.01 < -3$
   f. $-5 > -5$
   g. $\frac{1}{3} > \frac{1}{5}$
   h. $-\frac{1}{3} > -\frac{1}{5}$

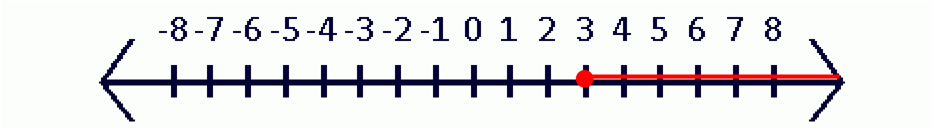
2. Which numbers are a solution of $x < -5$?
   a. 0
   b. -4.9
   c. -5.01
   d. $-\frac{1}{5}$

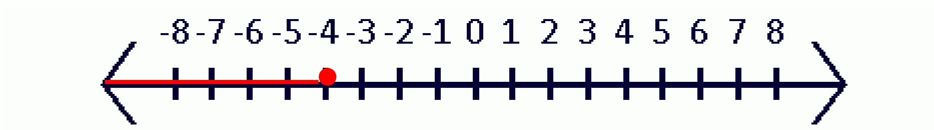
3. Write 4 numbers that are solutions of each inequality.
   a. $b < 4$
   b. $-6 \leq g$
   c. $4 < x$
   d. $y \geq -10$

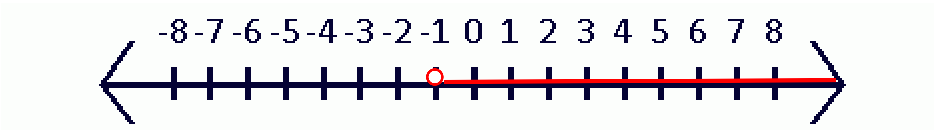
4. For which inequalities is 5 a possible value of x?
   a. $x > 6$
   b. $x < 5$
   c. $x \leq 5$
   d. $x \geq 5$

5. Define a variable and write an inequality to model each situation.
   a. The speed limit in school zones is 30 km/h
   b. The maximum number of people allowed in the hall is 600.
   c. In the wrestling class, you must be less than 50kg.
   d. In order to watch the movie you have to be at least 16 years old.

6. Write an inequality whose solution is graphed on the number line.

a. 

b. 

c. 

7. Graph the solution of each inequality on a number line.

a. $w > 5$

b. $-3 \geq r$

c. $c \geq 2.1$

d. $d \leq -\frac{1}{2}$

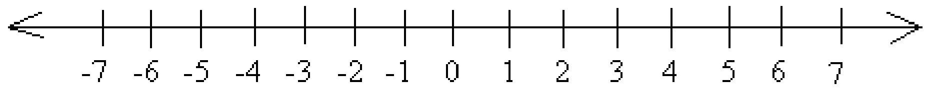
e. $-5 < x$

f. $y \geq 0$

Solving Linear Inequalities by Using Addition and Subtraction

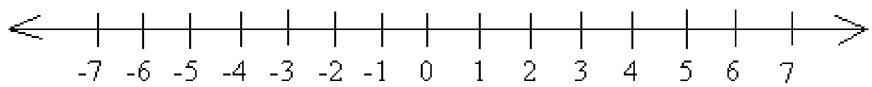
Let's see what happens when we add and subtract to an inequality

$-2 < 4$



Let's add 2 to each side. Does the inequality still hold true?

$-2 < 4$



Let's subtract 2 from each side. Does the inequality still hold true?

What happens when we add and subtract to both sides of an inequality?

Example 1:

a) Solve the inequality: $2.1 \geq x - 3.2$	b) Verify the solution	c) Graph

Example 2:

a) Solve the inequality: $x + 3 < 5$	b) Verify the solution	c) Graph

Example 3:

a) Solve the inequality: $2x + 5 < x - 10$	b) Verify the solution	c) Graph

Assignment: Lesson 3 **P9.3**

Solve each inequality. Check your solution and graph your answer.
Because there is one more important thing to learn tomorrow, it is really important for this assignment that you collect the variable on the side where it will be a $+$ value.

1. $a + 5 < 14$

2. $9k - 12 \geq 80 + 8k$

3. $6y > 14 - 2 + 7y$

4. $q + 10 > 3q - 7 - 3q$

5. $6c - (5c - 7) \leq 12$

6. $a - 12 < 6$

7. $4 < 1 + \frac{n}{7}$

8. $2x + 3 \geq x + 5$

9. $x + \frac{1}{8} < \frac{1}{2}$

10. $3x - 9 \leq 2x + 6$

11. $-0.17x - 0.23 \geq 0.75 - 1.17x$

12. $3(r - 2) < 2r + 4$

14. $1.8w + 4.5 \geq 0.8w - 12.2$

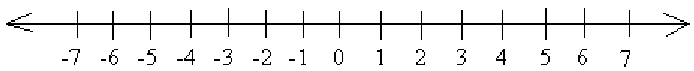
15. $3.8 < 2x - (9 - 1.2x)$

Solving Linear Inequalities Using Multiplication and Division

Let's look at what happens when we multiply and divide to an inequality.

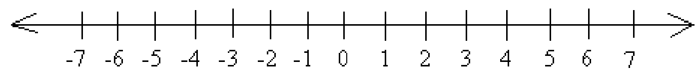
$3 > -1$

Let's **multiply** 2 to each side.
Does the inequality still hold true?



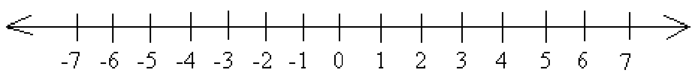
$3 > -1$

Let's **multiply** -2 to each side.
Does the inequality still hold true?



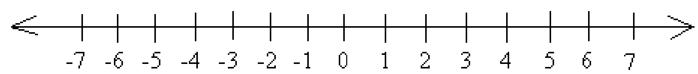
$-6 < 3$

Let's **divide** 2 into each side.
Does the inequality still hold true?



$-6 < 3$

Let's **divide** -2 into each side.
Does the inequality still hold true?



(1) What happens when we **Multiply** or **Divide** by a positive number?

(2) What happens when we **Multiply** or **Divide** by a negative number?

To solve an inequality, we use the same strategy as for solving an equation. However, when we multiply or divide by a negative number, we reverse/flip the inequality sign.

Example 1: Solve the following inequalities. Graph each solution.

a) $-10x \leq 50$ b) $10x \geq -50$ c) $\frac{x}{-3} > -2$ d) $\frac{x}{3} < -2$

e) $6 < 20 - 7x$

f) $2(x + 6) \geq 5x - 9$

g) $-\frac{x}{4} - \frac{7}{2} < \frac{x}{8} + \frac{1}{4}$

h) $3(x - 2) - 5 < 2(x - 1) + 2x$

i) $-2.6a + 14.6 > -5.2 + 10.7$

j) $21 > -7(x + 2)$

Example 2: A super-slide charges \$1.25 to rent a mat and \$0.75 per ride. Josh has \$10.25. How many rides can Josh go on?

a) Choose a variable, and then write an inequality to solve the problem.

b) Solve the problem.

c) Graph the solution.

Assignment: Lesson 4 **P9.3**

Solve, Check, and graph the solution

1. $-5k < 25$

2. $\frac{x}{-3} > -12$

3. $6x \leq -18$

4. $-2s \geq -4.8$

5. $\frac{x}{4} + 2.5 < 6.1$

6. $1 + \frac{3}{7}x > 13$

7. $12y + 23 \geq -1$

8. $-1 - \frac{m}{4} \leq 6$

9. $9n - 12n + 42 > 0$

10. $6y + 10 > 8 + (y + 14)$

11. $-6x - (2x + 3) \geq 1$

12. $m + 3 - 4m > 2m + 23$

13. $10 + y \leq 8 - (y - 2)$

14. $0.4x - 1.23 > x + 1.17$

15. $0.2(x - 3) < 0.5(2x + 4) - 1$

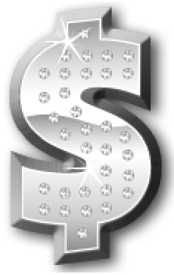
16. $\frac{x}{2} - \frac{3}{4}(2x - 5) < -\frac{1}{4}$

17. $12.5x - 3.2 \geq 14x - 6.2$

Solving Stories

S1

Jennifer has $35.50 and is saving $4.25/week.



Eva has $24.25
and is saving $5.50/week.

In how many weeks will Eva start having more in her savings than Jennifer?

Model, Solve, and Verify

S2

Two rental halls are considered for the 2014 graduation Reception

Hall A costs $50 per person

Hall B costs $2500, plus $30 per person.

Determine the number of people for which Hall A will cost less than Hall B

Model, Solve, and Verify

S3

Tamoor and Saba belong to different local fitness clubs.

Tamoor pays $35 per month And a one time registration Fee of $15.

Saba pays $25 per month but had to pay a $75 registration

After how many months will Tamoor's bill be less than Saba's bill?

Model, Solve, and Verify

S4

Jake plans to board his dogs while he is away on vacation.

House A charges a $90 one time fee plus $5 per day.

House B charges a $100 one time fee plus $4 per day.

For how many days must Jake board his dog for boarding house A to be more than boarding house B.

Model, Solve, and Verify

S5

Abdalla has offers for a position as a salesperson at two local electronic stores.

Store A will pay a flat rate of $55 per day plus 3% of sales.

Store B will pay a flat rate of $40 per day plus 5% of sales.

What do Abdalla’s sales need to be for store B to be a better offer?

Model, Solve, and Verify

S6

The basketball team here at school wants to buy new jerseys.

Jerseys Unlimited charges $40 per jersey plus $80 for a logo design.

Uniforms R Us charges $50 per jersey.

How many jerseys does the team need to buy for Jerseys Unlimited to be the cheaper option?

Model, Solve, and Verify

P9.3 Linear Inequalities

Key Terms:

1. Relation ____

____

2. Linear Relation ____

____

3. Table of Values ____

____

4. Input Values ____

____

5. Dependent Variable ____

____

6. Output Values ____

____

7. Independent Variable ____

____

8. Linear Graph ____

____

9. Linear Equation ____

____

10. Discrete Data ____

____

11. Interpolation ____

____

12. Extrapolation ____

____

Looking for Patterns

Why are patterns and trends important in mathematics?

____

____

Task #1: Without counting 1 by 1, how many shaded squares are in figure 10 (10 x 10 square)?

My Calculations:
Some of my classmates ideas:

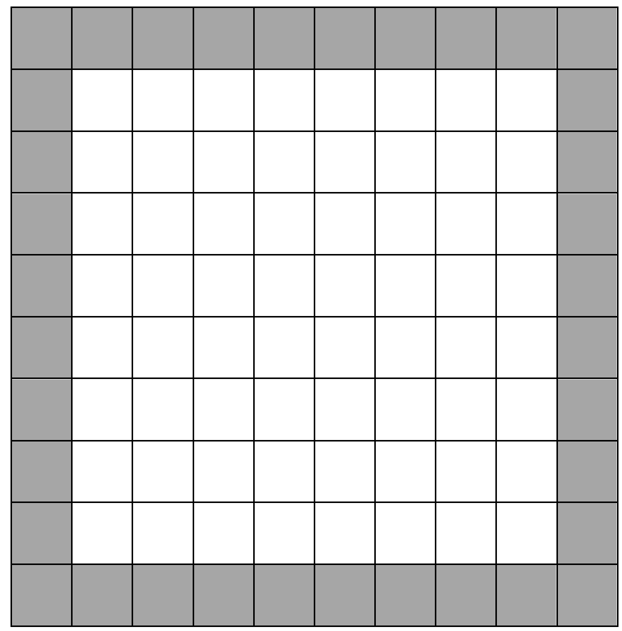


Figure 10

Assume that if figure 10 is a 10×10 grid, then figure 3 would be a 3×3 grid etc.

Using your own calculation idea from above, calculate how many shaded squares would be in

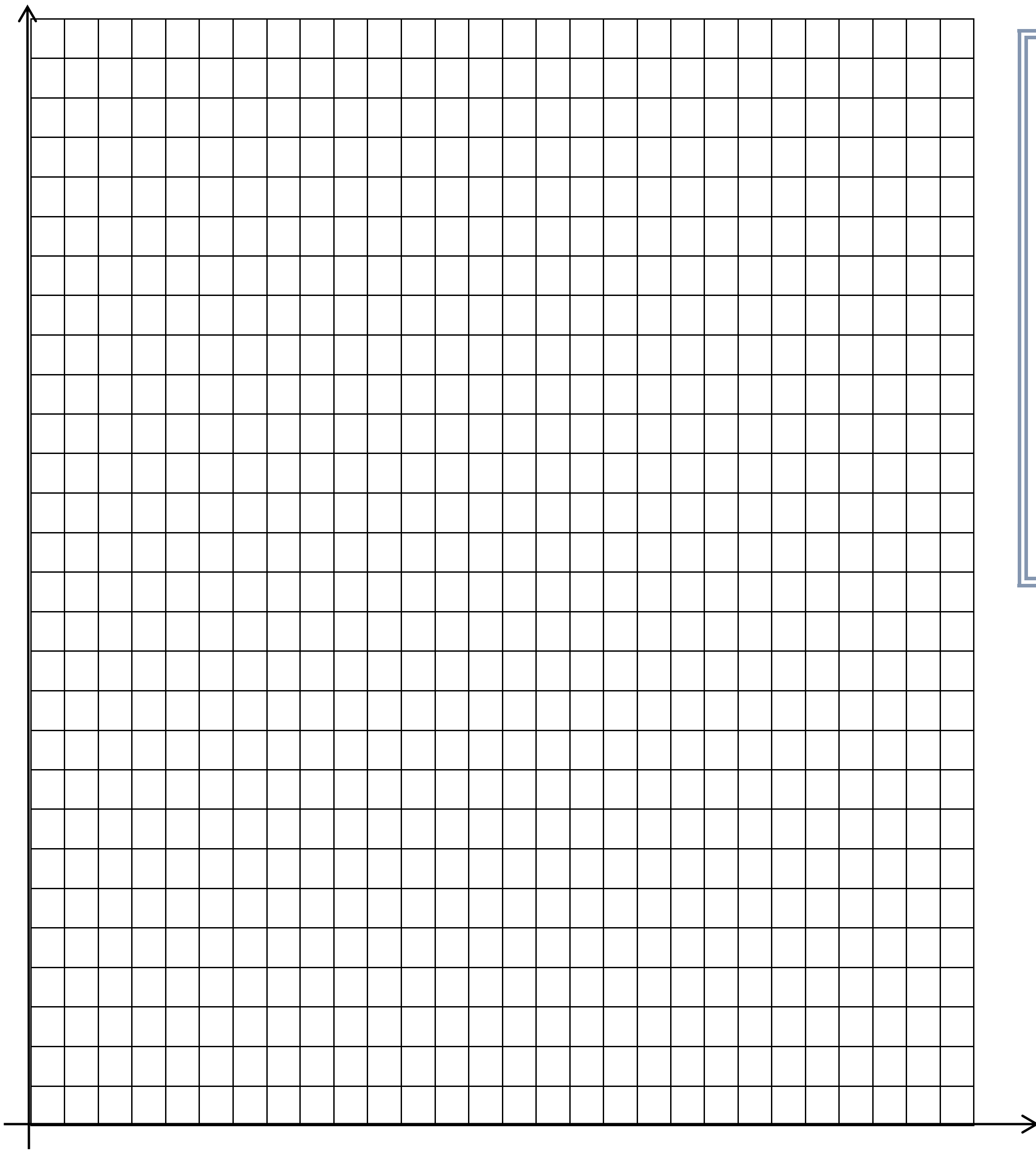
Figure ____	Figure ____

Using a classmate's calculation idea from above, calculate how many shaded squares would be in…

Figure ____	Figure ____

How can we determine the number of shaded squares will be in any figure? You need to be able to justify your thinking.

Figure #	Figure 1	Figure 2	Figure 3	Figure 4	Figure 5	Figure 6
Number of Shaded Squares						
Figure #	Figure 7	Figure 8	Figure 9	Figure 10	Figure 11	Figure 12
Number of Shaded Squares						



Describe the pattern in words…

Task #2: Draw the next L figure in the pattern.

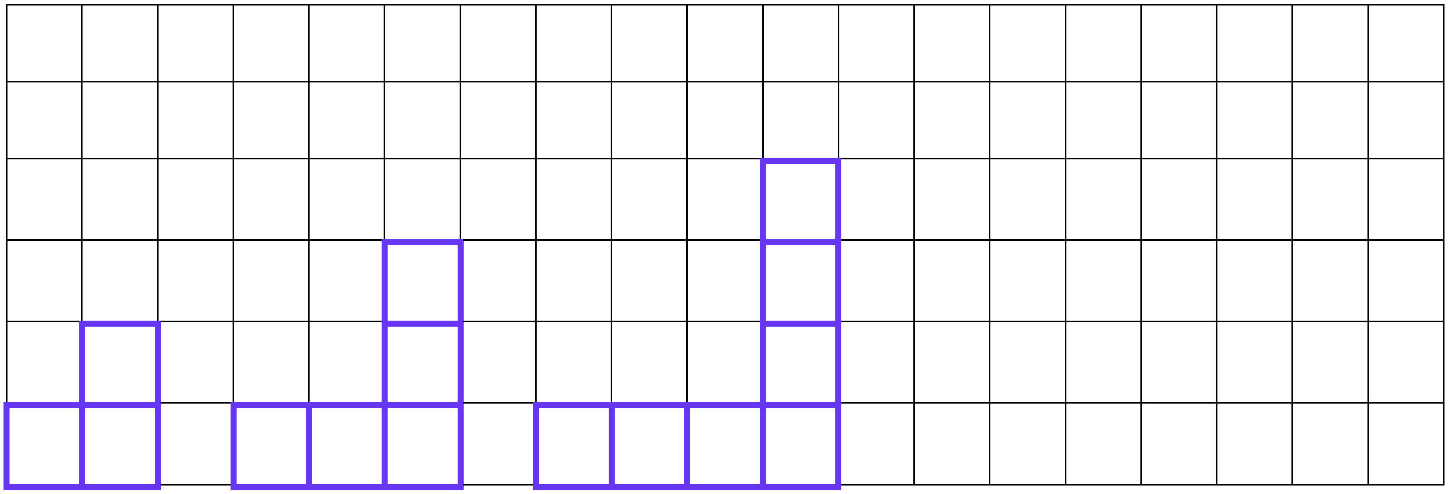


Figure 1 **Figure 2** **Figure 3** **Figure 4?**

How did you know what Figure 4 was going to look like?

How does it look like the pattern is growing?

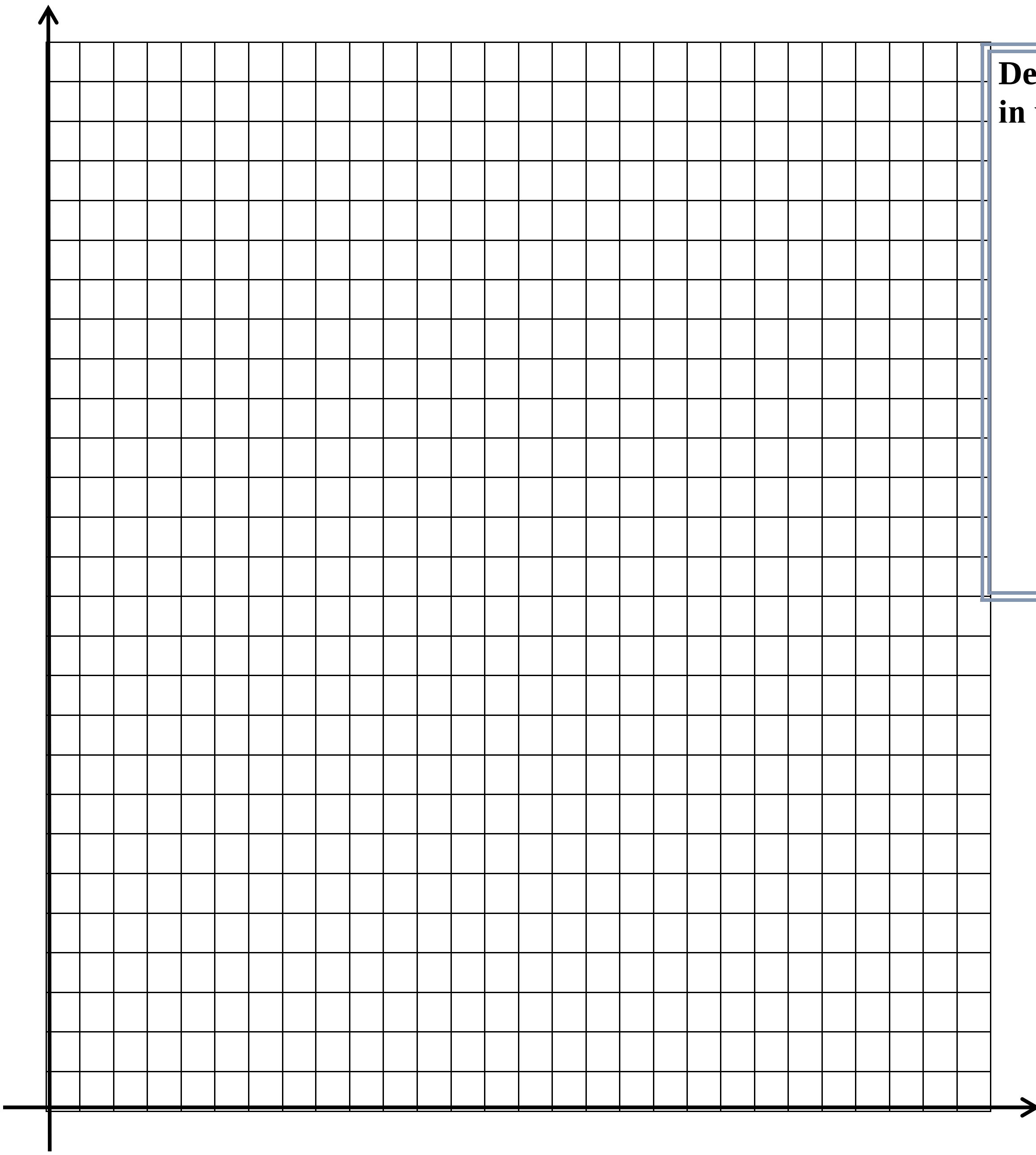
Count how many squares are represented by each figure.

Figure 1	Figure 2	Figure 3	Figure 4

If we weren't allowed to count by 1's, what are some of the calculations that could be used to find the number of squares in each figure?

How can we determine the number of shaded squares will be in any figure? You need to be able to justify your thinking.

Figure #	Figure 1	Figure 2	Figure 3	Figure 4	Figure 5	Figure 6
Number of Shaded Squares						
Figure #	Figure 7	Figure 8	Figure 9	Figure 10	Figure 11	Figure 12
Number of Shaded Squares						



Describe the pattern in words…

Task #2: Draw the next L figure in the pattern.

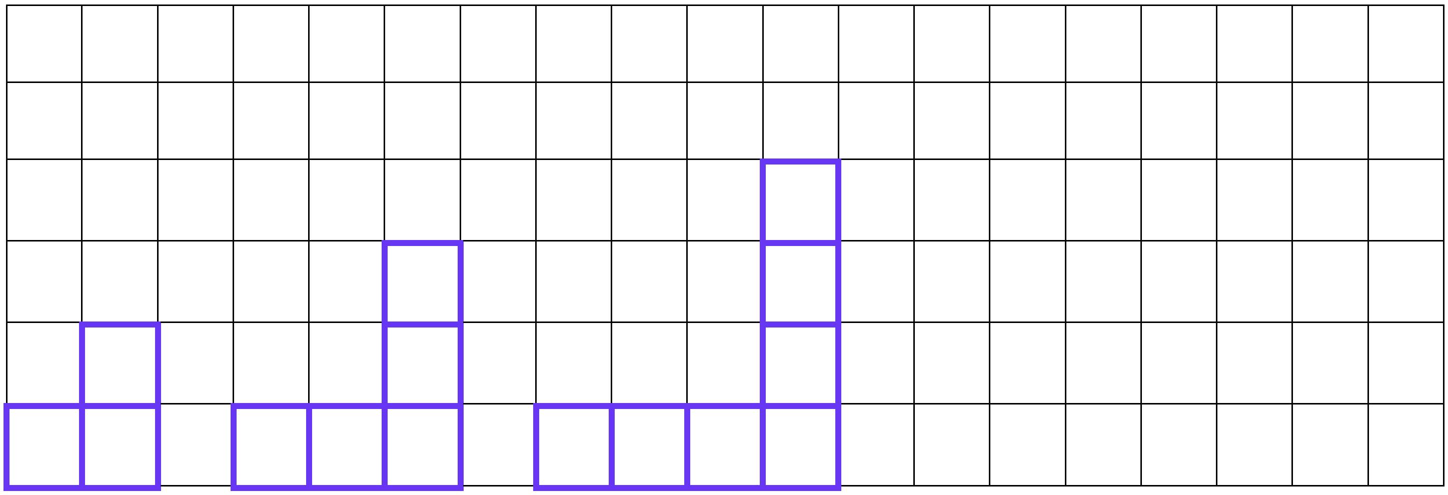


Figure 1 **Figure 2** **Figure 3** **Figure 4?**

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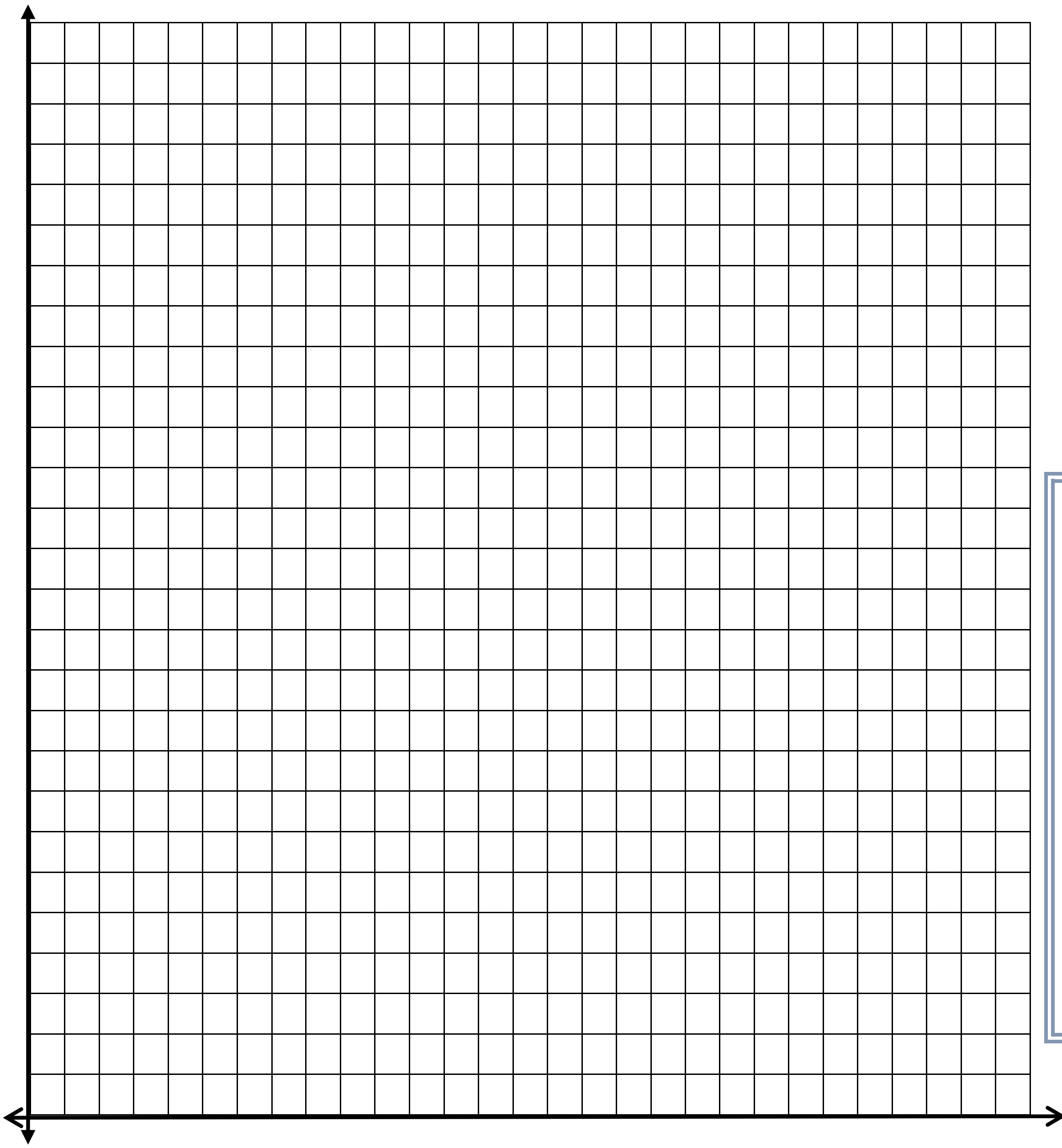
Figure 1	Figure 2	Figure 3	Figure 4

If we weren't allowed to count by 1's, what are some of the calculations that could be used to find the number of squares in each figure?

How can we determine the number of shaded squares will be in any figure? You need to be able to justify your thinking.

Figure #	Figure 1	Figure 2	Figure 3	Figure 4	Figure 5	Figure 6
Number of Squares						
Figure #	Figure 7	Figure 8	Figure 9	Figure 10	Figure 11	Figure 12
Number of Squares						



Describe the pattern in words…

Lesson 1: Assignment **P9.1**

1. The square represents tables, the circles represents seats at the table.

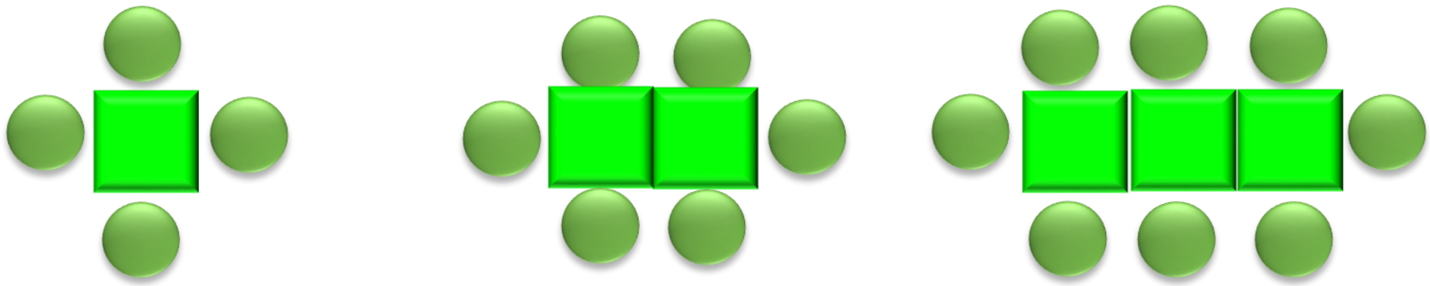


Fig 1 Fig 2 Fig 3

   a. Draw the next figure in the pattern (fig. 4)

   b. If you couldn't count by 1's, is there a technique to find out how many seats there are in each of the figures?

   c. How many seats would be in figure 103?

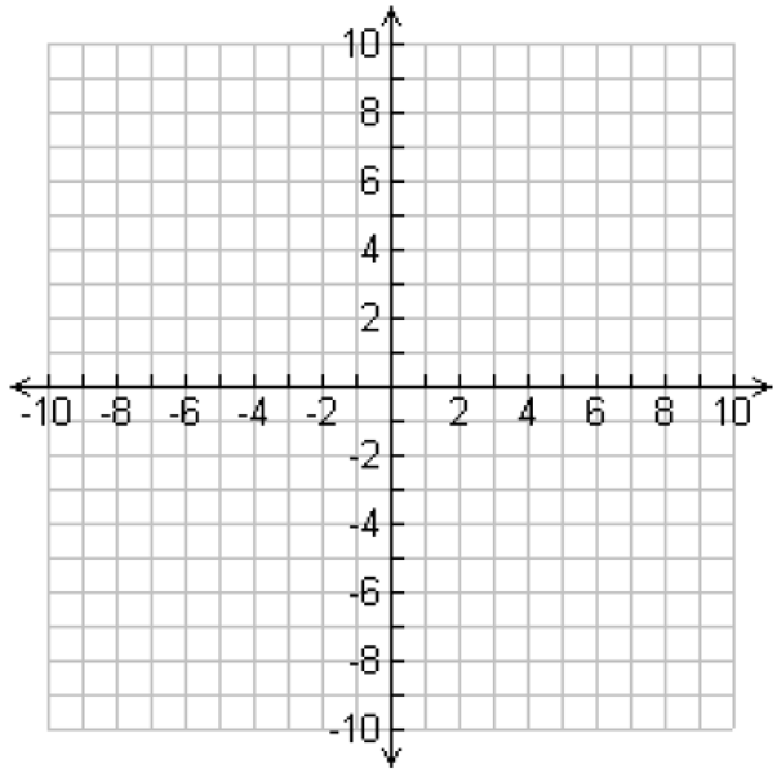
   d. Would you be able to fit exactly 17 seats in the pattern?

   e. What figure number has exactly 2694 seats? Show how you achieved your answer.

How to Graph on a Cartesian Plane/Coordinate Plane

Often, mathematicians graph their data on Cartesian Planes to determine patterns. Let's review how to graph ordered pairs before we apply it to linear relations.

A. Terminology



1. Quadrant:

2. Horizontal Axis:

3. Vertical Axis:

4. Coordinate Axes:

5. Origin:

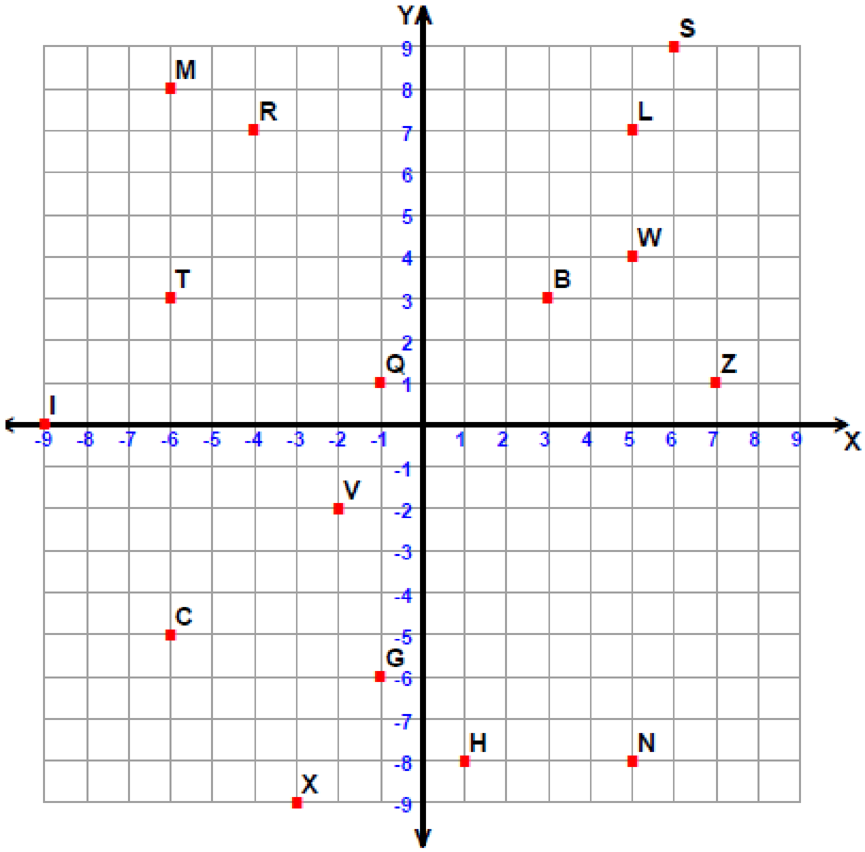
6. Coordinates:

7. Ordered Pair:

8. Plot the following points on the Cartesian plane above.

a) A$(5,-3)$ b) $B(0,9)$ c) $C(3,-10)$ d) $D(-3,0)$

e) $E(10,8)$ f) $F(-3,3)$ g) $G(-5,-5)$ h) $H(0,-4)$



9. Find what point is located at each ordered pair.

1) $(7,1)$ ________	3) $(-6,-5)$ _______	5) $(-1,1)$________	7) $(5,7)$ ________
2) $(-6,3)$ ________	4) $(1,-8)$ ________	6) $(-3,-9)$ _______	8) $(-9,0)$ ________

10. Write the ordered pair for each given point.

9) **N** ________	11) **R** ________	13) **G** ________	15) **W** _________
10) **V** ________	12) **S** ________	14) **B** ________	16) **M** ________

11. Plot the following points on the coordinate grid.

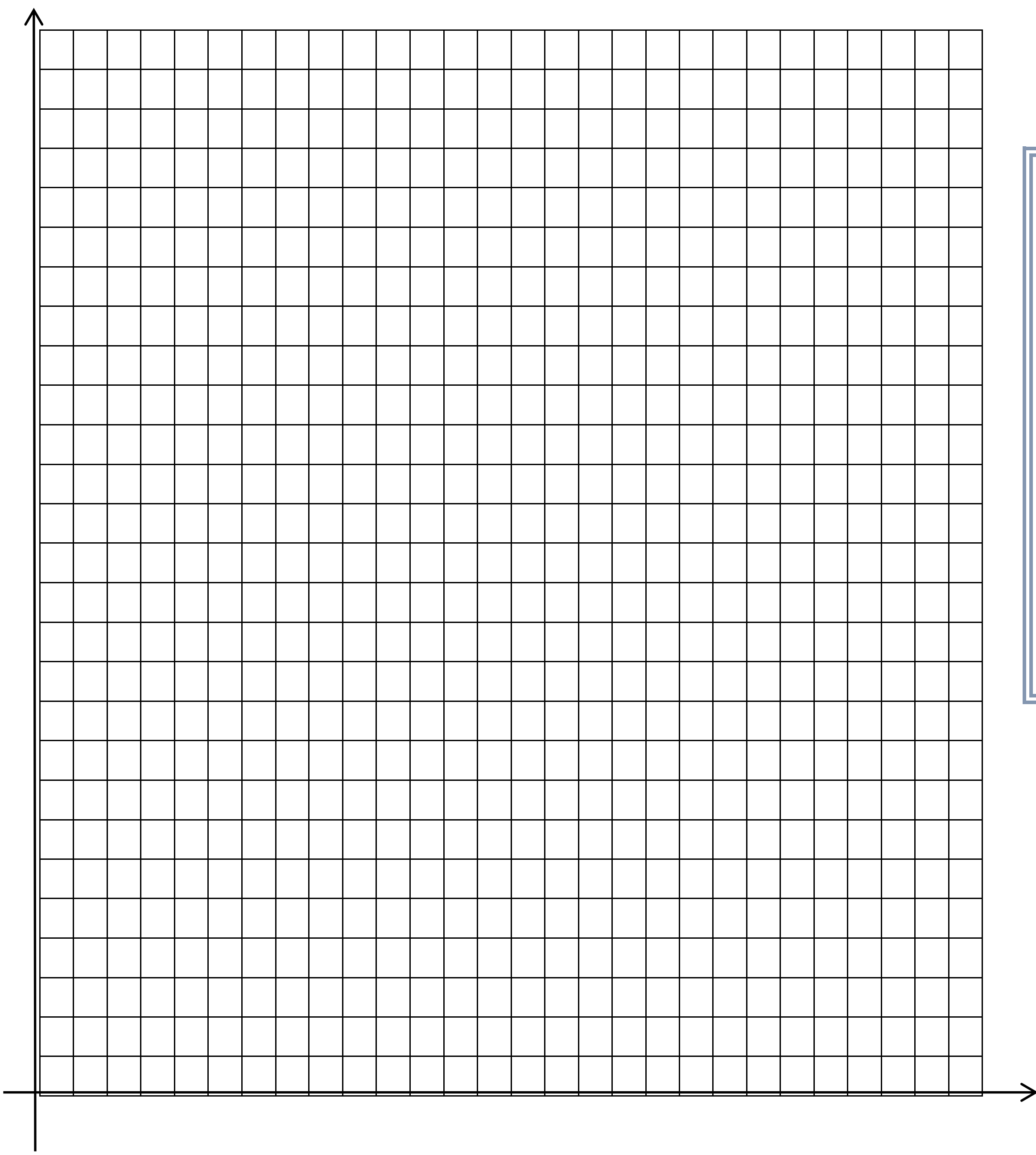
17) $\boldsymbol{E}$ $(-4,3)$	19) $\boldsymbol{F}$ $(-3,5)$	21) $\boldsymbol{P}$ $(8,4)$	23) **A** $(8,8)$
18) $\boldsymbol{Y}$ $(6,-1)$	20) $\boldsymbol{D}$ $(-3,-2)$	22) $\boldsymbol{O}$ $(-6,-9)$	24) **J** $(-2,1)$

Lesson 3: Graphing Patterns P9.1

Let's go back and plot the data from our first two patterns that we encountered last class.

Task #1: 10 × 10 Square

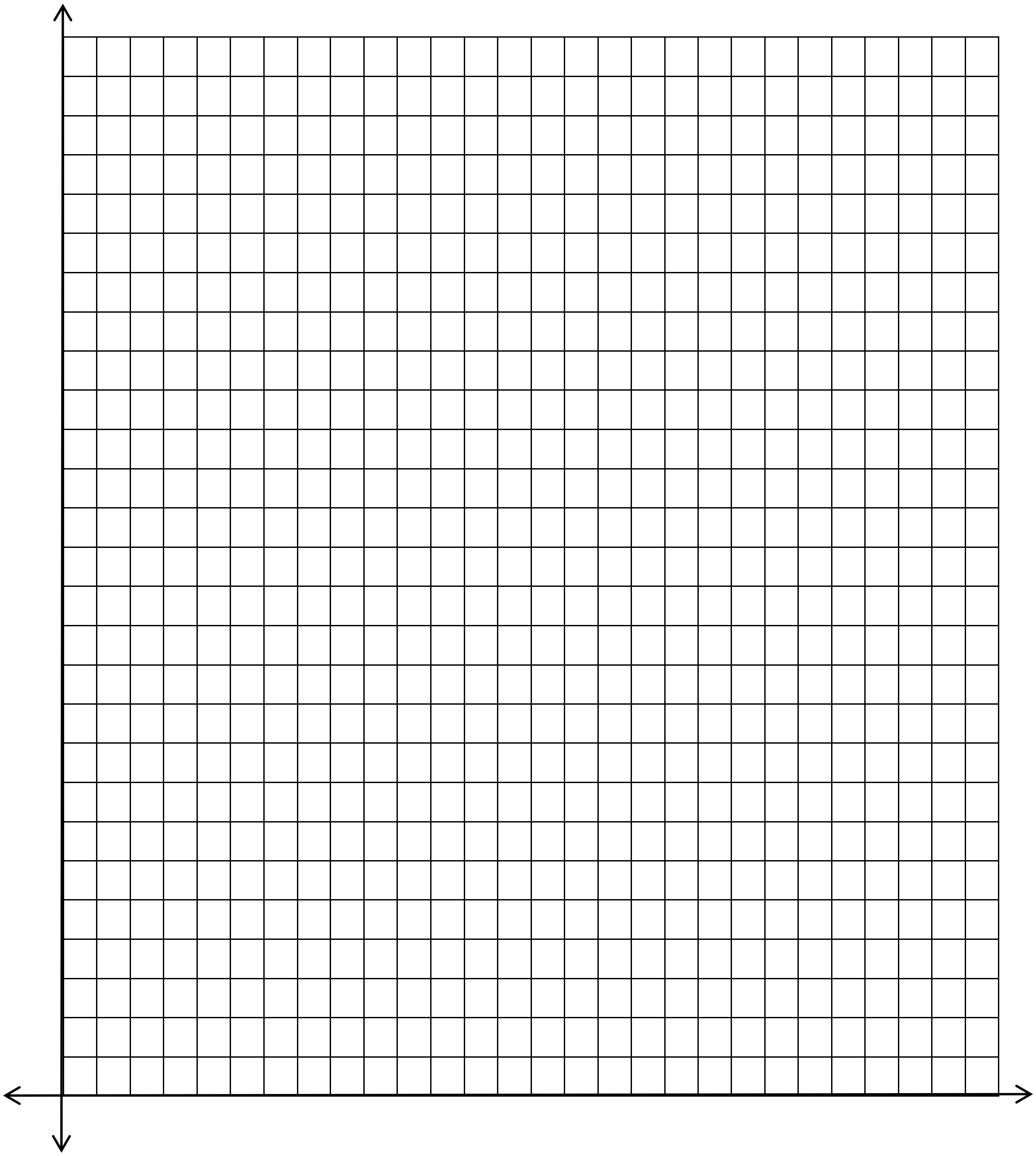
Figure #	Figure 1	Figure 2	Figure 3	Figure 4	Figure 5	Figure 6
Number of Shaded Squares						
Figure #	Figure 7	Figure 8	Figure 9	Figure 10	Figure 11	Figure 12
Number of Shaded Squares						



Describe the pattern in words…

Task #2: L Figures

Figure #	Figure 1	Figure 2	Figure 3	Figure 4	Figure 5	Figure 6
Number of Squares						
Figure #	Figure 7	Figure 8	Figure 9	Figure 10	Figure 11	Figure 12
Number of Squares						

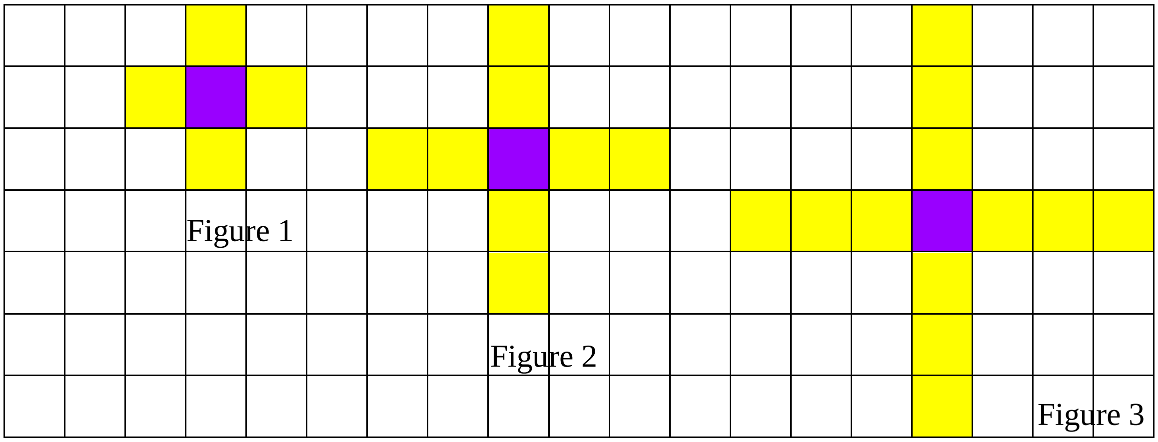


Describe the pattern in words…

What do we notice about the way the numbers look on the graph? What kind of pattern is this?

Graphing Patterns

Task #3: Lower Case t's



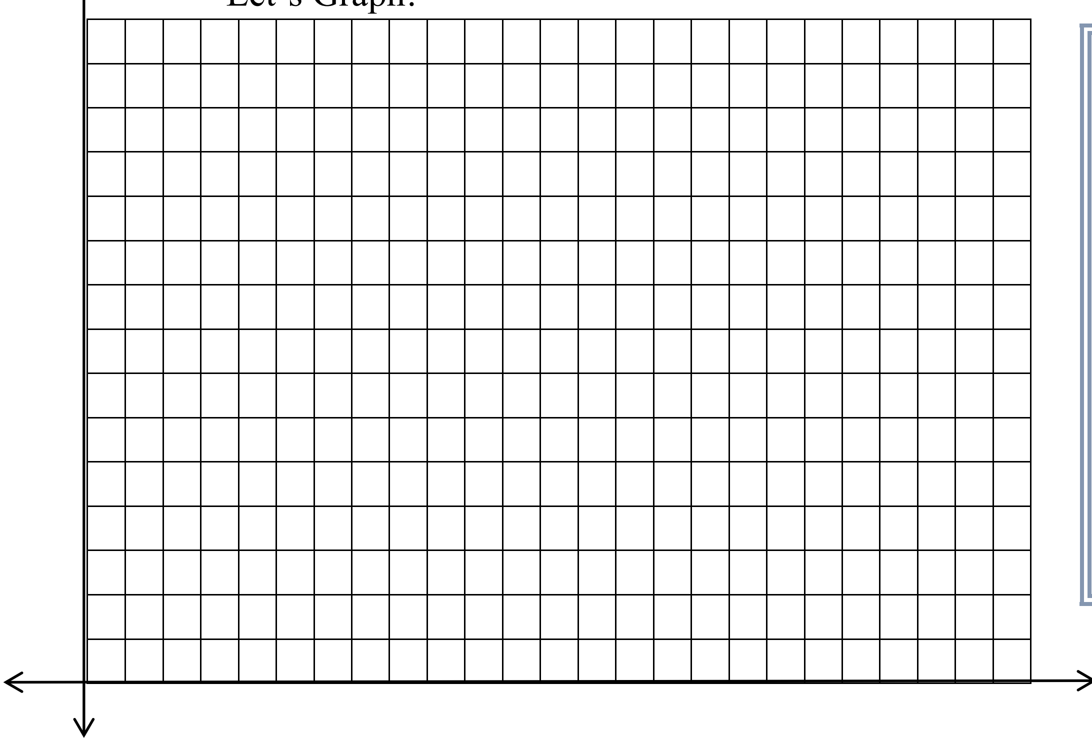
How many squares would figure 4 need? Draw a sketch of what you think it will look like.

What part is staying the same (constant) and what part is changing (multiplier)?

Let's make a chart

Figure #	Figure 1	Figure 2	Figure 3	Figure 4	Figure 5	Figure 6
Number of Squares						
Figure #	Figure 7	Figure 8	Figure 9	Figure 10	Figure 11	Figure 12
Number of Squares						

Let's Graph!



Describe the pattern in words…

Task #4: Toothpicks

Figure 1

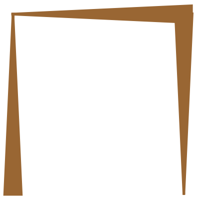


Figure 2

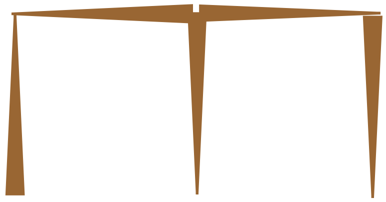
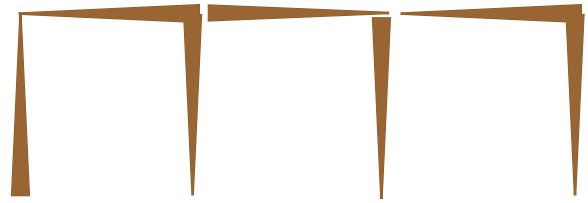


Figure 3



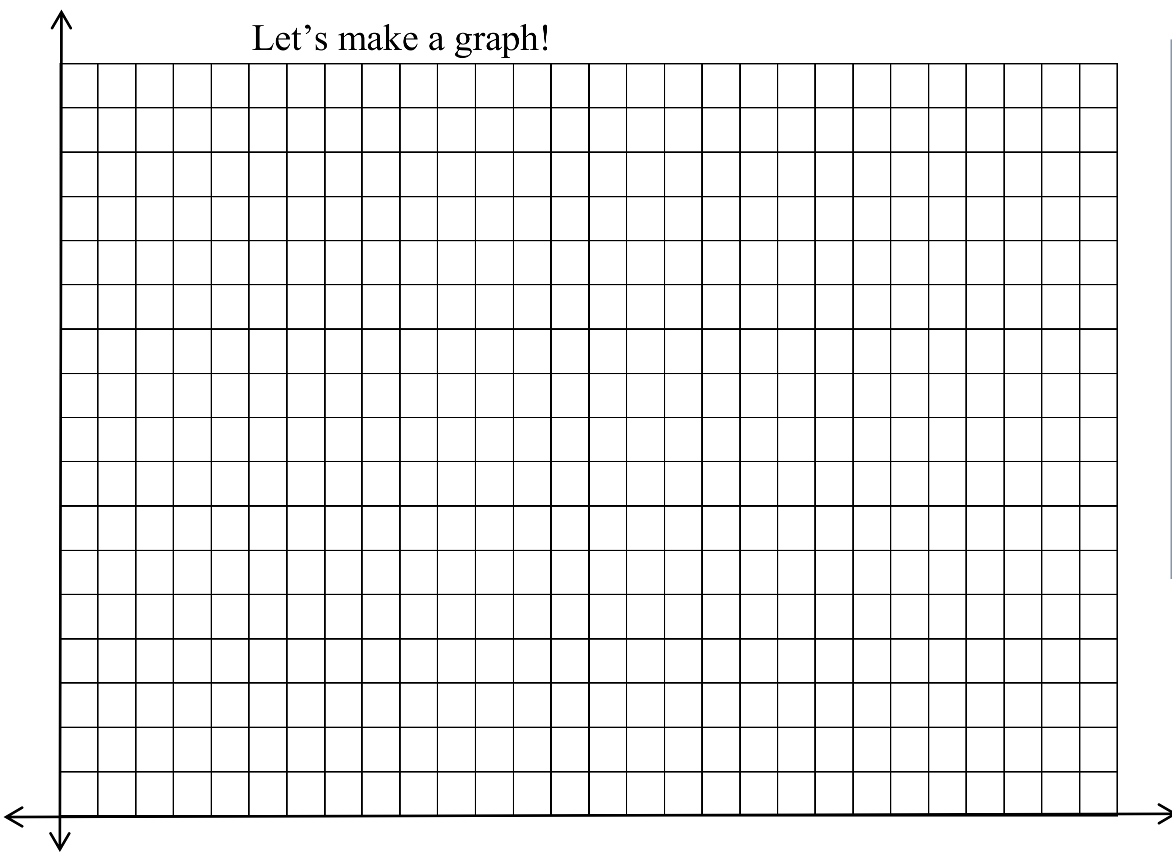
What part is staying the same (constant)? ____

What part is changing (multiplier)? ____

Let's make a chart!

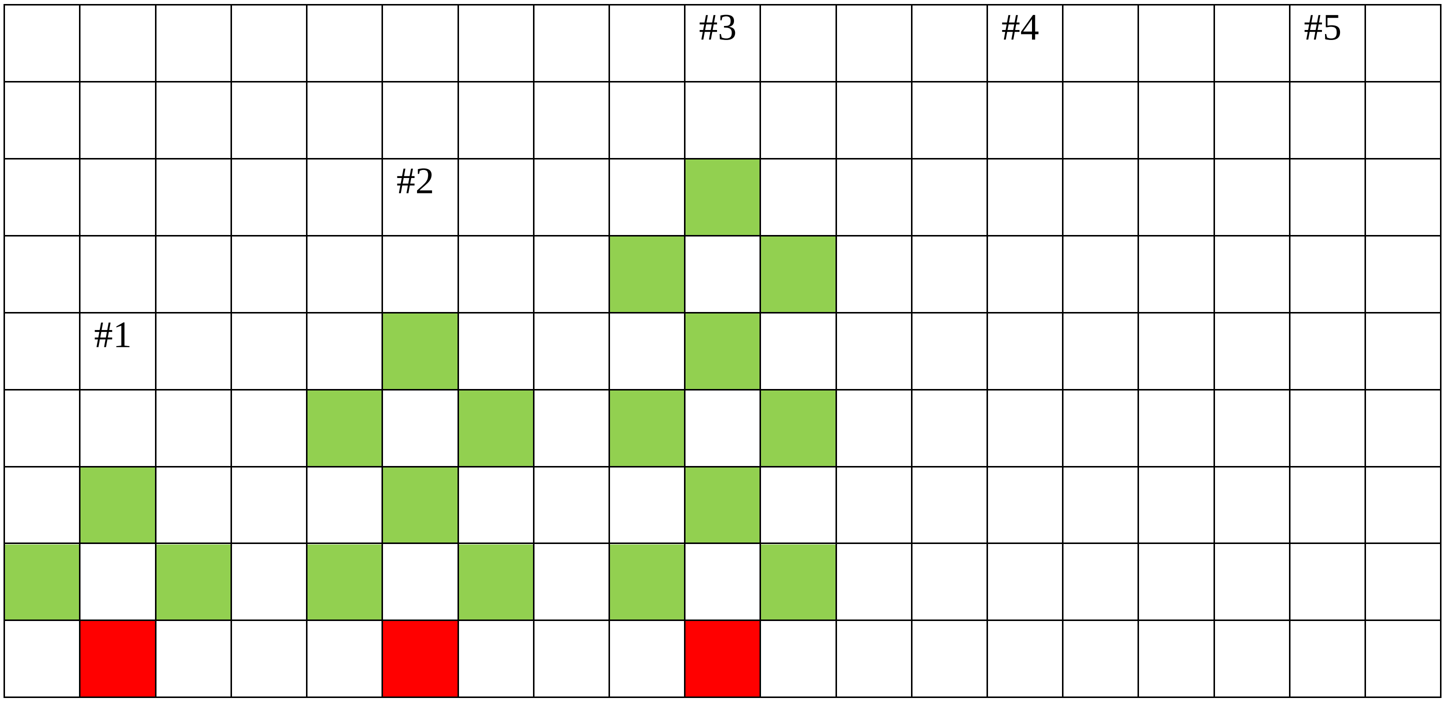
Figure #	Figure 1	Figure 2	Figure 3	Figure 4	Figure 5	Figure 6
Number of toothpicks						
Figure #	Figure 7	Figure 8	Figure 9	Figure 10	Figure 11	Figure 12
Number of toothpicks						

Let's make a graph!



Describe the pattern in words…

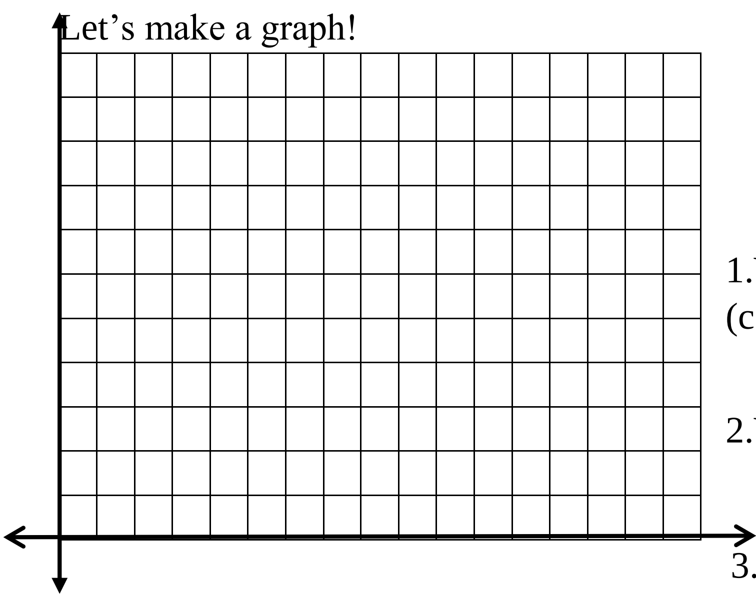
Task #5:



Let's make a chart!

Figure #	Figure 1	Figure 2	Figure 3	Figure 4	Figure 5	Figure 6
Number of squares						

Let's make a graph!



Describe the pattern in words…

1.What part of the pattern is staying the same (constant?)

2.What part of the pattern is changing (multiplier)?

3.What would position 30 in the pattern look like?

4. If you had 40 tiles, could you build position 12 in the pattern? Why or why not?

5. If you had 100 tiles, which position in the pattern could you build?

Assignment: Lesson 3 **P9.1**

1. Here is a pattern of squares, each square has a side length of 1 cm. The pattern continues.

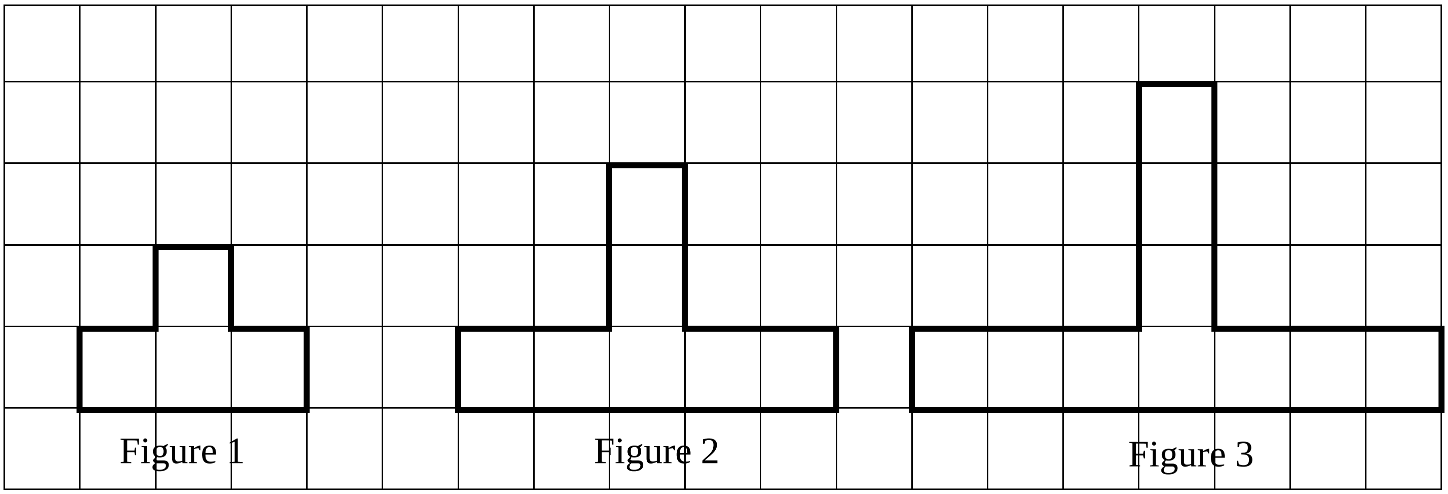
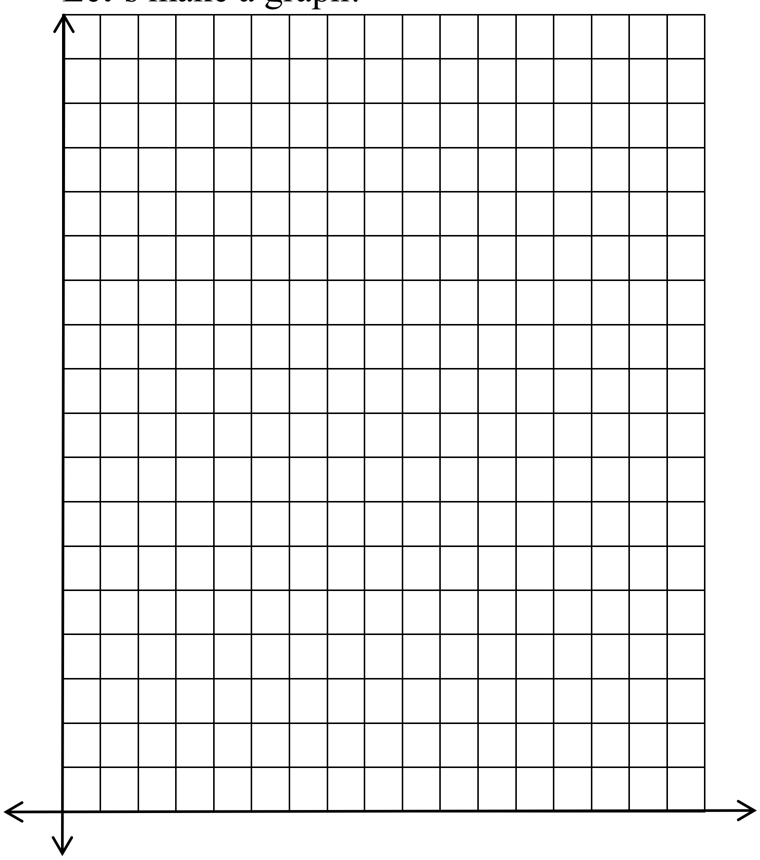


Figure #	Figure 1	Figure 2	Figure 3	Figure 4	Figure 5	Figure 6	Figure 7	Figure 8
Perimeter								

Let's make a graph!



Describe the pattern in words…

What stayed the same in the pattern?

What changed in the pattern?

2. Assuming that the pattern continues. Answer the following questions.

Figure 1 Figure 2 Figure 3

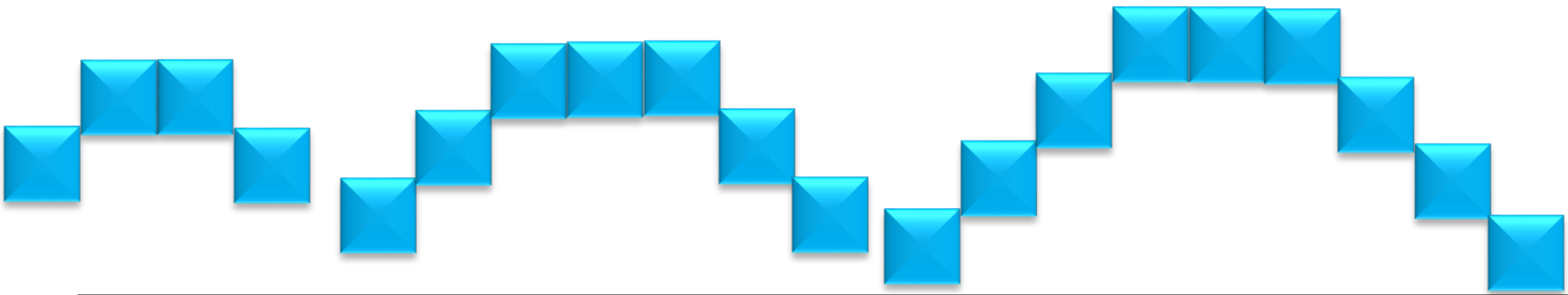
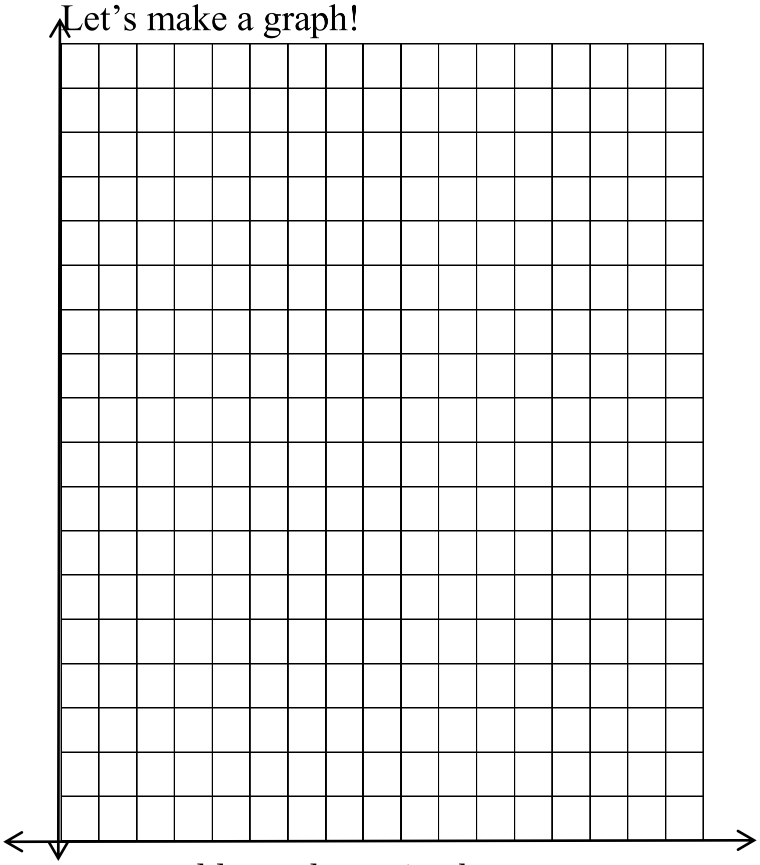


Figure #	Figure 0	Figure 1	Figure 2	Figure 3	Figure 4	Figure 5	Figure 6	Figure 7
Number of Tiles								

Let's make a graph!



How could you determine how many squares are in figure 2133?

What figure number has 645 squares?

Describe the pattern in words…

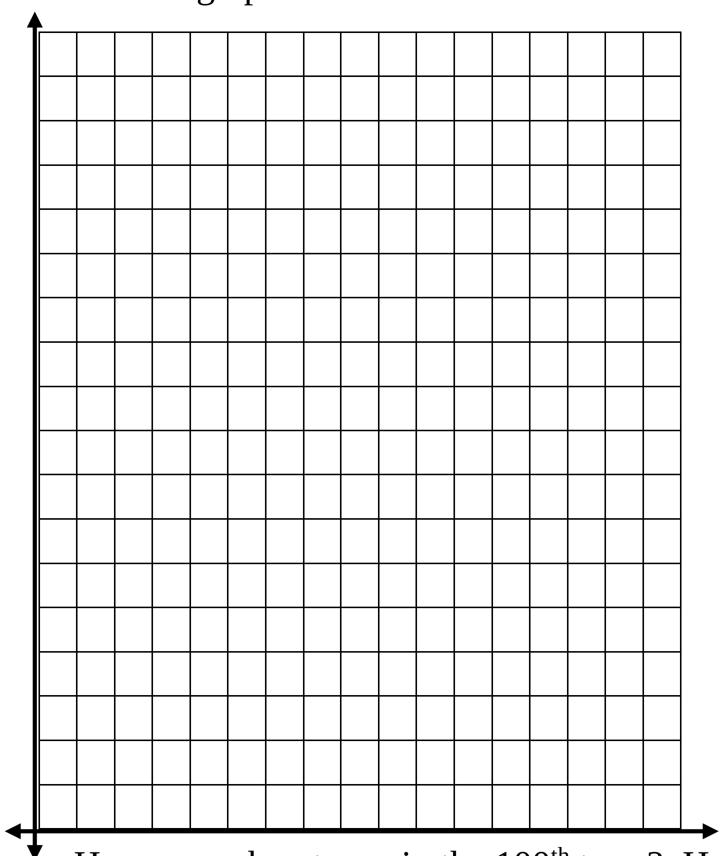
What stayed the same in the pattern?

What changed in the pattern?

3. Consider this pattern

a. Organize the pattern into a table of values

b. Make a graph



c. How many hearts are in the 100th term? How did you find your answer?

Describe the pattern in words…

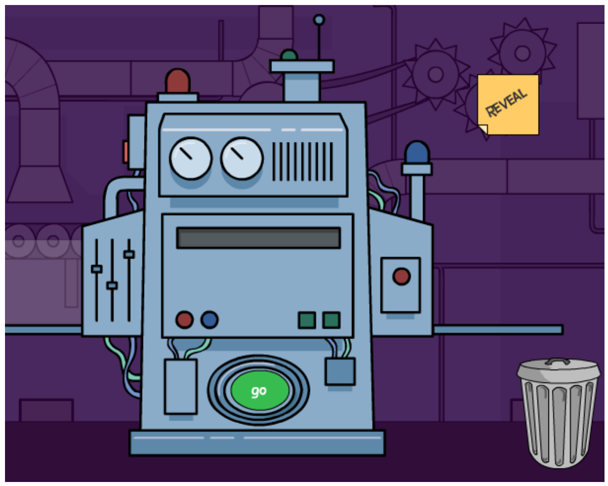
What stayed the same in the pattern?

What changed in the pattern?

d. What term has 3199 hearts? How did you find your answer?

Introducing the Number Transformer

In mathematics there exists a number transformer that can change one number into another. Often in mathematics the transformer code is hidden and must be found. In this unit, your most important job is to find that code. In order for it to be the right code, it must work for all the numbers in the question.



Task #6 Number Transformer: This table represents clues. The original number is called the input (what is put into the number transformer) and the resulting number is called the output (what comes out of the number transformer).

Challenge: Work with your group to find out what operations are being used on the top numbers to create the bottom numbers (it has to be the same for all of the numbers). You are allowed to use a combination of operations.

Input x	0	1	2	3	4
Output y		3		7	

a. If the number 15 was the input number, what would be the output?

b. Describe the pattern between the input and output values in words.

c. Describe the pattern between the input and output values using numbers and operations. What operations to we apply to x, in order to get y?

d. Write a general rule or equation to find the output (y) given any input (x).

$y =$______________

Task #7 Number Transformer: This table represents clues. The original number is called the input and the resulting number is called the output.

Challenge: Work with your group to find out what operations are being used on the top numbers to create the bottom numbers (it has to be the same for all of the numbers). You are allowed to try a combination of operations.

Input x	-3	-2	-1	0	1
Output y		-11			4

a. If the number 10 was the input number, what would be the output?

b. Describe the pattern between the input and output values in words.

c. Describe the pattern between the input and output values using numbers and operations.

d. Write a general rule or equation to find the output (y) given any input (x).

e. If I wanted to output the number 15, what number would I have to input?

The Importance of Finding a Mathematical Equation & Key Terms

Task #1: Let's look back at our shaded square. Will 256 squares work as a shaded square border? What would be the dimensions of that square? Get your poster and add this question in. Work as a team!

Insert your work here for your own personal notes:

Often, we are asked to find information about the future based on current data. In order to do that, we need to unlock the mathematical code from the number transformer. Let's find a way to do that!

Code #1:

Input x	1	2	3	4	5	6
Output y	4	6	8	10	12	14

Code #2:

Figure f	1	2	3	4	5	6
Perimeter P	4	6	8	10	12	14

Code #3

Input x	-2	-1	0	1	2	3
Output y	5	2	-1	-4	-7	-10

Code #4

Input x	-2	-1	0	1	2	3
Output y	-6	-2	2	6	10	14

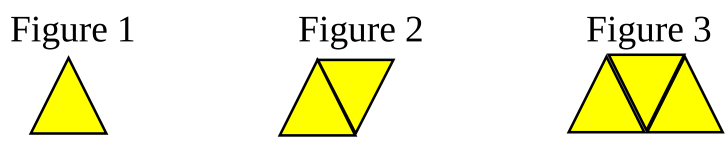
How do we crack the code?

Key Terms:
Before we move through another lesson, it is important to define some of the terms that we will be using. Let's do it in the context of a question.

Task #8 Find the Perimeter of a Triangle Train

Things to know:

- Each side length is 1 cm
- Interior lines don't count as part of a perimeter

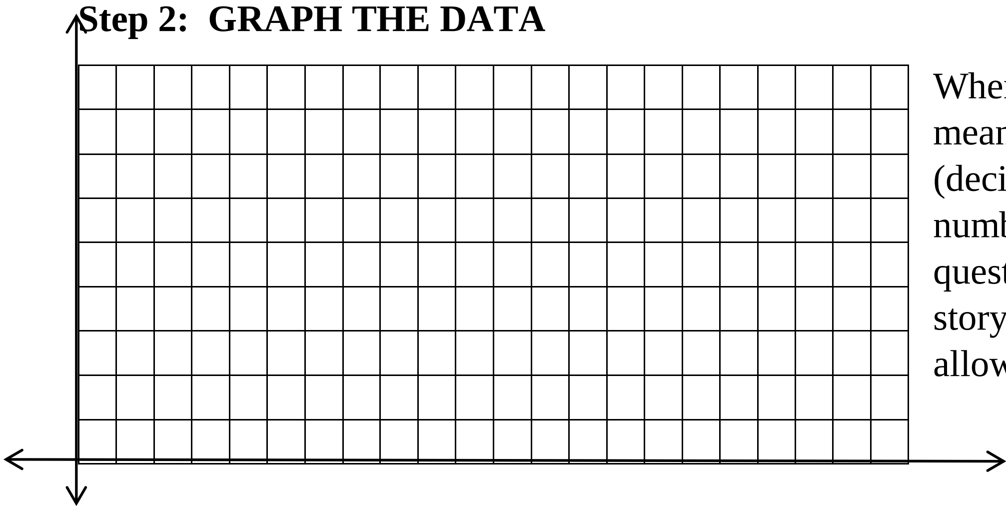


Step 1: TABLE OF VALUES

Input	**Figure #**					
Output	**Perimeter**					

* In a table of values we always put the input number on the top, and the output on the bottom.
* The input represents the dependent variable (goes on the vertical axis of the graph)
* The output represents the independent variable (goes on the horizontal axis of the graph)

Step 2: GRAPH THE DATA



When we join the dots with a line, that means that the numbers in between (decimals and fractions) the whole numbers are allowed to be part of the question. You need to think about the story and if decimals and fractions are allowed.

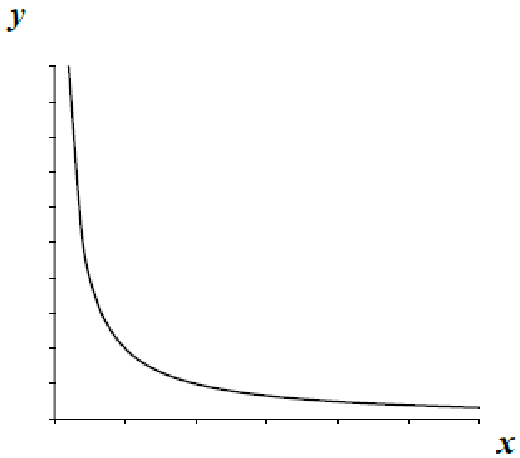
STEP 3: DESCRIBE THE PATTERN IN WORDS

STEP 4: WRITE THE GENERAL EQUATION

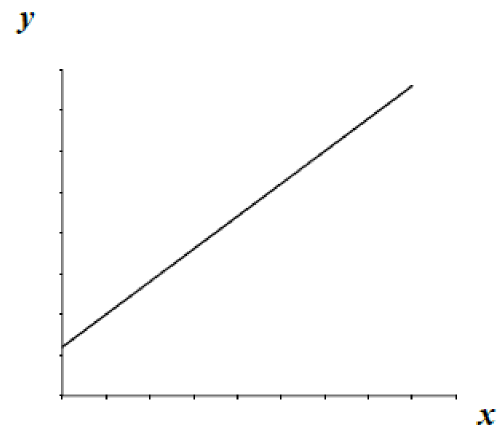
Use the equation to find out what figure will have a perimeter of 522?

Which of the following graphs ARE linear?

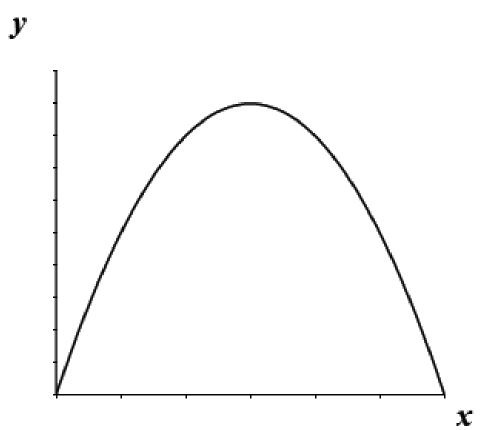
a.



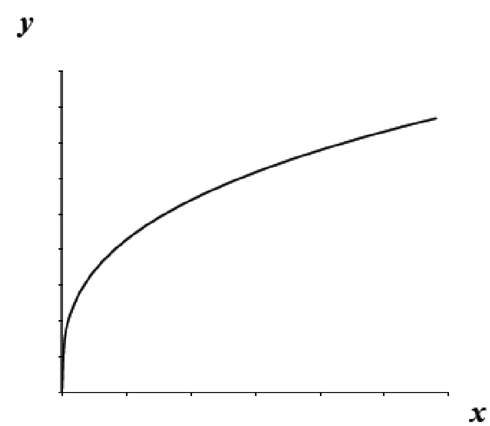
b.



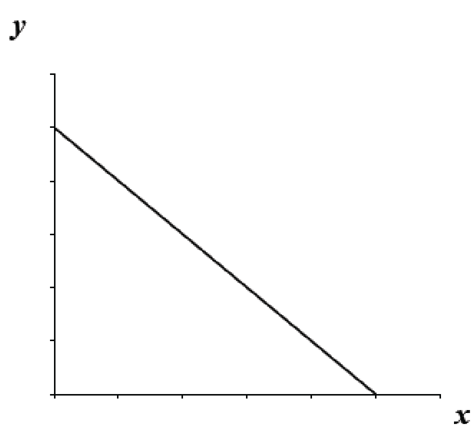
c.



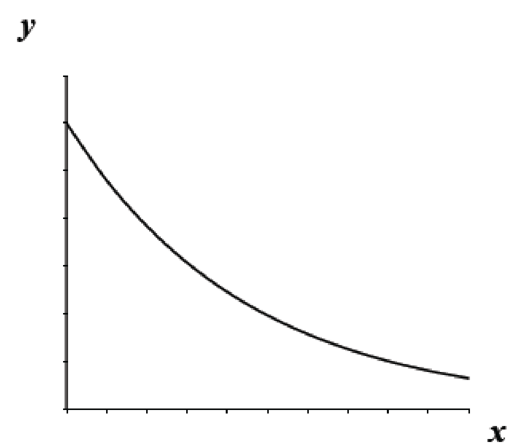
d.



e.



f.



Have you noticed, that all of the graphs we have drawn have gone up by a constant amount? The graphs have always been a straight line. When this happens, it is called a

LINEAR RELATION

Characteristics of a linear **graph**…

With a yes or no, state whether the following equations are linear (yes) or not (no).

a. $2x + y = -3$	b. $x + y = x^2 - 3$
c. $x = 3$	d. $2x^2 + 3x + 2 = 0$
e. $y^3 + 2y = -2$	f. $4x = 2y + 3$
g. $y = -10$	h. $x^2 = 3$
i. $y = 3x + 4$	j. $2x + y = 5 - 2y + 3x$
k. $6y = 2y^2 - 10$	l. $12x = 0$
Characteristics of a linear **equations**…	

With a yes or no, state whether or not the following table of values are linear (yes) or not (no).

a.

x	0	1	2	3	4
y	5	8	11	15	18

b.

x	-3	-2	-1	0	1
y	-8	-7	-6	-5	-4

c.

x	10	8	6	4	2
y	12	8	12	8	12

d.

x	-1	0	1	2	3
y	-6	-3	0	3	6

Characteristics of a linear **table of values**…

Now that we have established what a linear relation, linear table of values, and linear equation looks like, let's get to work!

Example 1: Complete the table of values for each function

a. $y = 3x + 8$

x	y
9	
-6	
3	
-1	
-2	

b. $y = -2x - 1$

x	y
-3	
-1	
0	
4	
10	

c. $y = \frac{1}{2}x - 3$

x	y

d. $y = -\frac{2}{3}x + 1$

x	y

Group Task #9

1. Create your own linear equation. All group members must agree that it is linear. Bonus marks are given for being creative (don't just use easy positive numbers).

2. Using your linear equation, create a table of values.

x						
y						

Show your work here:

3. Graph your linear relation.

Group Task #9

1. Linear Equation

2. Table of values.

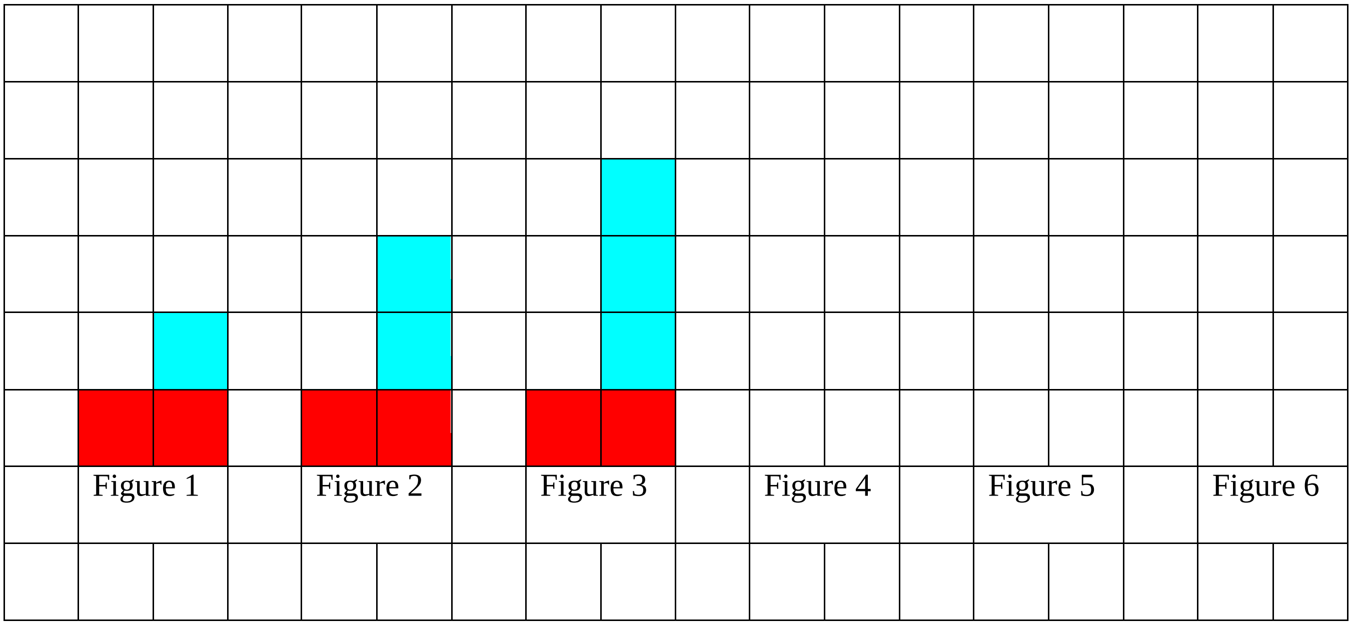
x						
y						

Show your work here:

3. Graph the linear relation.

Assignment: Lesson 5 **P9.1**

1. Using the following figures



a. Draw in the next 3 figures.

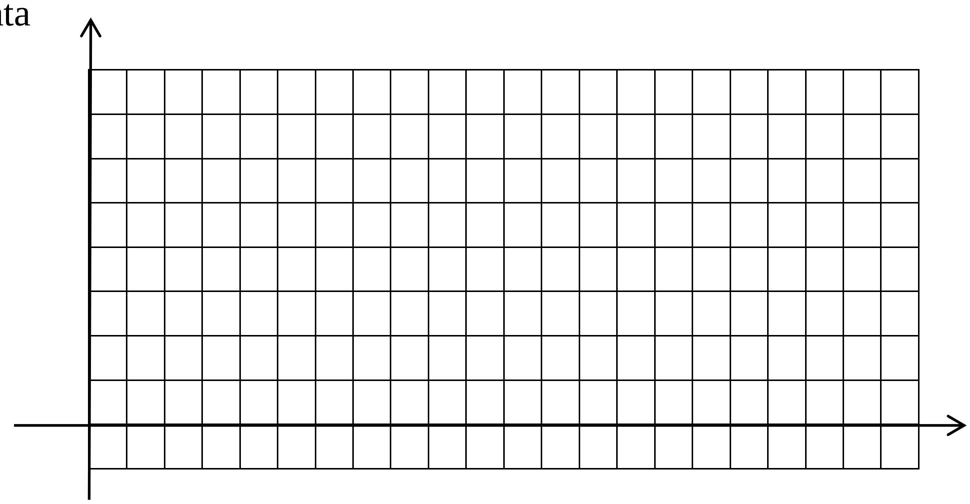
b. Create a table of values using the data from above.

c. Describe how the pattern is working using words.

d. Crack the code.

e. How many squares will be in figure 132?

f. Graph the data



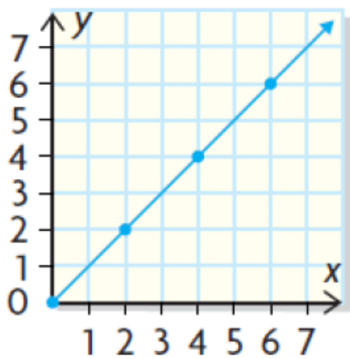
g. Is this graph linear? How can you tell?

2. State whether the given functions are linear or nonlinear:

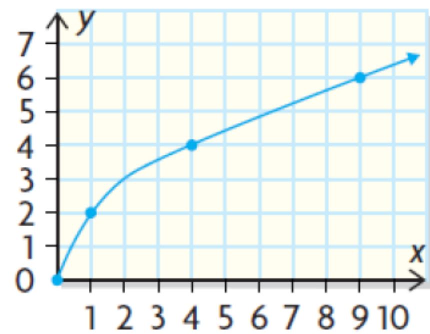
Function	Linear/Nonlinear
a. $y = -6x + 8$	
b. $y = -2x^2 + 1$	
c. $y = 2 + 4x$	
d. $y = 5$	
e. $y = x^3 + 4x$	
f. $2x = 4y + 3$	
g. $y = 2x^2 - 3x + 1$	

3. Identify each relation as linear or nonlinear. Explain how you know.

a.



b.



4. Determine if the table of values represent a linear function or non-linear function

a.

x	y
-2	-5
0	1
4	13
10	31

b.

x	y
-1	-3
0	2
1	6
2	12

5. Crack the code.

a.

r	0	1	2	3	4
p	-3	2	7	12	17

b.

q	z
0	-5
1	-4
2	-3
3	-2
4	-1

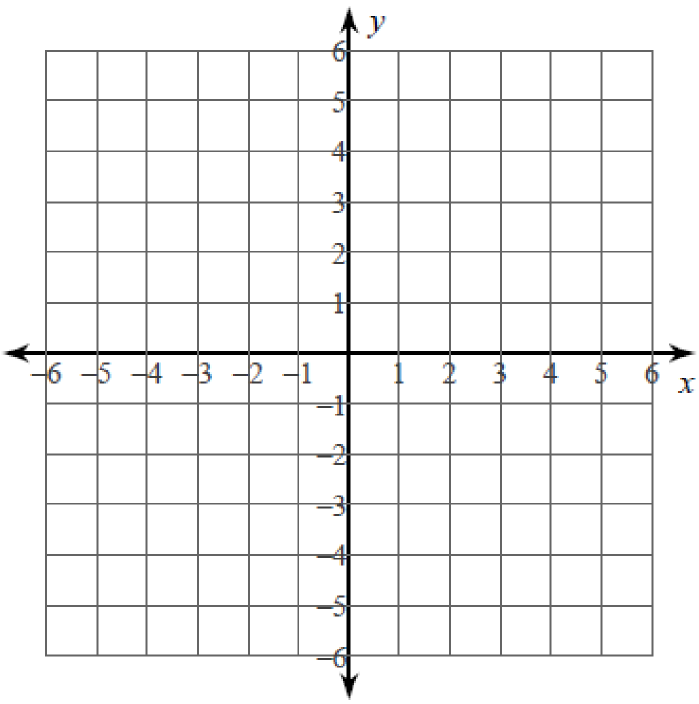
c.

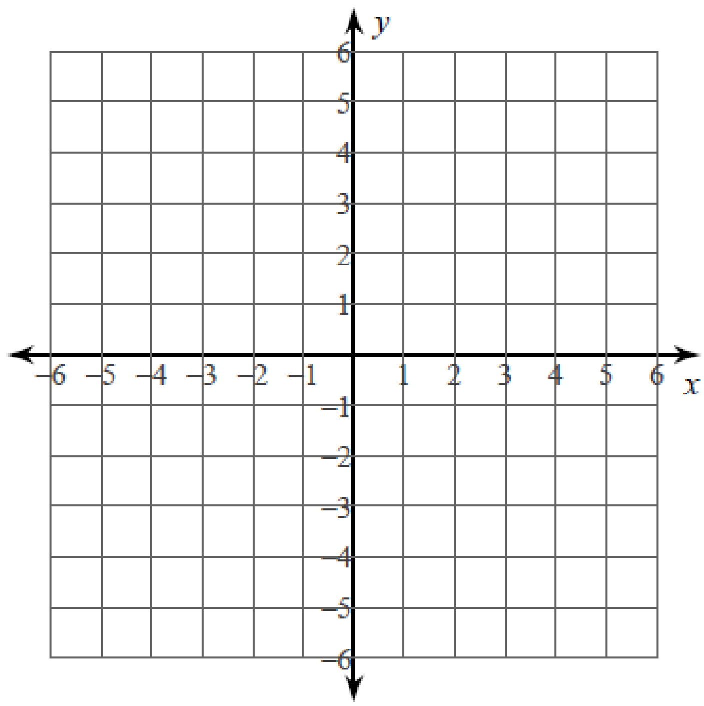
x	0	1	2	3	4
y	-11	-4	3	10	17

6. Create a table of values for each linear relation, then graph the relation.

a. $y = -x + 3$

b. $y = 2x - 3$





Graphing Linear Relations

A. From a Table of Values

You are given * **the table of values**

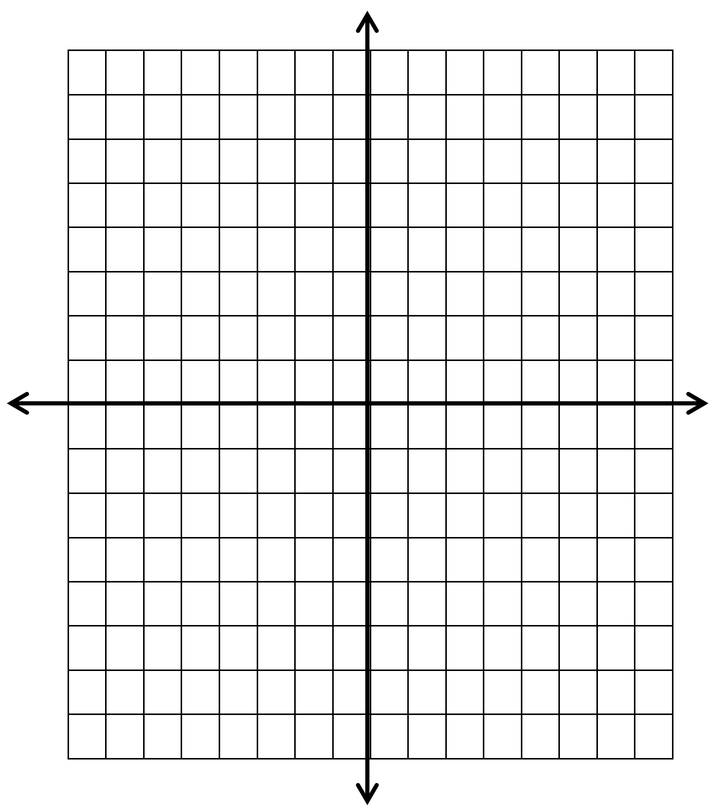
You need to find * **the equation**

* **the graph**

Example 1: Given the following table of values,

a) Graph the linear relation
b) Find the code (equation) that represents the table of values
c) Use the graph to estimate the value of y if $x = -8$
d) Use the equation to verify the value of y if $x = -8$
e) Use the graph to estimate the value of x if $y = -7$
f) Use the equation to verify the value of x if $y = -7$

x	y
-3	9
-2	7
-1	5
0	3
1	1
2	-1
3	-3



B. From an Equation

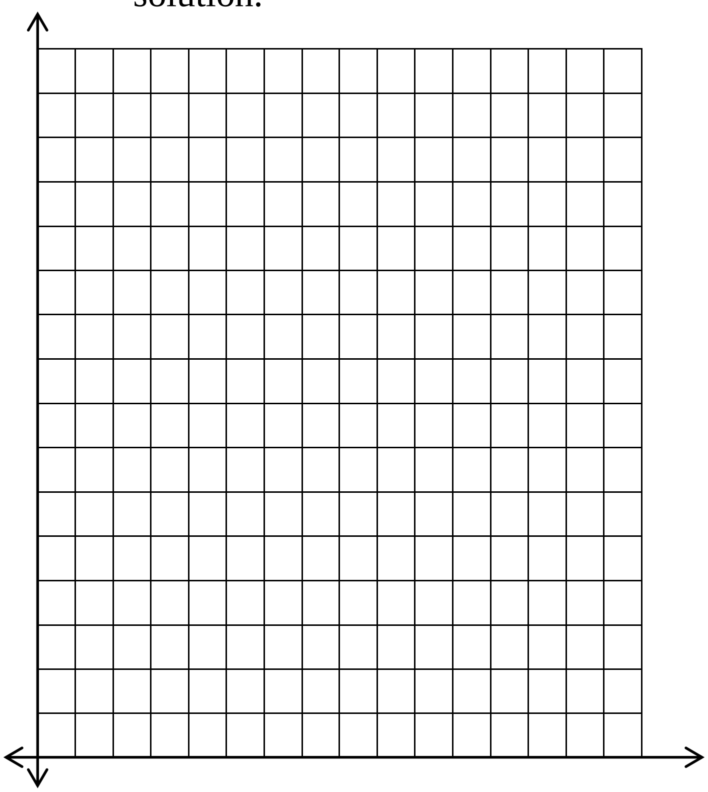
You are given * **the equation**

You need to find * **the table of values**

* **the graph**

Example 2: The world's largest cruise ship, *Freedom of the Seas*, uses fuel at a rate of $12800\ kg/h$. The fuel consumption, f, in kilograms, can be modelled using the equation $f = 12800t$, where t is the number of hours travelled.

a) Create a graph to represent the linear relation for the first $7h$.
b) Approximately how much fuel is used in $11\ h$? Verify your solution.
c) How long can the ship travel if it has approximately $122000\ kg$ of fuel? Verify your solution.



C. From a Story

You are given * **a story about a problem**

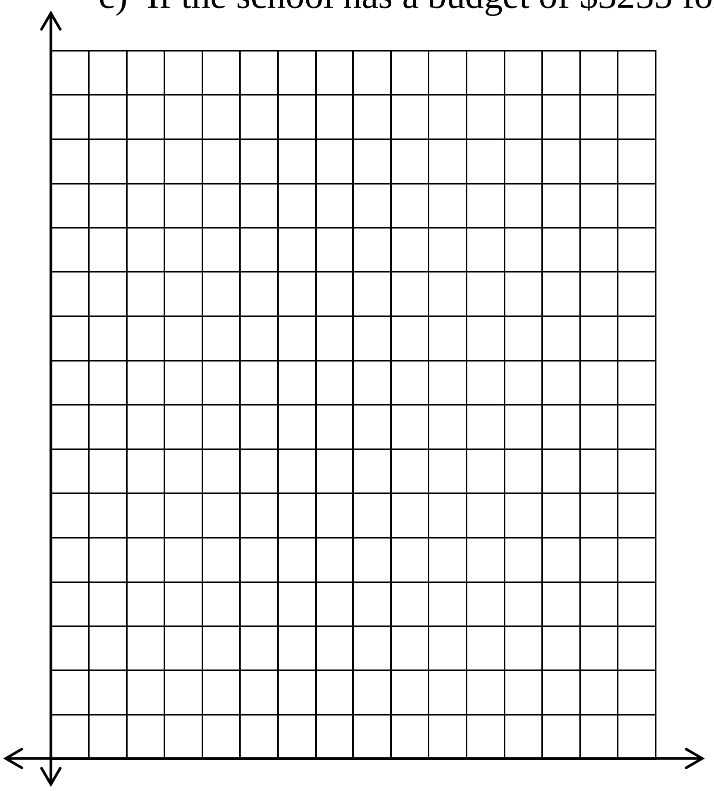
You need to find * **the equation**

* **the table of values** In this order!

* **the graph**

Example 3: A school pays a company $220 to design a gym T-shirt. It costs an additional $15 to make each T-shirt.

a) Develop an equation to determine the cost of the T-shirts. Make sure you state what your variables represent.
b) What would it cost to make 253 T-shirts for the grade 9's only?
c) If the school has a budget of $3255 for T-shirts, how many T-shirts can be ordered?



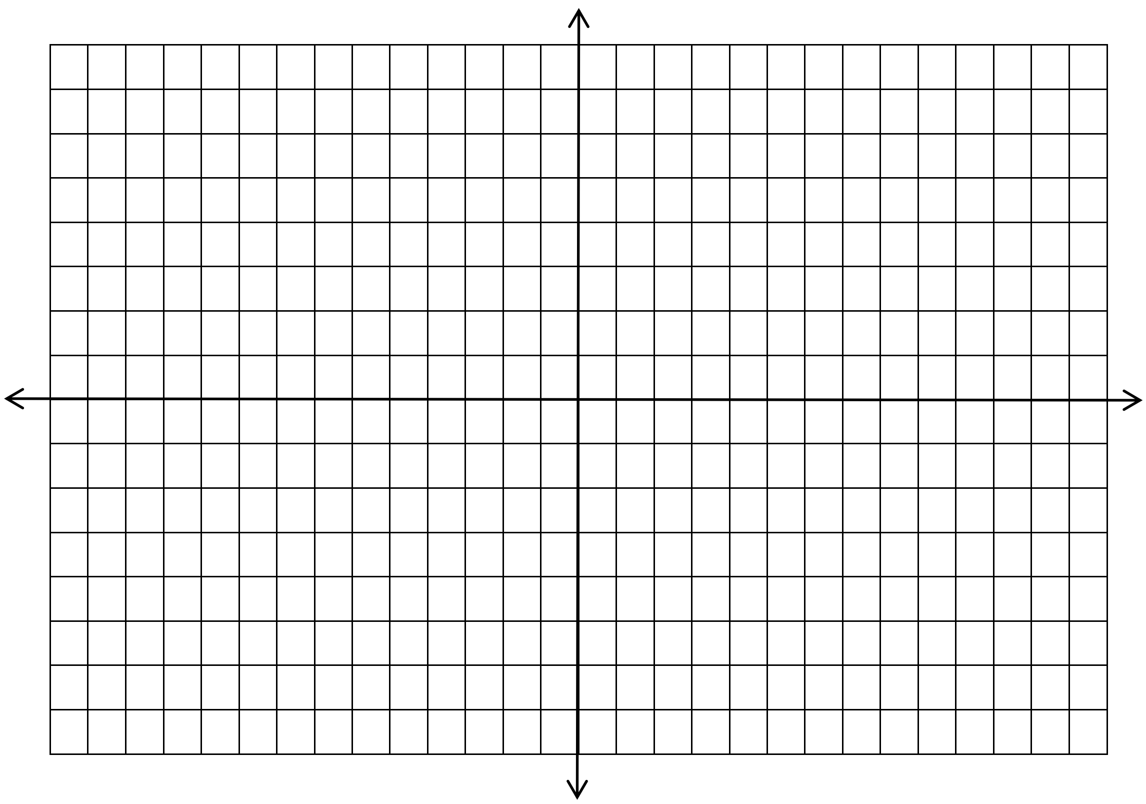
Graphing $y = a$, $x = a$, and $ax + by = c$

A. Graping $y = a$ and $x = a$

So far we have only tackled linear relations of the form $y = ax + b$, where a and b are rational numbers (but not 0).

Example 1: Graph the following

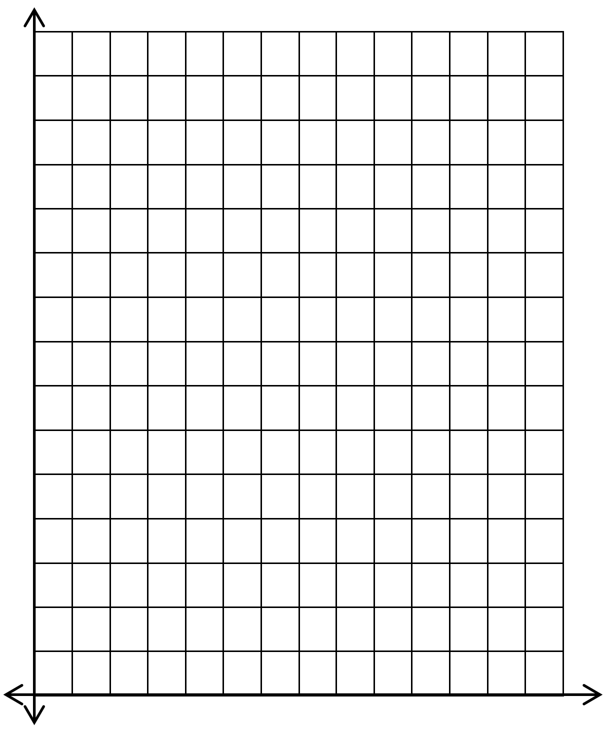
$y = -2x - 3$ $\quad$ $P = 3f - 2$ $\quad$ $s = 2f + 1$



All of these equations resulted in a ____________________________ graph

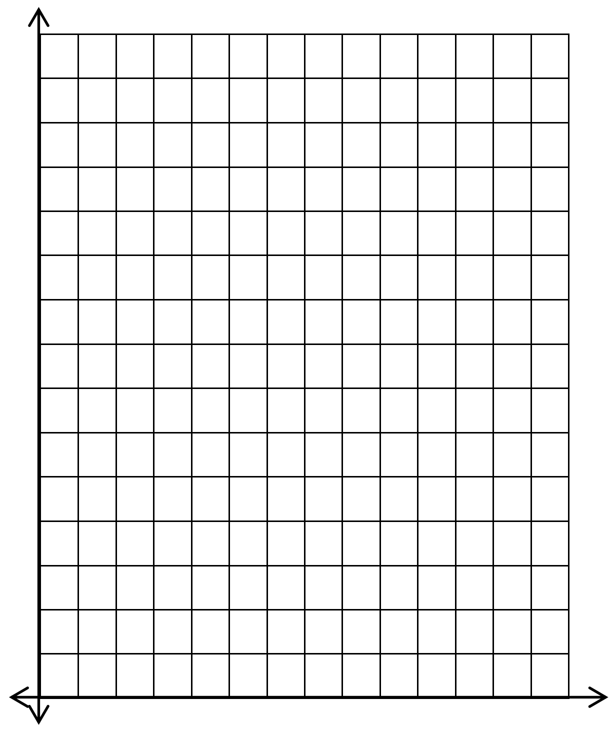
Task #10: Graph the following table of values

Time, x (s)	Distance, y (m)
0	8
20	8
40	8
60	8
80	8
100	8



Describe a situation that the graph might represent.

Distance, x (m)	Height, y (m)
4	1
4	2
4	3
4	4
4	5
4	6



Describe a situation that the graph might represent.

What is different about the above table of values and graphs from the ones we have been working with so far?

Horizontal and Vertical Lines

VERTICAL LINE $x = a$	HORIZONTAL LINE $y = a$

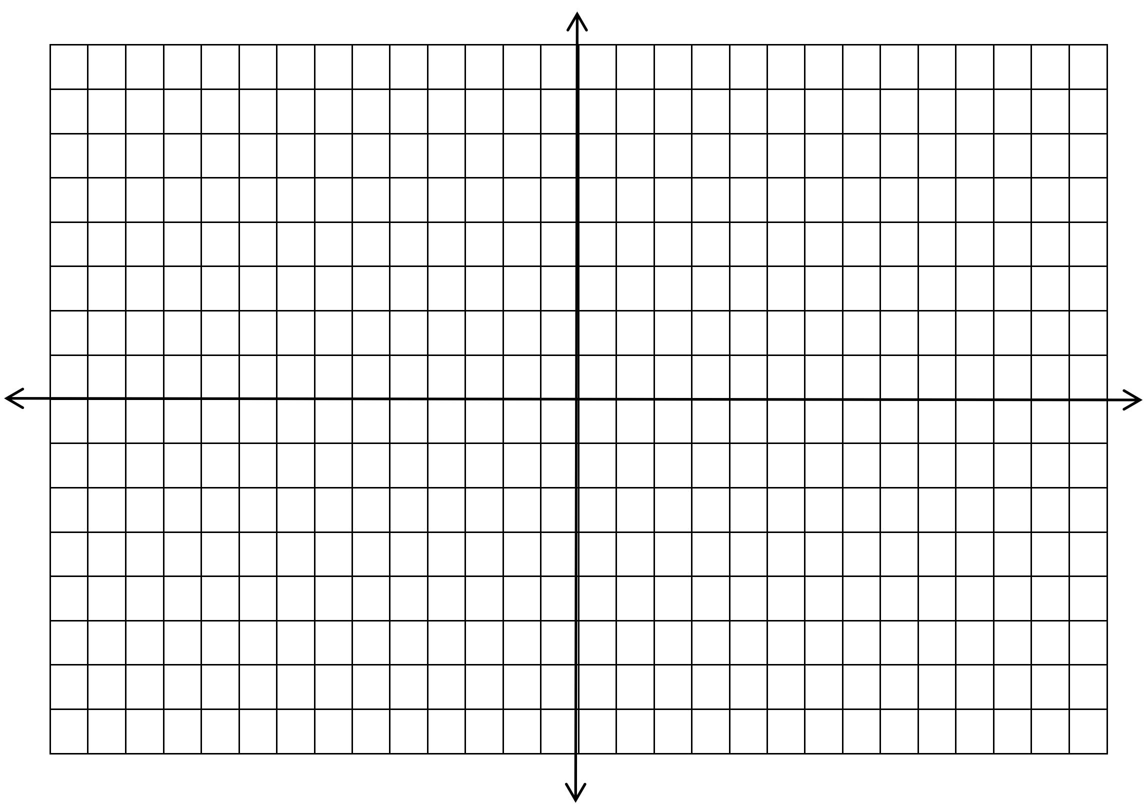
Example 2: Graphing and Describing Horizontal and Vertical Lines

For each equation below:

i) Graph the equation

ii) Describe the graph.

a. $x = -2$ b. $y + 4 = 0$ c. $2x = 7$



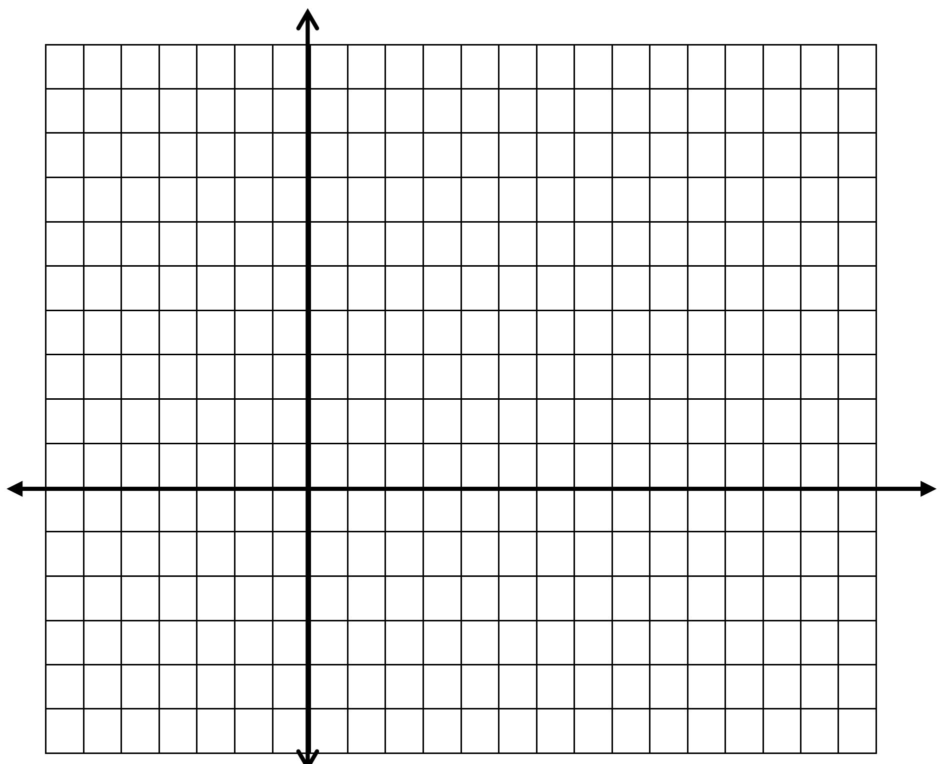
B. Graphing $ax + by = c$ (Diagonal Lines)

Anytime a linear equation has an x and a y, it will result in a diagonal line graph. All of the diagonal equations so far have been of the type $y = ax + b$ where the y has been isolated on one side, and the x on the other. Now we are going to work with an equation where all the pieces are jumbled up.

What to do? ____

Examples:

Example 3: Graphing and Equation in the Form
For the equation $5x - 2y = -10$
a) Make a table of values for $x = -2, 0, and\ 2$.
b) Graph the equation



Example: State whether the following equations result in a diagonal (oblique) line, vertical line, or a horizontal line.

Equation	$y = 3x + 2$	$x = -8$	$3y + 2x + 2 = 0$	$4 = y$
Horizontal/Vertical/ Diagonal				
Equation	$3x + 7y = -2$	$y = -2x$	$x = 10$	$2y = 2x + 5$
Horizontal/Vertical/ Diagonal				

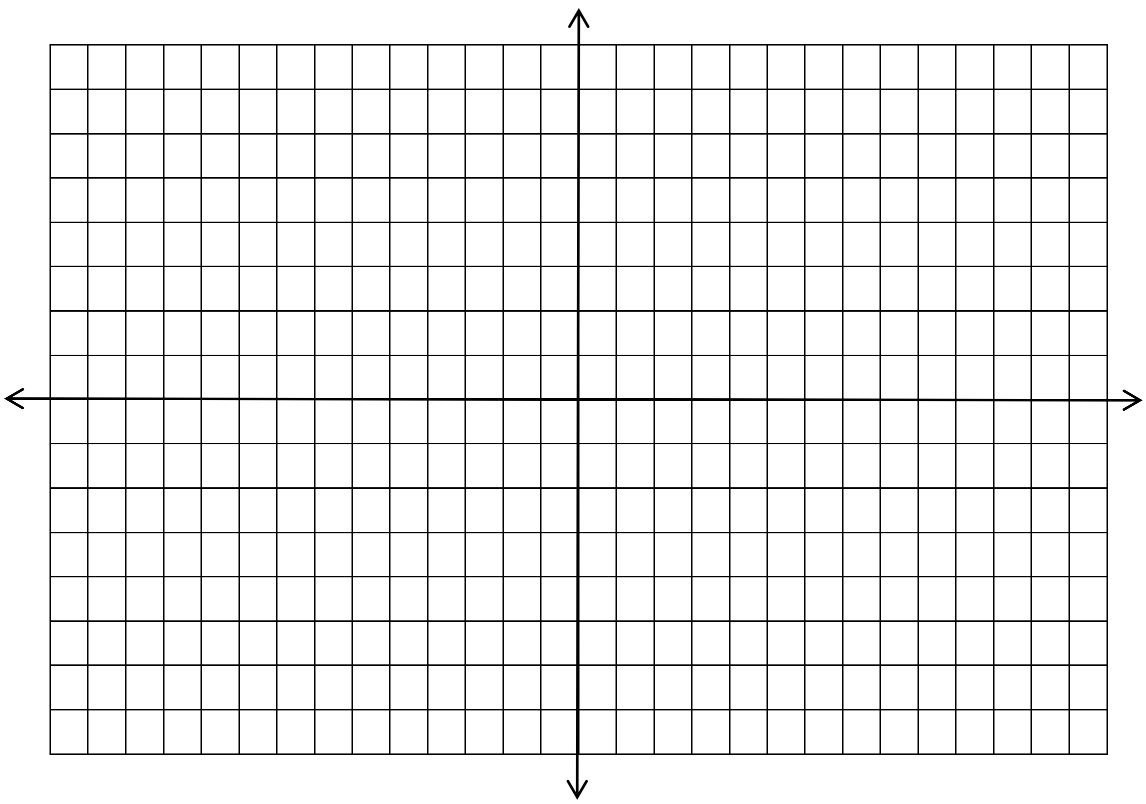
Graphing $y = a$, $x = a$, and $ax + by = c$

A. Graping $y = a$ and $x = a$

So far we have only tackled linear relations of the form $y = ax + b$, where a and b are rational numbers (but not 0).

Example 1: Graph the following

$y = -2x - 3$ $P = 3f - 2$ $s = 2f + 1$

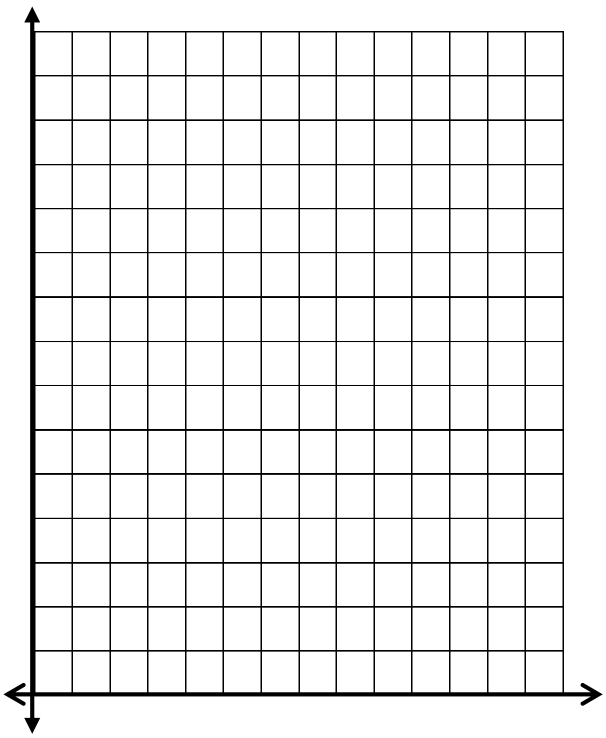


All of these equations resulted in a ______________________________ graph

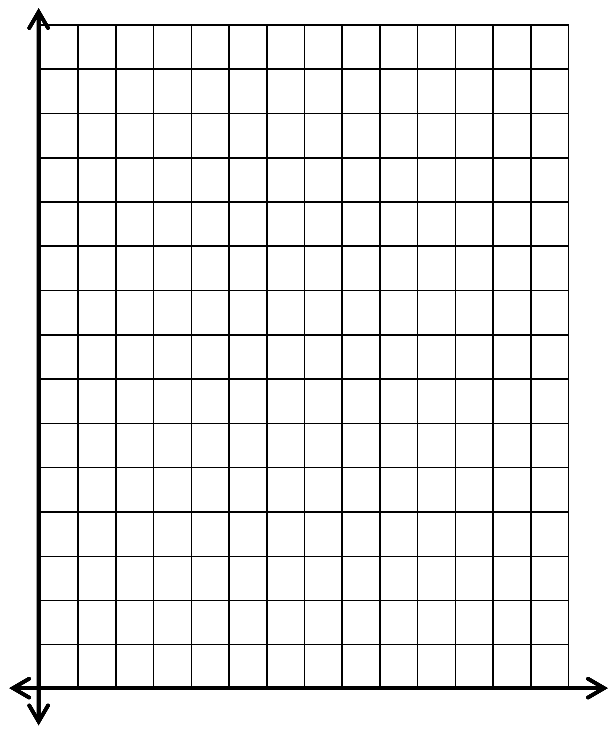
Task #10: Graph the following table of values

Time, x (s)	Distance, y (m)
0	8
20	8
40	8
60	8
80	8
100	8

Distance, x (m)	Height, y (m)
4	1
4	2
4	3
4	4
4	5
4	6



Describe a situation that the graph might represent.



Describe a situation that the graph might represent.

What is different about the above table of values and graphs from the ones we have been working with so far?

Horizontal and Vertical Lines

VERTICAL LINE $x = a$	HORIZONTAL LINE $y = a$

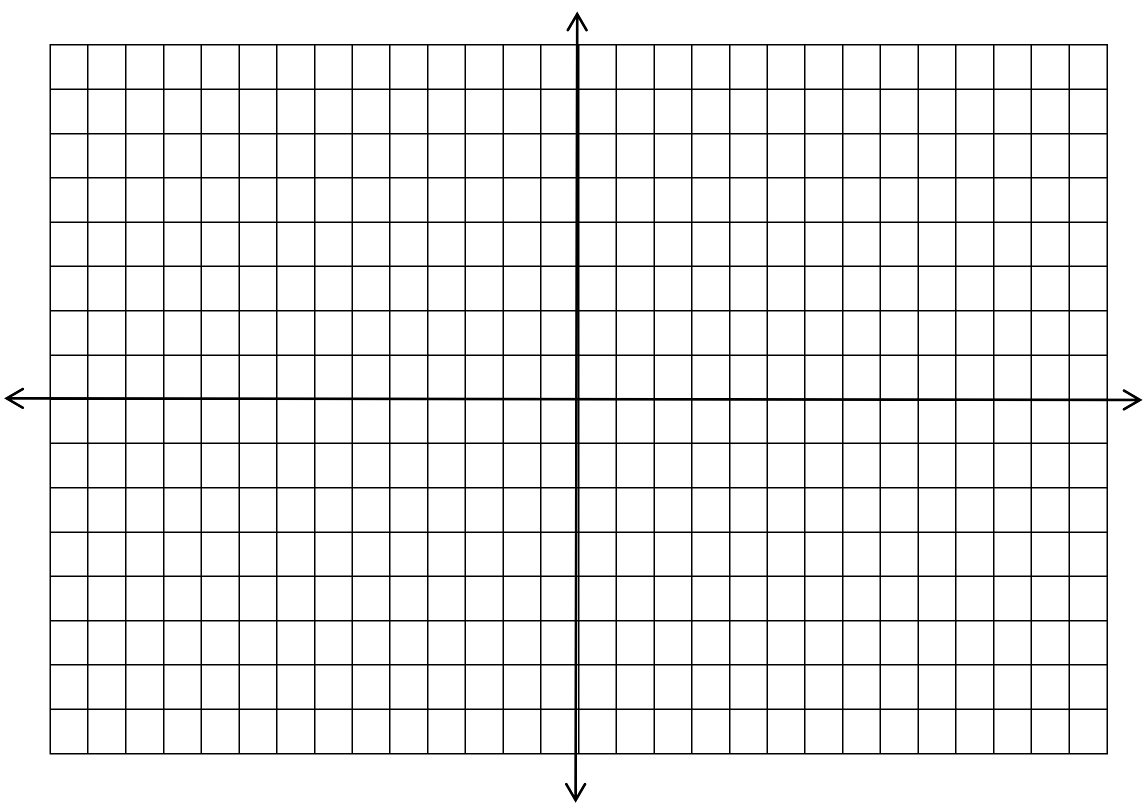
Example 2: Graphing and Describing Horizontal and Vertical Lines

For each equation below:

i) Graph the equation

ii) Describe the graph.

a. $x = -2$ b. $y + 4 = 0$ c. $2x = 7$



B. Graphing $ax + by = c$ (Diagonal Lines)

Anytime a linear equation has an x and a y, it will result in a diagonal line graph. All of the diagonal equations so far have been of the type $y = ax + b$ where the y has been isolated on one side, and the x on the other. Now we are going to work with an equation where all the pieces are jumbled up.

What to do? ____

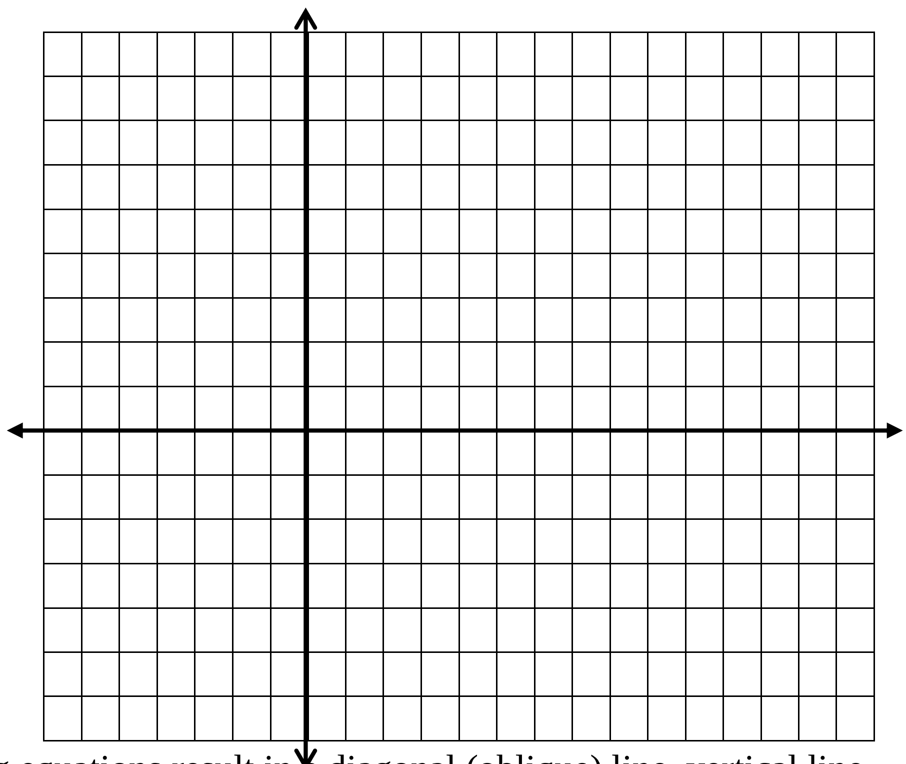
Examples:

Example 3: Graphing and Equation in the Form

For the equation $5x - 2y = -10$

a) Make a table of values for $x = -2, 0, and\ 2$.

b) Graph the equation



Example: State whether the following equations result in a diagonal (oblique) line, vertical line, or a horizontal line.

Equation	$y = 3x + 2$	$x = -8$	$3y + 2x + 2 = 0$	$4 = y$
Horizontal/Vertical/ Diagonal				
Equation	$3x + 7y = -2$	$y = -2x$	$x = 10$	$2y = 2x + 5$
Horizontal/Vertical/ Diagonal				

Interpreting Graphs

Example 1: The weather balloon recorded the air temperature at different altitudes.

Altitude (m)	350	750	1000	1500	1800
Temperature (℃)	11.4	5.7	2.1	−5.0	−10.0

a. What is the temperature at $700m$?
b. What is the height at -6.5℃?

c. Are we able to get an exact value for temperature at 992.76m? How come? What would we need?

INTERPOLATION:

Example 2: I often like to make predictions on how far I can run at certain paces.

At a speed of 4 minutes per kilometre (1000 metres):

Time (minutes)	2	4	6	8	10
Distance (metres)		1000		2000	

a. Predict how long it will take me to run $10{,}000m$ (10km)

b. Predict how far I will run in 25 minutes

c. What assumptions are we making in this situation?

d. Can we find the equation? If so, what is it?

Extrapolation:

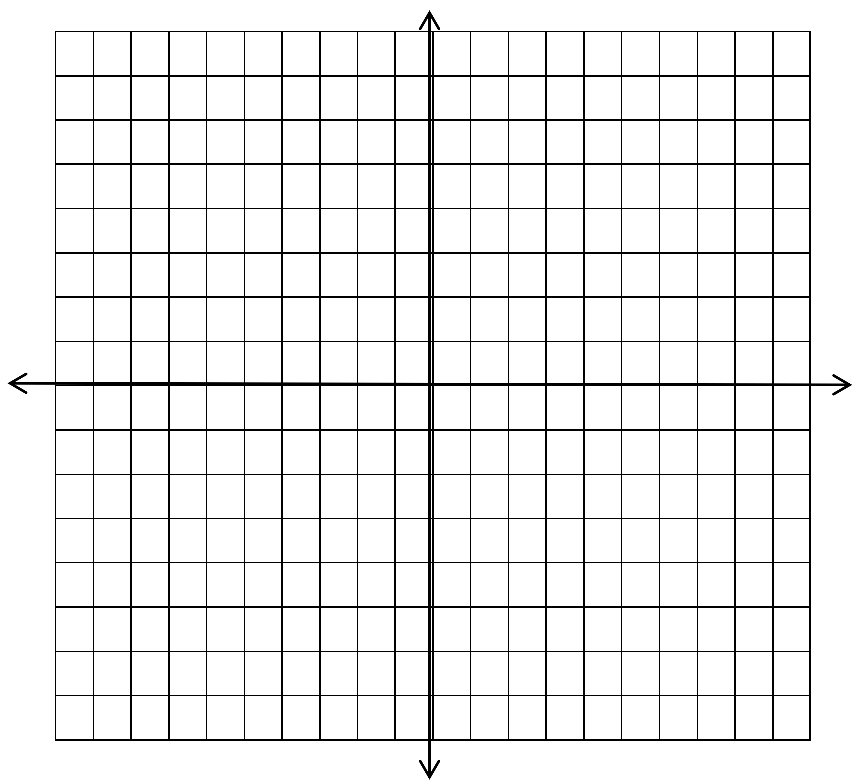
Example 3: Graph $y = 2x - 5$

a. If $x = 9$, what is y?

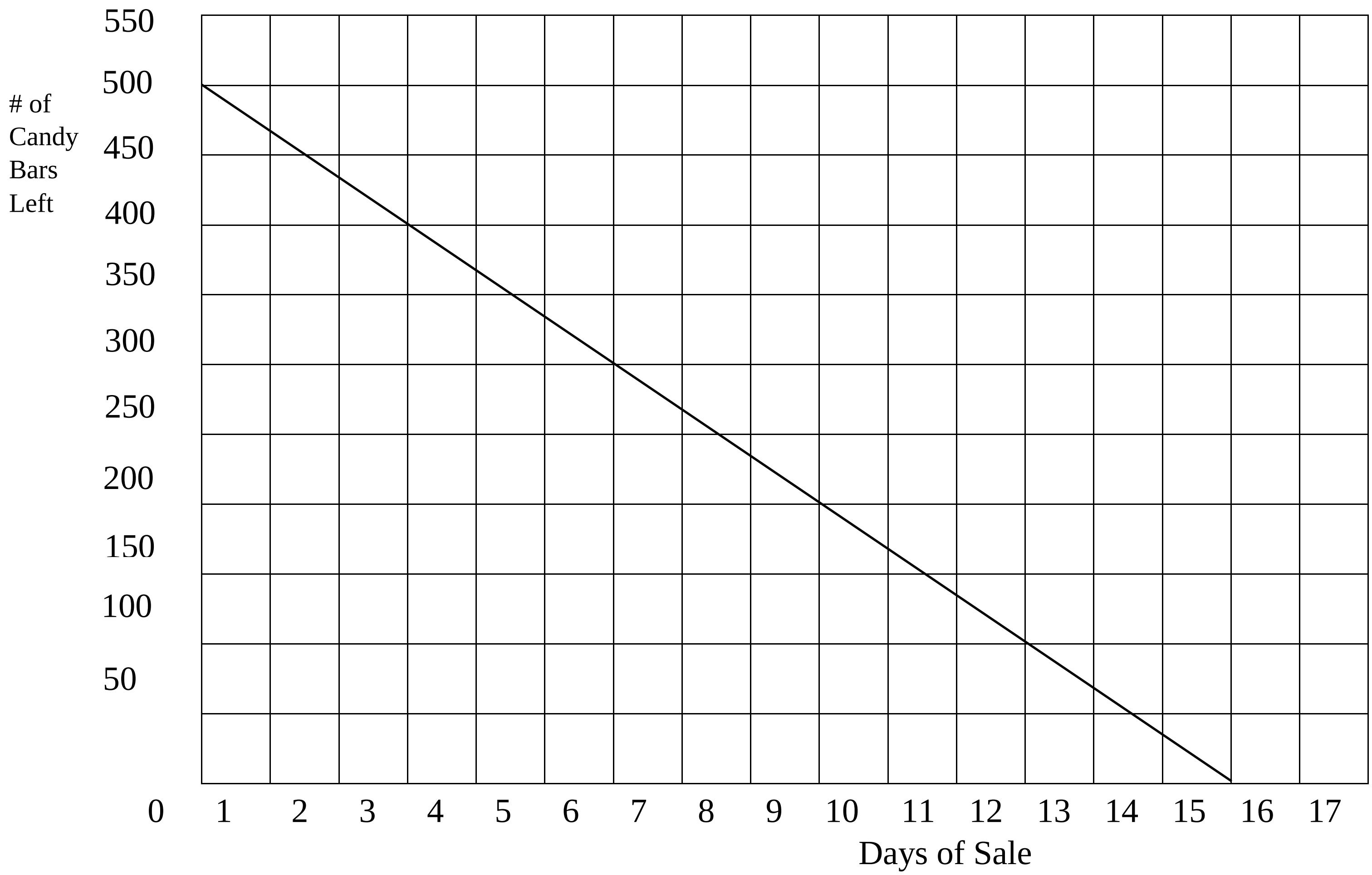
b. If $y = -3$, what is x?

c. If $x = 12$, what is y?

d. If $y = -10$, what is x?



Example 4: Sara is selling candy bars to raise money for a club she belongs to. Here is a graph that displays her efforts in selling the candy bars door to door.



1. Explain why the graph is going down.

2. How many candy bars were there to sell at the beginning?

3. How many days did it take for Sara to sell all of her chocolate bars?

4. Were the candy bars sold at a constant rate?

5. At what day was Sara half-way through her chocolate bars?

6. How many candy bars are left after 3 days?

7. How many days have gone by when there are 100 candy bars left?